A diffúziós egyenlet megoldása:

$$\frac{\partial \rho}{\partial t} = D\frac{\partial^2 \rho}{\partial x^2}$$

olyan megoldást keresünk, ami kielégíti:

$$\left(\frac{\partial}{\partial t} - D\frac{\partial^2}{\partial x^2}\right) G = \delta(x)\,\delta(t) \qquad -+$$

Fourier transzformálunk:

$$G(x,t) = \int\frac{d\omega}{2\pi}\int\frac{dq}{2\pi}\, e^{-i\omega t + iqx}\; G(\omega,q)$$

$$G(\omega,q) = \int dt \int dx\; e^{i\omega t}\, e^{-iqx}\; G(x,t)$$

$$\Rightarrow \quad (-i\omega + Dq^2)\, G(\omega,q) = 1$$

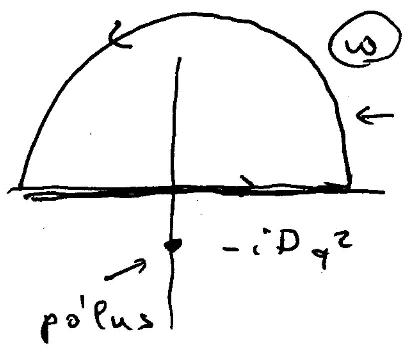
$$\Rightarrow \quad G(\omega,q) = \frac{1}{-i\omega + Dq^2}$$

Vissza transzformálunk:

$$G(x,t) = \int\frac{d\omega}{2\pi}\int\frac{dq}{2\pi}\, e^{-i\omega t}\, e^{iqx}\; \underbrace{\frac{1}{-i\omega + Dq^2}}_{\frac{i}{\omega + iDq^2}}$$

ω - integrál:

a) $t < \emptyset \Rightarrow \operatorname{Im}\omega > \emptyset$ felé le tudjuk zárni

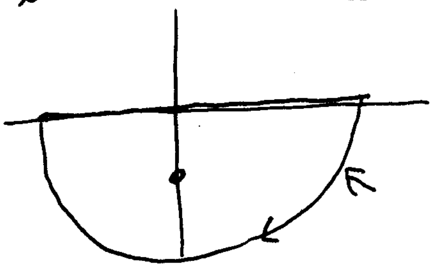


← az ív járuléka $= \emptyset$

$$\Rightarrow \int\frac{d\omega}{2\pi}\,\frac{i}{\omega + iDq^2}\, e^{-i\omega t} = \emptyset$$

b) $t > 0$ $\quad$ ω

Im $\omega < 0$ felé zárjuk le a kontúrt



ív integrálja 0

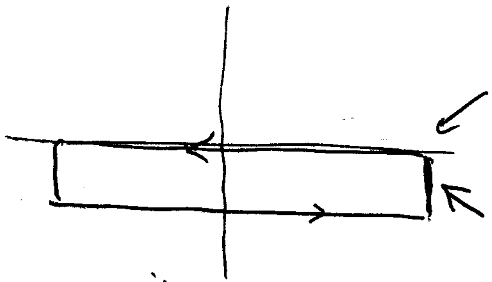
Reziduum tétel:

$$\oint \frac{i}{\omega + iDq^2} e^{-i\omega t} = (-2\pi i)\cdot i\, e^{-Dq^2 \cdot t}$$

$$\Rightarrow \quad G(x,t) = \int \frac{dq}{2\pi} \underbrace{e^{-Dq^2 t}\, e^{iqx}}_{e^{-Dt\left(q - \frac{ix}{2Dt}\right)^2} \cdot e^{-\frac{x^2}{4Dt}}} \cdot \Theta(t)$$

$$= \Theta(t)\, e^{-\frac{x^2}{4Dt}} \int_{-\infty - i\frac{x}{2Dt}}^{\infty - i\frac{x}{2Dt}} \frac{dz}{2\pi} e^{-Dtz^2}$$

de



erre

ez a szakasz 0-t ad

$$\oint dz\, e^{-Dtz^2} = 0$$

Res. tétel

$$\Rightarrow \int_{-\infty - i\frac{x}{2Dt}}^{\infty - i\frac{x}{2Dt}} \frac{dz}{2\pi} e^{-Dtz^2} = \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{-Dtz^2} = \frac{1}{\sqrt{4\pi Dt}}$$

$$\Rightarrow \quad \boxed{G(x,t) = \frac{1}{(4\pi Dt)^{1/2}} e^{-\frac{x^2}{4Dt}}\, \Theta(t)}$$

Általános megoldás:

$$G(x-x', t-t') \equiv \frac{1}{\sqrt{4\pi D(t-t')}} e^{-\frac{x^2}{4D(t-t')}} \Theta(t-t')$$

nyilván kielégíti

$$\left(\frac{\partial}{\partial t} - D\frac{\partial^2}{\partial x^2}\right) G = \delta(x-x')\,\delta(t-t') \quad -t$$

$\Rightarrow$

$$P(t,x) \equiv \int dx' \; G(x-x', t-\underset{t_0}{\underset{|||}{t'}}) \; P_0(\overset{t_0}{\overset{|||}{t'}}, x')$$

kielégíti

$$\left(\frac{\partial}{\partial t} - D\frac{\partial^2}{\partial x^2}\right) P(x,t) = 0 \quad -t$$

$P(x,t_0) = P_0(x,t_0)$ határ feltétellel.