

# Computer simulations in Physics

## Game models

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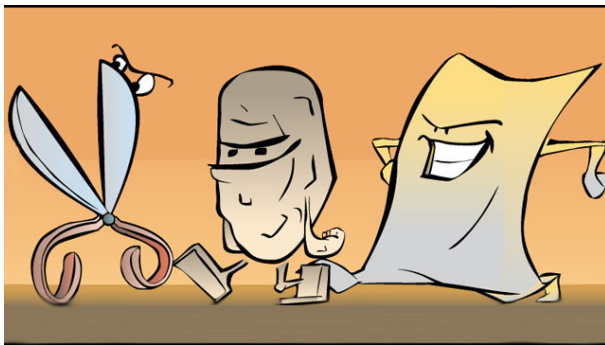
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## Game models:

- ▶ Rock-paper-scissors
- ▶ Prisoner's dilemma
- ▶ Chicken, hawk-dove game

# Rock-paper-scissors

- ▶ No winning strategy on (truly) random opponent
- ▶ E.g bacterian and antibiotics in mice
- ▶ Grass-rabbit-fox
- ▶ Popular in games



# Prisoner's Dilemma

- ▶ Two people playing the game
- ▶ Two options: Cooperate, Defect
- ▶ Cooperate: Confess the crime
- ▶ Defect: deny the crime
- ▶ Result: years in prison

	Cooperate	Defect
Cooperate	-1, -1	-3, 0
Defect	0, -3	-2, -2

# Prisoner's Dilemma

- ▶ Payoff matrix
- ▶ Reward for actions based on other player's actions

	Cooperate	Defect
Cooperate	2, 2	0, 3
Defect	3, 0	1, 1

	Cooperate	Defect
Cooperate	1, 1	0, 2
Defect	0, 2	0, 0

# Prisoner's Dilemma

- ▶ Each player with a preferred strategy that collectively results in an inferior outcome
- ▶ Dominating strategy regardless of the opponent's action
- ▶ Nash equilibrium, from which no individual player benefits from deviating

	Cooperate	Defect
Cooperate	2, 2	0, 3
Defect	3, 0	1, 1

	Cooperate	Defect
Cooperate	1, 1	0, 2
Defect	0, 2	0, 0

# Prisoner's Dilemma

- ▶ One game  $\rightarrow$  defect
- ▶ Fixed number of games  $\rightarrow$  defect

# Chicken game, Hawk-Dove game

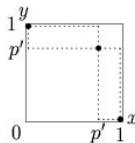
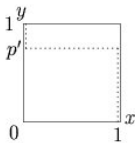
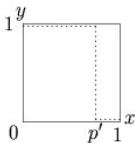


# Chicken game, Hawk-Dove game

- ▶ No preferred strategy
- ▶ The best strategy is to anti-coordinate with your opponent

	Cooperate	Defect
Cooperate	0, 0	-1, 2
Defect	2, -1	-5, -5

- ▶ Example: Cold war
- ▶ Solution: anti-correlated pure strategy
- ▶ Probabilistic, or mixed strategy (play Hawk with  $p'$ )



# Chicken game, Hawk-Dove game difference to Prisoner's dilemma

	Cooperate	Defect
Cooperate	Reward	S, T
Defect	T, S	Punish

	Hawk-Dove			Prisoner's dilemma	
	C	D		C	D
C	2, 2	1, 3	C	2, 2	0, 3
D	3, 1	0, 0	D	3, 0	1, 1

- ▶ Prisoner's dilemma:  
 $\text{Temptation}(T) > \text{Reward}(R) > \text{Punish}(P) > \text{Sucker}(S)$
- ▶ Chicken game:  
 $\text{Temptation}(T) > \text{Reward}(R) > \text{Sucker}(S) > \text{Punish}(P)$

# Stag game

Prisoner's dilemma

	C	D
C	2, 2	0, 3
D	3, 0	1, 1

Hawk-Dove

	C	D
C	2, 2	1, 3
D	3, 1	0, 0

Stag game

	C	D
C	3, 3	0, 2
D	2, 0	1, 1

- ▶ Prisoner's dilemma:  
Temptation(T) > Reward(R) > Punish(P) > Sucker(S)
- ▶ Chicken game:  
Temptation(T) > Reward(R) > Sucker(S) > Punish(P)
- ▶ Stag game:  
Reward(R) > Temptation(T) > Punish(P) > Sucker(S)

## Prisoner's dilemma: multiple agents

- ▶ Against all others
- ▶ Against itself
- ▶ Against a fully random agent
- ▶ Number of agents: 14, 62

## Prisoner's dilemma: multiple agents: Strategies

- ▶ Strategies for repeated games in Axelrod's tournament ( 1980):
- ▶ ALLD: choosing D always (unconditional defector, the bad guy, ...)
- ▶ ALLC: choosing C always („the good guy” or sucker)
- ▶ Random: chooses D or C with probabilities  $q$  or  $(1-q)$
- ▶ TFT (Tit-for tat): chooses C first, then she repeats/reciprocates the previous strategy of the co-player
- ▶ Generous TFT: TFT, but chooses C (instead of D) with a probability  $q$
- ▶ WSLS (win-stay-lose-shift): first C or D, then she changes it if her payoff is smaller than an aspiration level ( $U_x < a$ )
- ▶ Stochastic reactive strategies: Chooses C or D with probabilities dependent on the previous decision of the co-player
- ▶ Stochastic reactive strategies with longer memory: Etc.
- ▶ Go-by-Majority cooperates on the first round, then takes majority strategy.

## Multiple agents: Winning strategy

- ▶ The winner is: Tit-for-tat!
- ▶ Human law
- ▶ Note that Common good was not included
- ▶ Why not “always defect”(AD), which is the Nash equilibrium of the
- ▶ Prisoners' dilemma for any finite number of plays?
- ▶ Nash equilibrium means that AS is the best strategy against AD
- ▶ AS is not dominant strategy
- ▶ It is not the best strategy for all strategies

## Multiple agents: Best strategy

- ▶ Large pool of players (movie):
- ▶ It can be shown that for a repeated PD game there is no best strategy for all possible strategies
- ▶ But for a good strategy it has to be
  - ▶ Nice (do not defect first)
  - ▶ Punish others for being nasty
  - ▶ Forgive fast
  - ▶ Be efficient against yourself