Computer simulations in Physics

Agent Based Modelling

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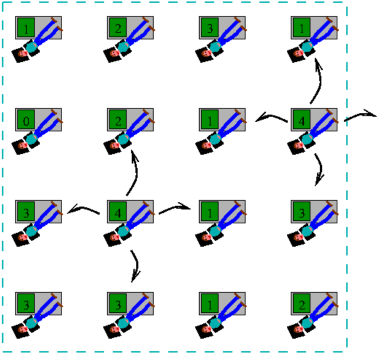
Self-Organized Criticality

- Critical state: inflection point in the critical isotherm
- Power law functions of correlation length, relaxation time
- Control parameter, generally temperature
- Critical point as an attractor?
- Why? Power law: We see many cases
  - $1/f$ noise (music, ocean, earthquakes, flames)
  - Lack of scales (market, earthquakes)
- Underlying mechanism?

Bak-Tang-Wiesenfeld model

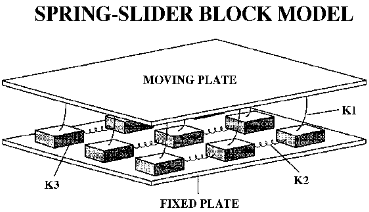
- Originally a sandpile model
- Better explained as a *Lazy Bureaucrat model*:
  - Bureaucrats are sitting in a large office in a square lattice arrangement
  - Occasionally the boss comes with a dossier and places it on a random table
  - The bureaucrats do *nothing* until they have less than 4 dossiers on their table



- Once a bureaucrat has 4 or more dossiers on its table starts to panic and distributes its dossiers to its 4 neighbors
- The ones sitting at the windows give also 1 dossier to its neighbors and throw the rest out of the window.

Bak-Tang-Wiesenfeld model

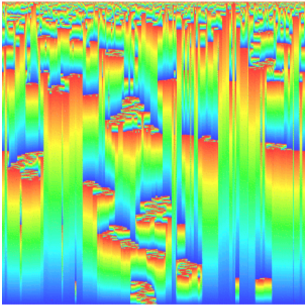
- Originally a sandpile model
- Better explained as a *Lazy Bureaucrat model*:
- Best application: Spring block model of earthquakes:
    - Masses sitting on a frictional plane in a grid are connected with springs to eachother and to the top plate



- Top plate moves slowly, increasing the stress on the top springs slowly and randomly
- If force is large enough masses move which increases the stress on the neighboring masses

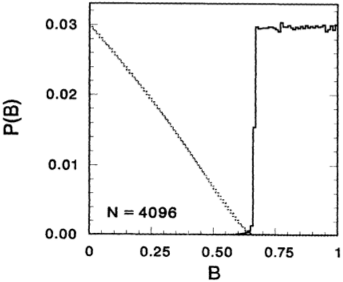
Bak-Sneppen model of evolution

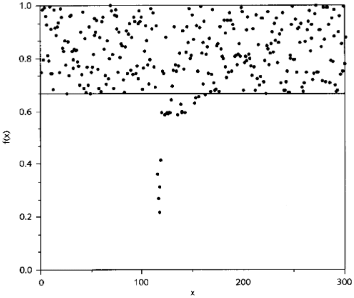
- N species all depends on two other (ring geometry)
- Each species are characterized by a single *fitness*
- In each turn the species with the lowest fitness dies out and with it its two neighbors irrespective of their fitness
- These 3 species are replaced by new ones with random fitness
- Inital and update fitness is uniform between $[0, 1]$



Bak-Sneppen model of evolution: Results

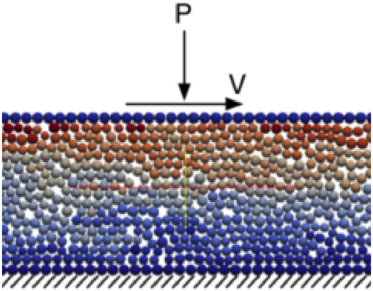
- Steady state with avalanches
- Avalanches start with a fitness $f > f_c \simeq 0.66$
- Avalanche size distribution power law
- Distance correlation power law

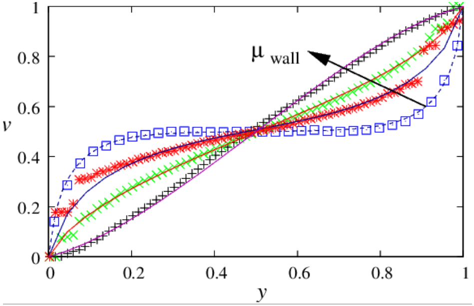




Bak-Sneppen model of evolution an application: Granular shear

- Fitness → Effective friction coefficient
- Specimen with lowest fitness dies out → block is sheared at weakest position (shear band)
- Neighbors, related species die out and replaced by new species → structure gets random around the shear band.

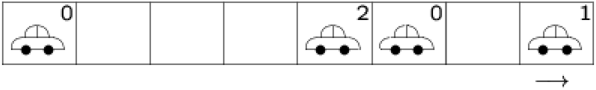




Traffic models

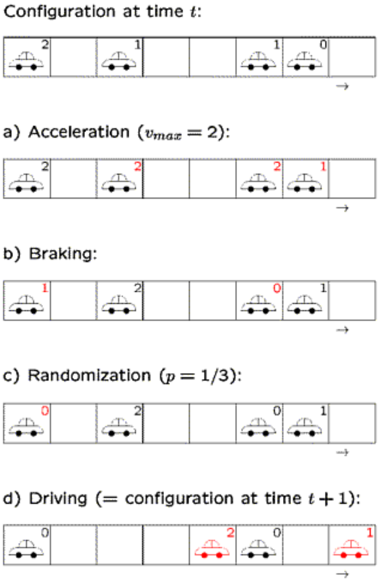
Nagel–Schreckenberg model

- Periodic 1d lattice (ring) Autobahn
- discretized in space and time
- Cars occupying a lattice moving with velocities $v = 0, 1, 2, 3, 4, 5$
- Remark, if max speed is 126 km/h, then lattice length is 7 m, a very good guess for a car in a traffic jam
- It uses parallel update: at each timestep all cars move v sites forward

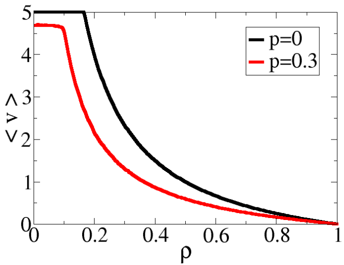


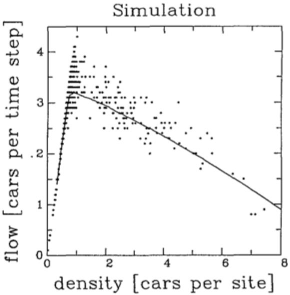
Nagel–Schreckenberg model

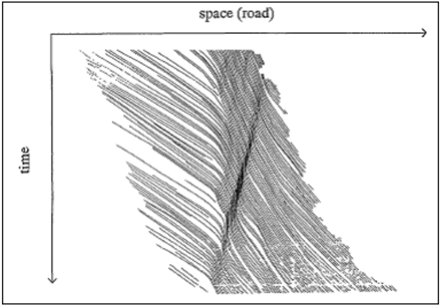
- Algorithm:
  1. **Acceleration**: All cars not at the maximum velocity increase their velocity by 1
  2. **Slowing down**: Speed is reduced to distance ahead (1 sec rule)
  3. **Randomization**: With probability p speed is reduced by 1
  4. **Car motion**: Each car moves forward the number of cells equal to their velocity.



Emergence of traffic jams





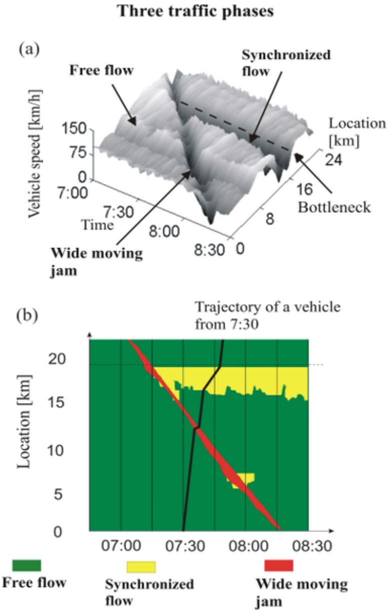


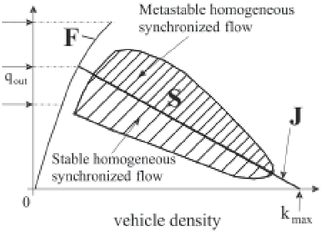
Nagel–Schreckenberg model

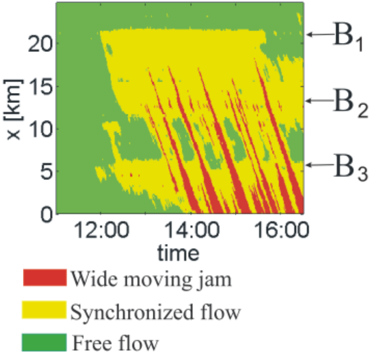
- Transition from free-flow to jammed state
- Jammed state is a phase-separated phase
- Without randomization a sharp transition
- Had been used in NRW to predict traffic jams


autobahn.NRW
Hochrechnung für Bundesautobahnen
Montag, 01.07.2013, 08:05 h
aktuelle Verkehrslage
Prognose 30 min
Prognose 60 min
+7
Langzeit Prognose
Fahrzeitprognose
Farbmodus: Standard
Baustelleninfo
Verkehrsdichten
frei
zähfließend
dicht
gestaut
Baustelle mit geringer Staugefahr
Baustelle mit hoher Staugefahr
Anschlussstelle
Sperrung der Autobahn
Vollsperrung der Anschlussstelle
Teilsperrung der Anschlussstelle
Informationen
Impressum
Version 3.6.1
0 km
15 km
Wesel
Recklinghausen
Lünen
Gelsenk.
Bottrop
Herne
Dortmund
Kamp-Lintfort
Oberhsn.
Essen
Moers
Bochum
Mülheim
Witten
Duisburg
Kempen
Krefeld
Hagen
Velbert
Gevelsberg
Viersen
Düsseldorf
Mönchengladbach
Neuss
Wuppertal
Lüdenscheid

Three-phase traffic theory







Predator prey model

- $N(t)$ number of predators
- $E(t)$ number of prey
- Model (Lotka 1925, Volterra 1926):

$$\begin{aligned} \dot{E}(t) &= \beta_E E(t) - [\mu_E N(t)]E(t) \\ \dot{E}(t) &= [\beta_N E(t)]N(t) - \mu_N N(t) \end{aligned} \tag{1}$$

- Solution $\dot{E} = \dot{N} = 0$:

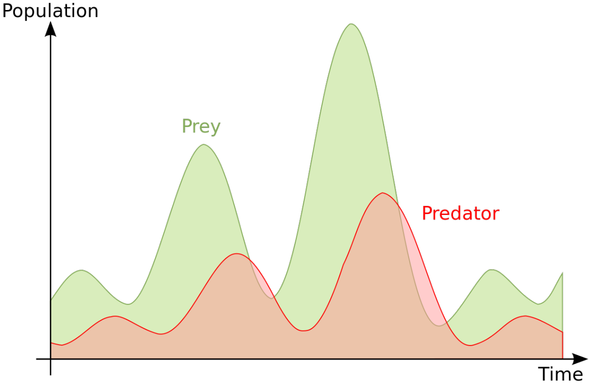
$$\begin{aligned} N = E = 0 \\ N = \beta_E/\mu_E, \quad E = \nu_N/\beta_N \end{aligned} \tag{2}$$

Predator prey model

- Solution $\dot{E} = \dot{N} = 0$:

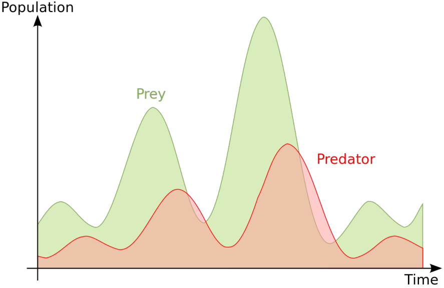
$$N = E = 0$$
$$N = \beta_E/\mu_E, \quad E = \nu_N/\beta_N \tag{3}$$

- Numerical solution:

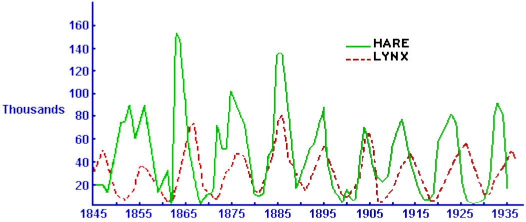


Predator prey model

- Numerical solution:



- Reality:



Other agent based models

- Agents are nodes
- Interactions through links
- Any network:
  - Lattices
  - Random networks
  - Scale-free
  - Fully connected graphs
- Examples:
  - Opinion models (not this time)
  - Minority models

Flocking Model

Flocking model

- Birds move with constant velocity (v_0)
- Align themselves to neighbors
- Some noise due to inaccurate averaging
- Differential equation

$$\theta_i(t + \Delta t) = \langle \theta(t) \rangle_{|\mathrm{r}_i - \mathrm{r}_j| < R} + \xi$$

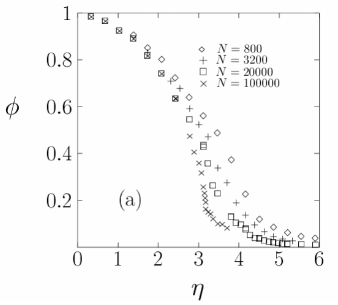
- Upgrade position:

$$\mathrm{r}_i(t + \Delta t) = \mathrm{r}_i(t) + v_0 \mathrm{e}(\theta_i(t)) \Delta t$$

where $\mathrm{e}(\theta)$ is a unit vector in the direction of θ.

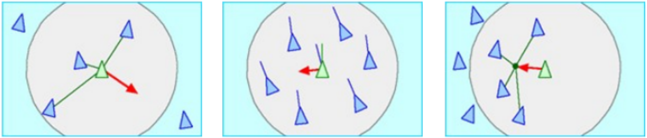
Flocking model

- Birds move with constant velocity (v_0)
- Align themselves to neighbors
- Some noise due to inaccurate averaging
- Phase diagram 1d:



Flocking model

- Birds move with constant velocity (v_0)
- Align themselves to neighbors
- Some noise due to inaccurate averaging
- Non-physicist model:



Separation: steer to avoid crowding local flockmates

Alignment: steer towards the average heading of local flockmates

Cohesion: steer to move toward the average position of local flockmates

http://www.red3d.com/cwr/boids/

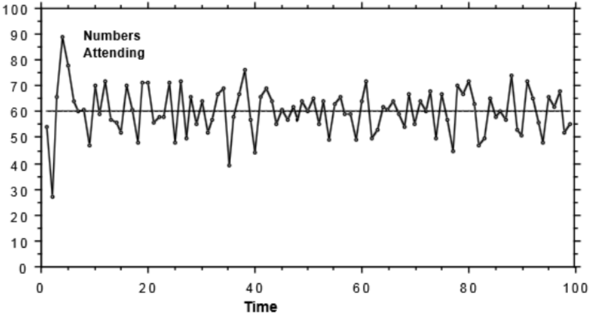
Minority models

"It is not worth an intelligent man's time to be in the majority. By definition, there are already enough people to do that."
Godfrey Harold Hardy
"Csak a döglött hal úszik az árral" - "Only dead fish swim with the tide"

- El Farol Bar problem
- Irish Music Thursdays
- Music is unenjoyable if more than 60 people go

Minority models

- El Farol Bar problem
- Irish Music Thursdays
- Music is unenjoyable if more than 60 people go
- After a transient attendance fluctuates around 60%
- In late stages regularities (cycles) are *arbitraged* away



Memory

- Intentionalism: I know that he know that I know what he . . .
- Intelligent animals: 2 levels
- Children: 2 levels
- Strong authists: 1 level
- Humans 5-7 levels

El Farol problem, strategy

- Attendance was: 44 78 56 15 23 67 84 34 45 76
- Should I stay or should I go now?
- N agents with strategies
- Agents change their strategy with respect to performance
- Similar problems:
  - Traffic decisions
  - Animals food/water
  - Shopping times

Minority model

- N players (odd for simplicity)
- Each player has $S \geq 2$ strategies
- Action of player i at time t is $a_i(t) = \{+1, -1\}$
- Total action: $A(t) = \sum_i a_i(t)$
- Payoff: $p_i(t) = -a_i(t)g[A(t)]$, $g(a)$ is an odd function, e.g. $\mathrm{sign}(x)$
- Information: $W(t+1) = g[A(t)] = \mathrm{sign}[A(t)]$
- Memory: limited to the last m values of W
- Strategy: A table from the 2^m possible inputs to the corresponding output
- Agent evaluates its strategies and plays the best one

Esteban Moro: The Minority Game: an introductory guide

Minority model: strategy

- Memory: limited to the last m values of W
- Strategy: A table from the 2^m possible inputs to the corresponding output

input			output
-1	-1	-1	-1
-1	-1	$+1$	$+1$
-1	$+1$	-1	$+1$
-1	$+1$	$+1$	-1
$+1$	-1	-1	$+1$
$+1$	-1	$+1$	-1
$+1$	$+1$	-1	$+1$
$+1$	$+1$	$+1$	-1

Random strategy

- Having N agents, the probability of having n +1 follows a binomial distribution

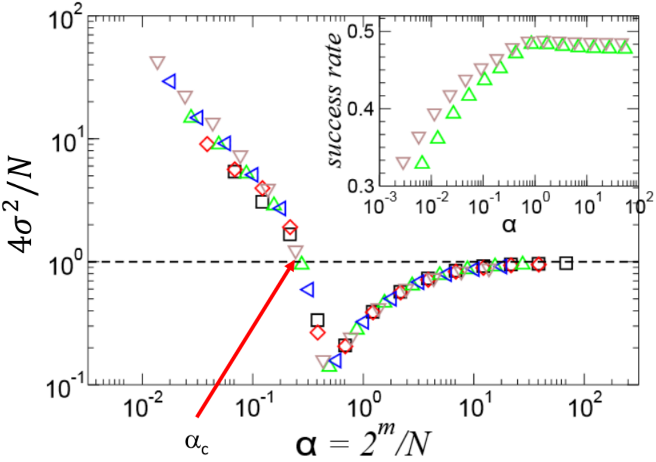
$$P(n) = \binom{N}{n} p^n (1-p)^{N-n}$$

- Average: $\langle n \rangle = pN$, $\langle n \rangle (p = 1/2) = N/2$
- Variance: $\sigma^2 = Np(1-p)$, $\sigma^2(p = 1/2) = N/2$
- Minority game:
- Average: $\langle A(t) \rangle = 0$
- Variance: σ^2/N is function of $\alpha = 2^m/N$ with

$$\lim_{\alpha \to \infty} \sigma^2/N = 1/4$$

  So the strategy with infinite memory becomes random
- At low values of α the variance increases as a power law $\sigma^2/N \sim \alpha^{-1}$

Minority model: variance



N= 101, 201, 301, 501, 701 (□, ♦, △, ▷, ▽)

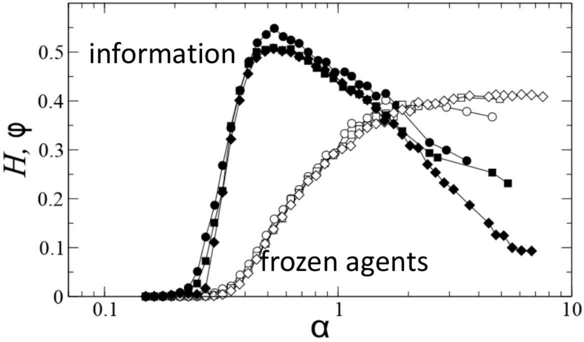
Minority model: order parameter

- Can we predict the sign of $A(t)$?
  - $\alpha < \alpha_c$: No, we have not enough information, agents are random
  - $\alpha > \alpha_c$: Yes, strong dependence, in market this can be exploited (arbitrage)
- Order parameter: information

$$H = \frac{1}{2^m} \sum_{\nu} \langle W(t+1) | W(t) = \nu \rangle$$

Minority model: variance

- Can we predict the sign of $A(t)$?
  - $\alpha < \alpha_c$: No, we have not enough information, agents are random
  - $\alpha > \alpha_c$: Yes, strong dependence, in market this can be exploited (arbitrage)



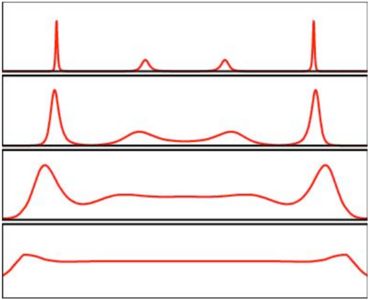
Bounded confidence opinion model: Deffuant model

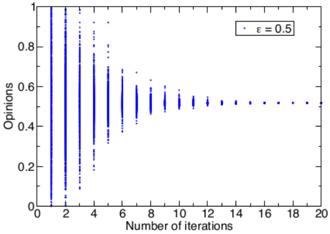
- Agents have opinion x_i
- if $|x_i(t) - x_j(t)| < \varepsilon$ then
  - $x_i(t+1) = x_i(t) - \mu[x_i(t) - x_j(t)]$
  - $x_j(t+1) = x_j(t) + \mu[x_i(t) - x_j(t)]$
- μ compromise parameter $\mu = 1/2$ complete compromise
- ε tolerance parameter
- Methods:
  - Monte-Carlo simulation
  - Master equation:

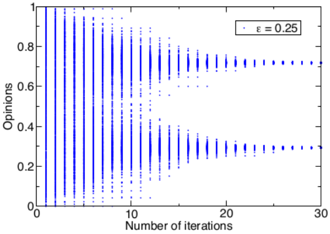
$$\frac{\partial P(x,t)}{\partial t} = \int_{|x_1 - x_2| < \varepsilon} dx_1 dx_2 P(x_1,t) P(x_2,t) \times$$

$$\times \left[\delta\left(x - \frac{x_1 + x_2}{2} \right) - \delta(x - x_1) \right]$$

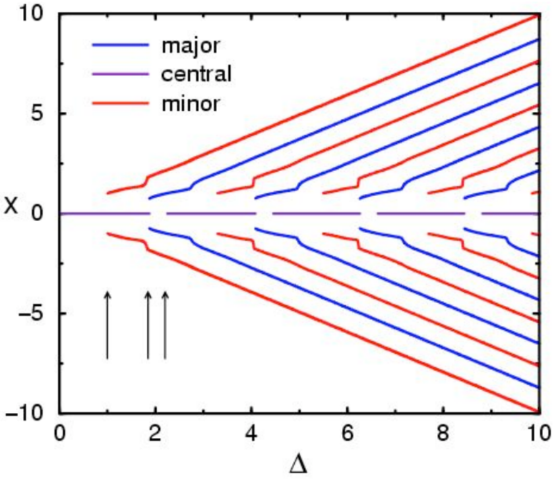
Deffuant model: Opinion groups (fully connected graph)







Deffuant model: Bifurcation diagram

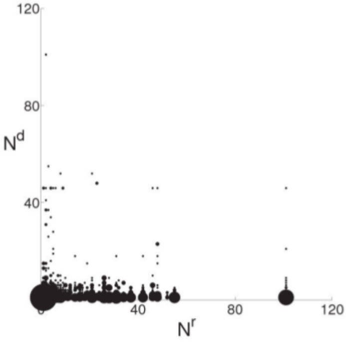


$\Delta = 2/\varepsilon$, $\mu = 1/2$

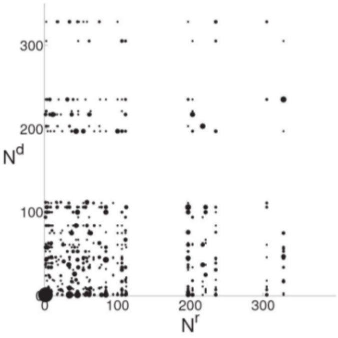
Application to Wikipedia edit wars

- Article has an opinion bieas A
- People who do not like the bias may edit the article
- Editors argue for a while then simply revert the other edits
- expeiremce of the editor helps to distinguish between edit war and vandalism

Benjamin Franklin



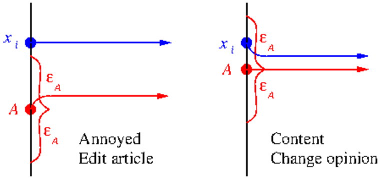
Israel and the apartheid analogy



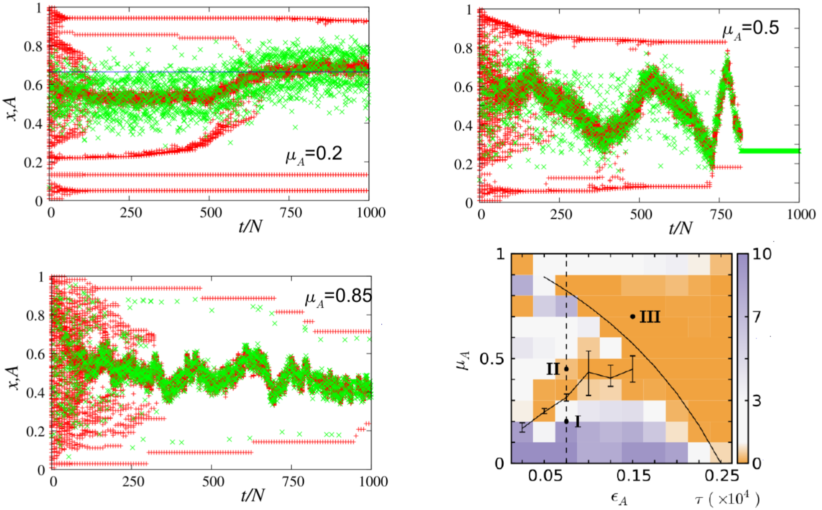
Controversial words in article titles in 2009

Bounded confidence model for Wikipedia

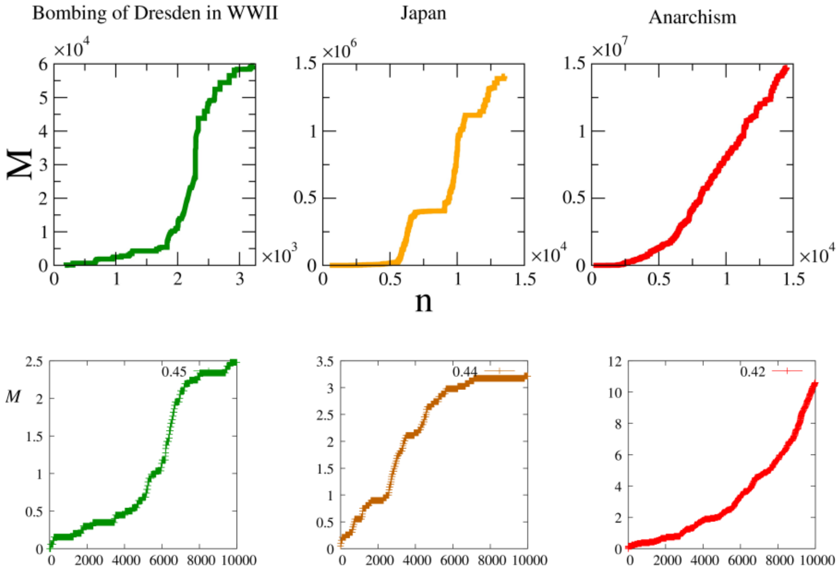
- Agents have opinion x_i, Article A
- if $|x_i(t) - A(t)| < \varepsilon_A$ then
  - $x_i(t+1) = x_i(t) + \mu_A[A(t) - x_i(t)]$
- Article bias is close to the editor's no edit but agent is a bit influenced by the article
- if $|x_i(t) - A(t)| < \varepsilon_A$ then
  - Article bias is far from the editor's edit!
  - $A(t+1) = A(t) + \mu_A[x_i(t) - A(t)]$
- μ_A article weight (probably amount of content or reasoning)
- ε_A tolerance parameter



Results of the Wikipedia model



Results of the Wikipedia model



Practice: Minority game

- (If you find it too difficult you can choose any other model)
-