Statistical physics 2, homework 1

Density operator for thermalized Spin-1/2 System: Consider a system of spin-1/2 particles on a rigid lattice. The interaction between the spins is negligible, though it serves the thermalization of the system. In a homogeneous magnetic field $\mathbf{B} = B_0\mathbf{e}_z$, the Hamiltonian of a spin is:

$$H = -\boldsymbol{\mu}\mathbf{B} = -\frac{1}{\beta_0}\sigma_z, \tag{1}$$

The particle charge is q, the mass is m thus, with the appropriate g-factor,

$$\beta_0 = \frac{4m}{\hbar g q B_0} \tag{2}$$

The one particle density operator has a 2×2 matrix representation.

1. Compute the one particle thermal density operator for a given temperature T!
2. Compute the partition function of the entire system of N particles, and use it to compute the mean energy per particle as a function of T!
3. What is the temperature T_X, below which the probability that a particle is not at the minimum energy level is smaller than 10^{-3}?
4. Show that as $T \to \infty$, the thermal state goes to the maximum entropy state $\rho = \frac{1}{2}\mathbf{I}$, where $\mathbf{I}$ is the identity operator!

Deadline: September 22, 2016