Computer simulations in Physics

Game models

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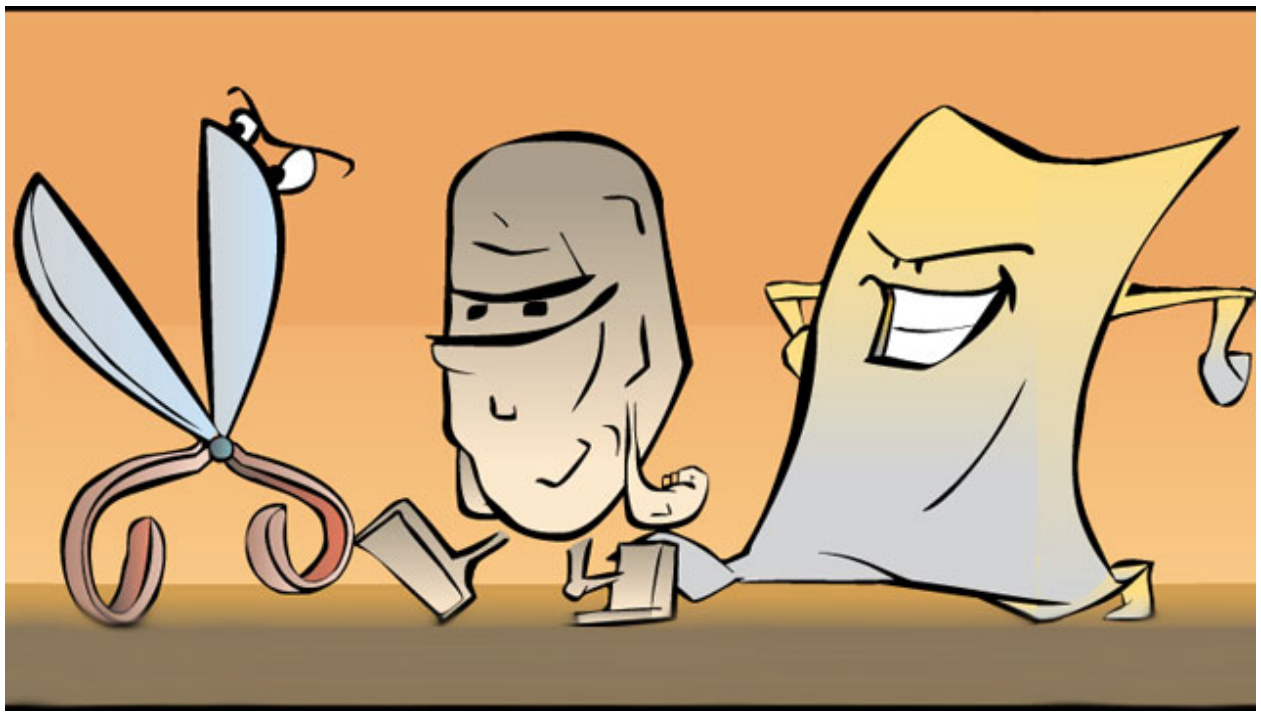
Game models:

- Rock-paper-scissors
- Prisoner's dilemma
- Chicken, hawk-dove game

Rock-paper-scissors

- No winning strategy on (truly) random opponent
- E.g bacterian and antibiotics in mice
- Grass-rabbit-fox
- Popular in games



Prisoner's Dilemma

- Two people playing the game
- Two options: Cooperate, Defect
- Cooperate: Confess the crime
- Defect: deny the crime
- Result: years in prison

	Cooperate	Defect
Cooperate	-1, -1	-3, 0
Defect	0, -3	-2, -2

Prisoner's Dilemma

- Payoff matrix
- Reward for actions based on other player's actions

	Cooperate	Defect
Cooperate	2, 2	0, 3
Defect	3, 0	1, 1

	Cooperate	Defect
Cooperate	1, 1	0, 2
Defect	0, 2	0, 0

Prisoner's Dilemma

- Each player with a preferred strategy that collectively results in an inferior outcome
- Dominating strategy regardless of the opponent's action
- Nash equilibrium, from which no individual player benefits from deviating

	Cooperate	Defect
Cooperate	2, 2	0, 3
Defect	3, 0	1, 1

	Cooperate	Defect
Cooperate	1, 1	0, 2
Defect	0, 2	0, 0

Prisoner's Dilemma

- One game $\rightarrow$ defect
- Fixed number of games $\rightarrow$ defect

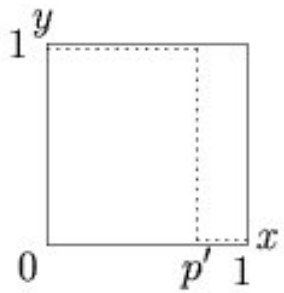
Chicken game, Hawk-Dove game

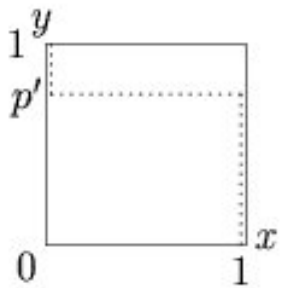
Chicken game, Hawk-Dove game

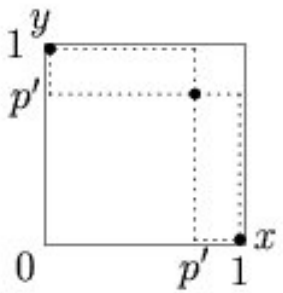
- No preferred strategy
- The best strategy is to anti-coordinate with your opponent

	Cooperate	Defect
Cooperate	6, 6	2, 7
Defect	7, 2	0, 0

- Example: Cold war
- Solution: anti-correlated pure strategy
- Probabilistic, or mixed stratedy (play Hawk with p')





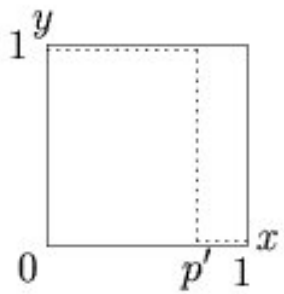


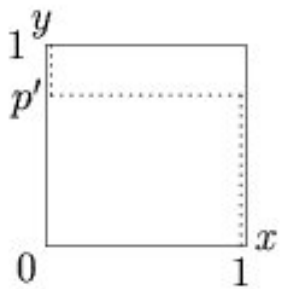
Chicken game, Hawk-Dove game

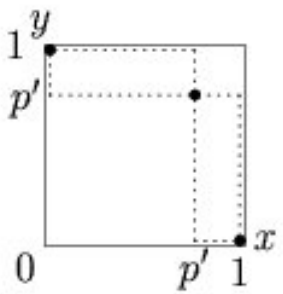
- Nash-equilibrium

	Cooperate	Defect
Cooperate	6, 6	2, 7
Defect	7, 2	0, 0

- $p_D = 1/3$
- gain: $14/3$







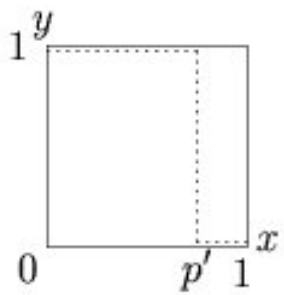
Chicken game, Hawk-Dove game

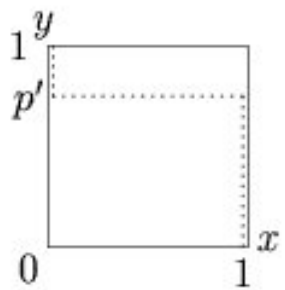
- Nash-equilibrium

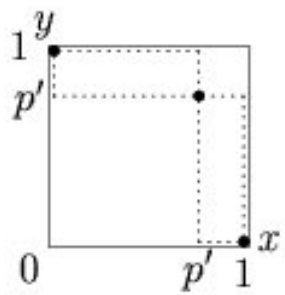
	Cooperate	Defect
Cooperate	R, R	S, T
Defect	T, S	P, P

- Defect with probability p
- To maximize:

$$p^2 P + p(1-p)(T+S) + (1-p)^2 R$$







Chicken game, Hawk-Dove game difference to Prisoner's dilemma

	Cooperate	Defect
Cooperate	Reward	S, T
Defect	T, S	Punish

Hawk-Dove

	C	D
C	2, 2	1, 3
D	3, 1	0, 0

Prisoner's dilemma

	C	D
C	2, 2	0, 3
D	3, 0	1,1

- Prisoner's dilemma:
  Temptation(T)>Reward(R)>Punish(P)>Sucker(S)
- Chicken game:
  Temptation(T)>Reward(R)>Sucker(S)>Punish(P)

Stag game

Prisoner's dilemma

	C	D
C	2, 2	0, 3
D	3, 0	1,1

Hawk-Dove

	C	D
C	2, 2	1, 3
D	3, 1	0, 0

Stag game

	C	D
C	3, 3	0, 2
D	2, 0	1,1

- Prisoner's dilemma:
  Temptation(T)>Reward(R)>Punish(P)>Sucker(S)
- Chicken game:
  Temptation(T)>Reward(R)>Sucker(S)>Punish(P)
- Stag game:
  Reward(R)>Temptation(T)>Punish(P)>Sucker(S)

Prisoner's dilemma: multiple agents

- Against all others
- Against itself
- Against a fully random agent
- Number of agents: 14, 62

Prisoner's dilemma: multiple agents: Strategies

- Strategies for repeated games in Axelrod's tournament (1980):
- AllD: choosing D always (unconditional defector, the bad guy, . . .)
- AllC: choosing C always („the good guy" or sucker)
- Random: chooses D or C with probabilities q or (1-q)
- TFT (Tit-for tat): chooses C first, then she repeates/reciprocates the previous strategy of the co-player
- Generous TFT: TFT, but chooses C (instead of D) with a probability q
- WSLS (win-stay-lose-shift): first C or D, then she changes it if her payoff is smaller than an aspiration level (Ux<a)
- Stochastic reactive strategies: Chooses C or D with probabilities dependent on the previous decision of the co-player
- Stochastic reactive strategies with longer memomy: Etc.
- Go-by-Majority cooperates on the first round, then takes majority strategy.
- . . . and many more

Multiple agents: Winning strategy

- The winner is: Tit-for-tat!
- Human law
- Note that Common good was not included
- Why not "always defect"(AD), which is the Nash equilibrium of the
- Prisoners' dilemma for any finite number of plays?
- Nash equilibrium means that AS is the best strategy against AD
- AS is not dominant strategy
- It is not the best strategy for all strategies

Multiple agents: Best strategy

- Large pool of players (movie):
- It can be shown that for a repeated PD game there is no best strategy for all possible strategies
- But for a good strategy it has to be
  - Nice (do not defect first)
  - Punish others for being nasty
  - Forgive fast
  - Be efficient against yourself