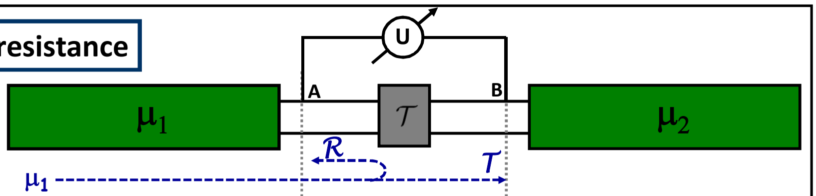
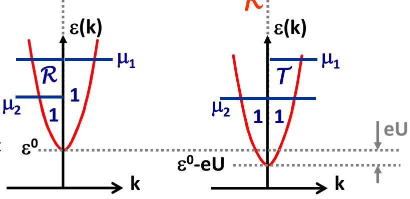
4-probe resistance



A single channel conductor with $\mathcal{T}$ transmission is considered. An $eV=\mu_1-\mu_2$ voltage is applied on the electrodes. At the right side of the scattering region all the states are occupied till μ_2, and at $\mu_2<E<\mu_1$ the +k states are occupied with $\mathcal{T}$ probability, and the –k states are unoccupied. At the left side the +k states are all occupied till μ_1, and the –k states till μ_2, whereas at $\mu_2<E<\mu_1$ the –k states are occupied with $\mathcal{R}$=1-$\mathcal{T}$ probability. Let us assume, that there is a U electric potential difference between point A and B (the dispersion curves are shifted with eU). U can be calculated from the charge neutrality condition, i.e. the electron density should be the same at A and B. Around the Fermi energy a constant DOS is assumed.



$$n_A=\int_{\varepsilon^0}^{\mu_1} g(\varepsilon)d\varepsilon+\int_{\varepsilon^0}^{\mu_2} g(\varepsilon)d\varepsilon+R\cdot\int_{\mu 2}^{\mu_1} g(\varepsilon)d\varepsilon$$

$$n_B=\int_{\varepsilon^0}^{\mu_2+eU} g(\varepsilon)d\varepsilon+\int_{\varepsilon^0}^{\mu_2+eU} g(\varepsilon)d\varepsilon+T\cdot\int_{\mu 2+eU}^{\mu_1+eU} g(\varepsilon)d\varepsilon$$

$$n_A=n_B\rightarrow g(\varepsilon_F)\cdot\{\mu_1-\mu_2+R\cdot(\mu_1-\mu_2)\}=g(\varepsilon_F)\cdot\{eU+eU+T\cdot(\mu_1-\mu_2)\}$$

$$\rightarrow eU=R\cdot\underbrace{(\mu_1-\mu_2)}_{eV}\qquad\rightarrow G_{4P}=\frac{I}{U}=\frac{(2e^2/h)\cdot T\cdot V}{R\cdot V}=\frac{2e^2}{h}\cdot\frac{T}{R}$$

special case: T = 1 -> $G_{4p}=\infty$, i.e. the 4-probe resistance is zero!

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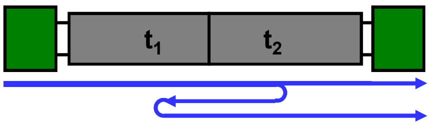
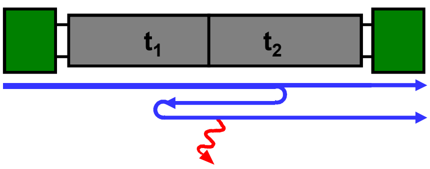
Coherent transport



$$t^0=|t^0|\exp\left(i\varphi^0+i\varepsilon t/\hbar\right)$$
$$t'=|t'|\exp\left(i\varphi'+i\varepsilon t/\hbar\right)$$

Only elastic scattering occurs, the energy of the electrons is preserved. Accordingly, the phase difference between different trajectories is constant in time. This is the case, if the studied structure is smaller than the characteristic lengthscale of inelastic scattring events.

Incoherent transport



$$t^0=|t^0|\exp\left(i\varphi^0+i\varepsilon t/\hbar\right)$$
$$t'=|t'|\exp\left(i\varphi'+i\varepsilon' t/\hbar\right)$$

The electrons suffer inelastic scattering events in the sample, i.e. their energy changes (e.g. they scatter on phonons). Due to the energy relaxation the phase difference between the different partial waves oscillates in time and therefore the interference vanishes.

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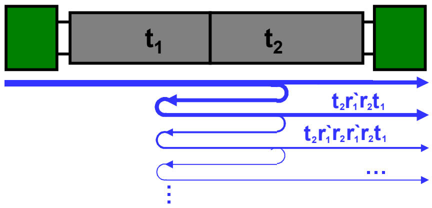
Coherent serial connection of scattering centers

$$t = t_2 t_1 + t_2 r_1' r_2 t_1 + t_2 r_1' r_2 r_1' r_2 t_1 + \ldots = \frac{t_2 t_1}{1 - r_1' r_2}$$

$$T = |t|^2 \qquad R = |r|^2$$

$$T = \frac{T_2 T_1}{1 + R_1 R_2 - 2\sqrt{R_1 R_2} \cdot \cos(\theta)}$$

$$\theta = \arg(r_1' r_2)$$



$$\langle R_{4p} \rangle = \frac{h}{2e^2} \left\langle \frac{1-T}{T} \right\rangle = \frac{h}{2e^2} \left\langle \frac{1 + R_1 R_2 - T_1 T_2 - 2\sqrt{R_1 R_2} \cdot \cos(\theta)}{T_1 T_2} \right\rangle = \frac{h}{2e^2} \left(\frac{R_1}{T_1} + \frac{R_2}{T_2} + 2 \frac{R_1}{T_1} \frac{R_2}{T_2} \right) \neq R_{4p}^{(1)} + R_{4p}^{(2)}$$

If coherence is satisfied an interference term appears, i.e. the four probe resistances of the two scattering centers is not additive.

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Incoherent serial connection of resistors (Ohm's law)

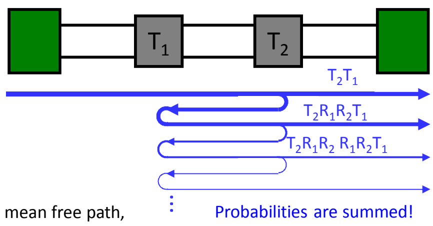
If the distance of the serial connected scattering centers is larger than the phase coherence lengh, then the phase factor of the various (multipally reflected) trajectories is random with respect to each other. In the series of multiple reflections we separate the absolute values, and the phases of the various trajectories:

$$t = t_2 t_1 + t_2 r_1' r_2 t_1 + t_2 r_1' r_2 r_1' r_2 t_1 + \ldots = |t_2||t_1| e^{i\phi_1} + |t_2||r_1'||r_2||t_1| e^{i\phi_2} + |t_2||r_1'||r_2||r_1'||r_2||t_1| e^{i\phi_3} + \ldots$$

In the expectation value of the total transmission the interference of different trajectories cancels due to the random phases.

$$\langle T \rangle = |t|^2 = |t_2|^2 |t_1|^2 + |t_2|^2 |r_1'|^2 |r_2|^2 |t_1|^2 + |t_2|^2 |r_1'||r_2||t_1|^2 \overbrace{\left\langle e^{i(\phi_1 - \phi_2)} + e^{i(\phi_2 - \phi_1)} \right\rangle}^{0} + \ldots =$$

$$= T_2 T_1 + T_2 R_1 R_2 T_1 + \ldots = \frac{T_2 T_1}{1 - R_1 R_2}$$



Summing the trajectories incoherently, the total resistance equals the sum of the two four probe resistances:

$$R_{4p} = \frac{h}{2e^2} \frac{R}{T} = \ldots = \frac{h}{2e^2} \left(\frac{R_1}{T_1} + \frac{R_2}{T_2} \right) = R_{4p}^{(1)} + R_{4p}^{(2)}$$

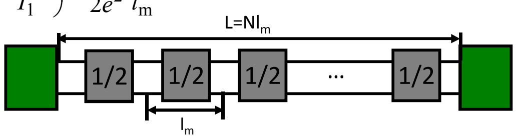
Let us connect N scattering centers, such that their distance is the mean free path, $L_1 = l_m$ and their transmission is $T_1 = 0.5$.

The total length: $L = N l_m$

The total resistance:

$$R_{4p}(L) = \frac{h}{2e^2} N \left(\frac{1 - T_1}{T_1} \right) = \frac{h}{2e^2} \frac{L}{l_m}$$

Ohm's law!



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