Excitations of superfluid

Schrödinger equation:

$$E(\psi) \equiv \mathcal{H}(\psi, \psi^*) = \int d^3\underline{r} \left(\frac{\hbar^2}{2m} |\nabla\psi|^2 + V(\underline{r})|\psi(\underline{r})|^2 \right)$$

$$\Rightarrow \quad i\hbar\dot{\psi} = \frac{\delta E}{\delta \psi^*} = -\frac{\hbar^2}{2m}\Delta\psi + V(\underline{r})\psi(\underline{r}) \quad \checkmark$$

Time dependent Gross - Pitaevskii:

$$E(\psi) = \int d^3\underline{r} \left\{ \frac{\hbar^2}{2m}|\nabla\psi|^2 + V|\psi|^2 + \frac{1}{2} g |\psi|^4 \right\}$$

$$\boxed{i\hbar\dot{\psi} = \frac{\delta E}{\delta\psi^*} = -\frac{\hbar^2}{2m}\Delta\psi + V\psi + g|\psi|^2\psi}$$

wave solutions: $V = -\mu \Rightarrow \rho_0 = |\psi_0|^2 = \frac{\mu}{g}$

$$\psi = \sqrt{\rho}\, e^{i\varphi} = \sqrt{\rho_0 + \delta\rho}\, e^{i\varphi} \approx \sqrt{\rho_0} + \frac{\delta\rho}{2\sqrt{\rho_0}} + i\sqrt{\rho_0}\cdot\varphi$$

small

$$i\hbar\dot{\psi} = i\hbar\left(\frac{1}{2\sqrt{\rho_0}}\delta\dot{\rho} + i\sqrt{\rho_0}\,\dot{\varphi}\right) = -\frac{\hbar^2}{2m}\left(\frac{1}{2\sqrt{\rho_0}}\Delta\delta\rho + i\sqrt{\rho_0}\,\Delta\varphi\right) + \dots$$

$$\left(\dots = -\mu\sqrt{\rho_0} - \mu\frac{\delta\rho}{2\sqrt{\rho_0}} - \mu i\sqrt{\rho_0}\,\varphi + g\,\rho_0^{3/2} + 2g\,\delta\rho\sqrt{\rho_0} + i g\,\rho_0^{3/2}\cdot\varphi \right)$$

$$\Rightarrow \quad \boxed{\hbar\dot{\varphi} = \frac{\hbar^2}{4m\rho_0}\Delta\delta\rho - 2g\,\delta\rho}$$

$$\boxed{\hbar\delta\dot{\rho} = -\frac{\hbar^2}{m}\rho_0\,\Delta\delta\varphi}$$

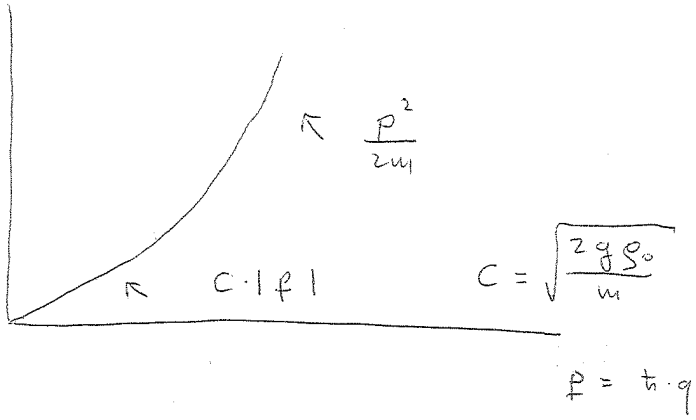
$$\Rightarrow \quad \hbar^2\ddot{\varphi} = -\left(\frac{\hbar^2}{2m}\right)^2\Delta^2\varphi + \frac{\hbar^2}{m}2g\rho_0\Delta\varphi$$

F.T.:

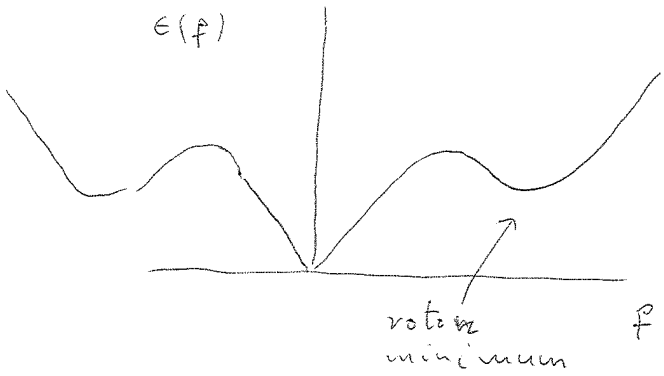
$$\Rightarrow \quad \boxed{\hbar\omega(q) = \sqrt{\frac{\hbar^2}{2m} q^2 \left(\frac{\hbar^2 q^2}{2m} + 4 g \rho_0 \right)}}$$

$\epsilon(\underline{p}) = \hbar\omega$

$\Rightarrow$

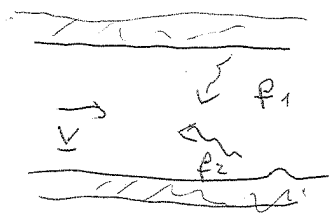


^{4}He : $g(\underline{r}) \Rightarrow g(q)$



statement : superfluidity $\longleftrightarrow$ interactions !

moving liquid is damped by creating excitations



"kicks" by wall

$\Rightarrow$ momentum reduction

energy needed to create excitation in moving condensate (Galilei transform.)

$$\boxed{\epsilon(p) = \Delta E(p) = \epsilon_0(p) + \underline{v} \cdot p}$$

container gets momentum $-\underline{p}$

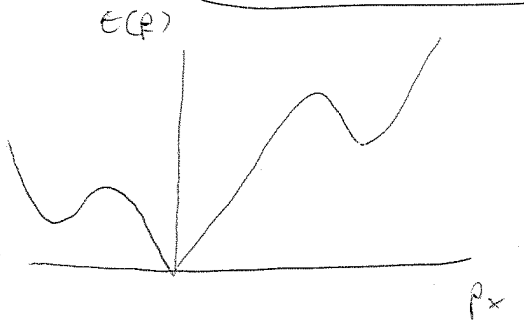
$$\Rightarrow \quad \text{energy} = \Delta E_{container} = \frac{p^2}{2M_{cont}} = 0$$

huge!

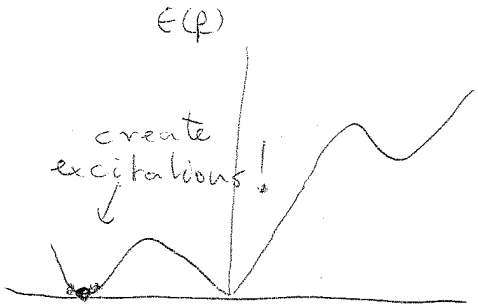
energy conservation satisfied if

$$\boxed{\epsilon(\underline{p}) = \epsilon_0(\underline{p}) + \underline{v}_s \cdot \underline{p} = 0} \quad !$$

$\underline{v}_s \parallel \hat{x}$



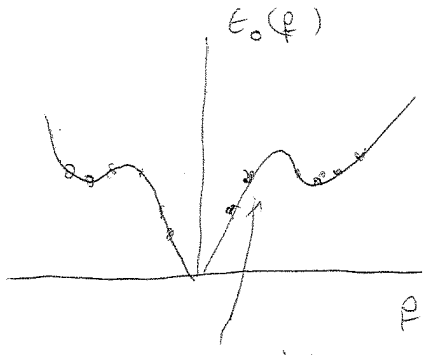
If $v_s = v_c$



$$\boxed{\begin{array}{l} v_s > v_c \\ \Rightarrow \text{ dissipation !} \end{array}}$$

What is "<u>normal part</u>" ??

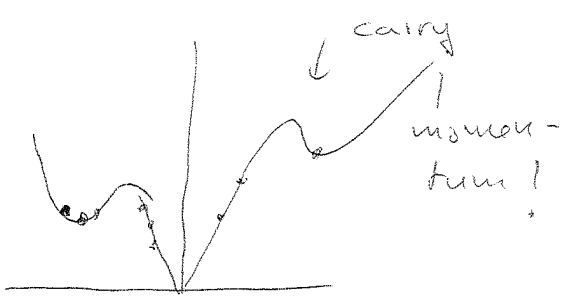
$\underline{v}_s = 0 \qquad T \neq 0$



occupation

$$n(\underline{p}) = \frac{1}{e^{\beta\epsilon_0(\underline{p})} - 1}$$

$\underline{v}_s \neq 0$



$$n(\underline{p}) = \frac{1}{e^{\beta\epsilon(\underline{p})} - 1}$$

$\underline{j}$ = current density $= \frac{1}{V}\,\underline{P}_{total} =$

$$\underline{j} = \rho \cdot \underline{v}_s + \underbrace{\frac{1}{V}\sum_{\underline{p}} \underline{p}\; n(\underline{p})}_{\underline{P}^{Q.P.}}$$

$\leftarrow$ "quasiparticles" = excitations

after equilibrialion:

$$\frac{1}{V}\,\underline{P}^{Q.P.} = \int \frac{d^3\underline{p}}{h^3}\,\underline{p}\,\frac{1}{e^{\beta\epsilon(\underline{p})}-1} = -\beta\int \frac{d^3\underline{p}}{h^3}\,\frac{\underline{p}\,e^{\beta\epsilon_0(\underline{p})}}{\left(e^{\beta\epsilon_0(\underline{p})}-1\right)^2}\cdot(\underline{p}\,\underline{v}_s)$$

$\epsilon_0(\underline{p}) + \underline{v}_s\cdot\underline{p}$ expand in $\underline{v}_s$

$$= -\,\underline{v}_s \underbrace{\int \frac{d^3\underline{p}}{h^3}\,\frac{p^2}{3k_BT}\,n_0(\underline{p})\left(1+n_0(\underline{p})\right)}_{\rho_n\ !}$$

$$\underline{j} = (\rho-\rho_n)\,\underline{v}_s = \rho_s\,\underline{v}_s$$

normal current died out already !

initially

$$\underline{j} = \rho\,\underline{v}_s$$

wall thermalizes the excitations $\rightarrow$ $\underline{j} = \underbrace{(\rho-\rho_n)}_{\rho_s}\underline{v}_s$

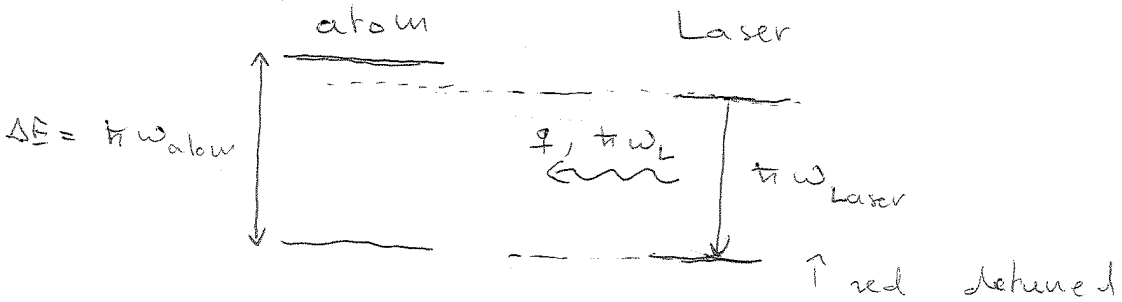
$$T \to 0 : \begin{cases} \rho_n \sim T^2 \to 0\,! \\ \rho_s \to \rho \end{cases}$$

Creating ultracold atoms

ultracold ~ $T \sim 1\,nK$!

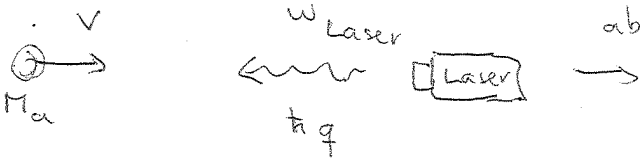
$$\text{room} \xrightarrow{10^{-2}} {}^4He \xrightarrow{10^{-2}} \text{dilution fridge} \xrightarrow{10^{-7}/10^{-8}} \text{ultracold gases}$$

Key: Doppler cooling



if atom moves toward Laser

→ can see Doppler-shifted $\omega' = \left(1+\frac{v}{c}\right)\omega_L$



$p' = M_a v + \hbar q$

gets a "kick"

$\Delta p_{atom} = \hbar q$

very small!

⇒ optical molasses $T \sim 1\,\mu K$, $N \sim 10^9$ atoms in $d \approx 0{,}1\,mm$

⇓

evaporative cooling

⇓

$N \sim 10^6$ atoms $T \sim nK$ ⇒ Bose condensation!

<u>Imaging:</u> CCD camera !

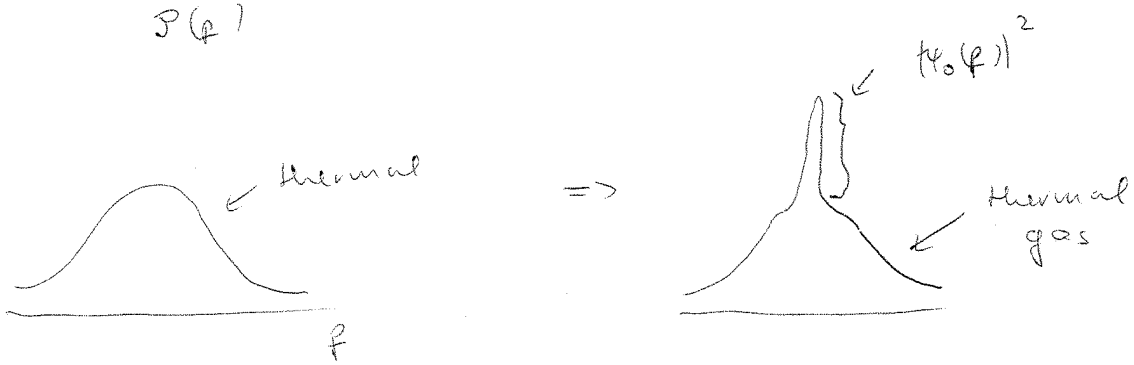
Time of Flight (ToF)

let atoms fly $\rightarrow$ $\underline{r}(t) = \frac{\underline{p}}{m} \cdot t$

$\Rightarrow$ $\mathcal{P}(\underline{r}, t) \propto \mathcal{P}\left(\underline{p} = \frac{r}{t} \cdot m, t=0\right) = \mathcal{P}(\underline{p}, t=0)$

Condensates wave function $\psi_0(\underline{r})$ $\Rightarrow$

$\Rightarrow$ $\mathcal{P}(\underline{p}) \sim |\psi_0(\underline{p})|^2$ $\rightarrow$ image it

$T > T_c$ $\qquad\qquad$ $T < T_c$

$\mathcal{P}(\underline{p})$