Artificial intelligence in data science

Game models

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Learn to play games

- Rules
- Observables
- Possible moves
- Aim: choose best move from observables
- Two methods:
  - Genetic algorithm
  - Reinforcement learning

Reinforcement learning

- Agent gathers information about environment (explores its states): $s_0,\ s_1, \ldots$
- Agent interacts with environment via actions $t_0,\ t_1, \ldots$
- Agent gets reward depending on the actions $r_0,\ r_1, \ldots$
- Modify agent's policy based on reward
- Agent moves to the next state

Ideas from: Fei-Fei Li, Justin Johnson, Serena Yeung

Q-value function

- Policy produces sample trajectories (or paths) $s_0, a_0, r_0, s_1, a_1, r_1 \dots$
- How good is a state? Value function (fitness) V, cumulative reward from a policy
- How good is a state-action pair? The Q-value function at state s and action a, is the expected cumulative reward from taking action a in state s and then following the policy. This is a conditional expected value

Bellman equation

- The optimal Q-value function Q^* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s,a) = \max_\pi \mathbb{E}\left(\sum_t \gamma^t r_t | s_0 = s, a_0 = a, \pi\right),$$

  where π is the actual policy
- Q^* satisfies the following Bellman equation:

$$Q^*(s,a) = \mathbb{E}_{s' \sim \varepsilon}\left(r + \gamma \max_{a'} Q^*(s',a') | s,a\right)$$

- if the optimal state-action values for the next time-step $Q^*(s',a')$ are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma \max_{a'} Q^*(s',a')$
- iterative solution

Reinforcement learning algorithm

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
    Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

Reinforcement learning algorithm

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N

Initialize action-value function Q with random weights

Initialize replay memory, Q-network

for episode $= 1, M$ **do**

  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

  for $t = 1, T$ **do**

    With probability ϵ select a random action a_t

    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

    Execute action a_t in emulator and observe reward r_t and image x_{t+1}

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end for

Reinforcement learning algorithm

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do** ← Play M episodes (full games)
    Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
    for $t = 1, T$ **do**
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Initialize state (starting game screen pixels) at the beginning of each episode

Reinforcement learning algorithm

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For each timestep
of the game

Reinforcement learning algorithm

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  end for
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With small probability select a random action (explore), otherwise select greedy action from current policy

Reinforcement learning algorithm

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    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

Take the action (a_t and observe the reward r_t and next state s_{t+1}

Reinforcement learning algorithm

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end for

Store transition in replay memory

Reinforcement learning algorithm

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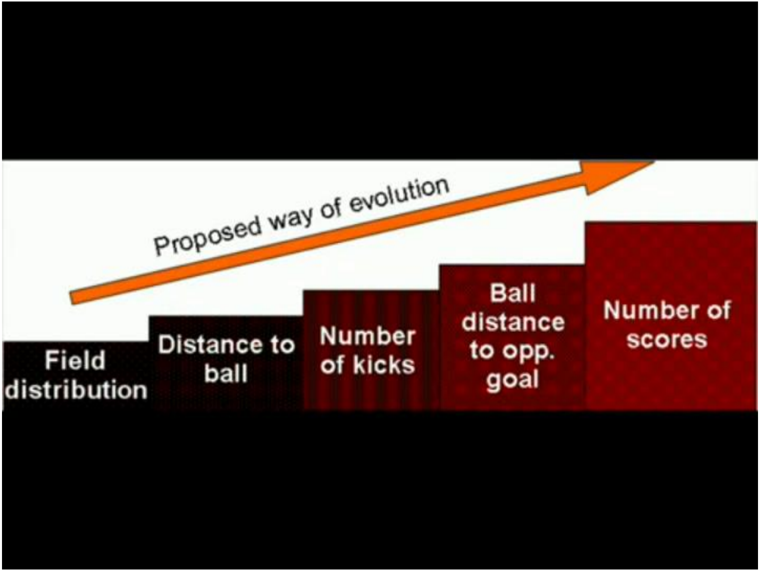
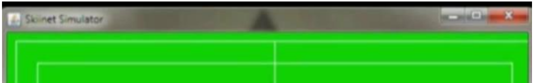
  end for
end for

Experience Replay: Sample a random minibatch of transitions from replay memory and perform a gradient descent step

Reinforcement learning scoring





Reinforcement learning rewards

- Instantaneous, the move is scored by a global function
- Cumulative, sum up the points along the actions
- Cumulative with forget rate.