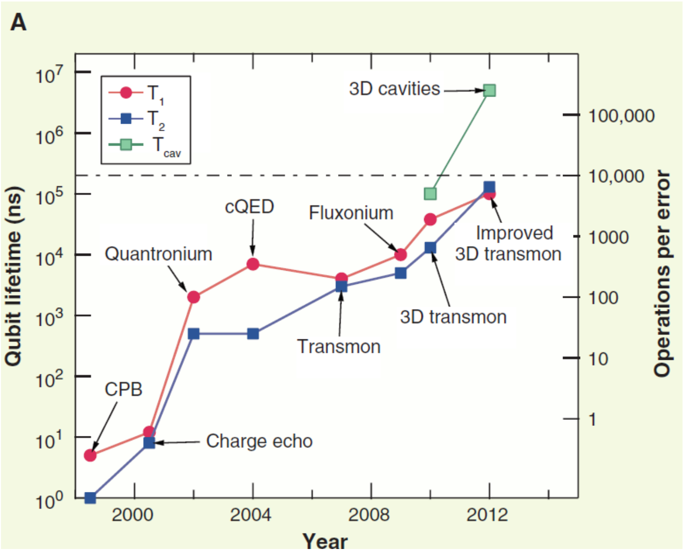
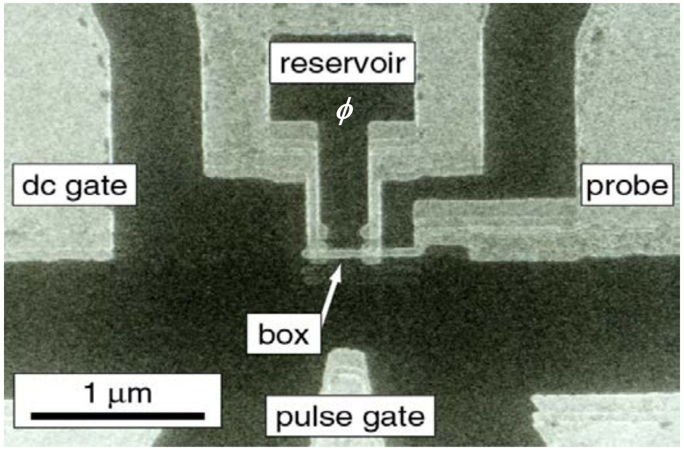
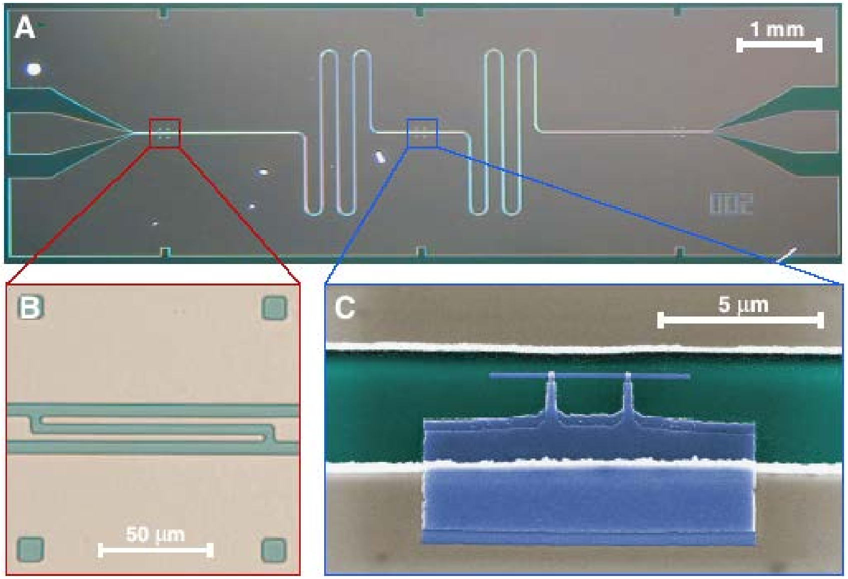
Quantum Computing Architectures

Budapest University of Technology and Economics 2018 Fall



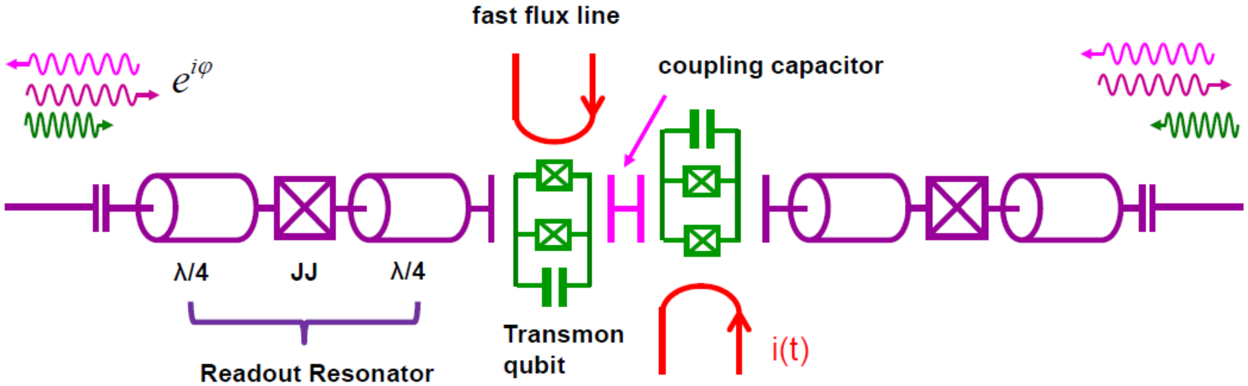


Lecture 9: Coupling qubits
State tomography



Coupling qubits
Capacitive coupling

Fix coupling – not tuneable
Separate readout resonator for both of them
Can perform swap operation



Coupling term

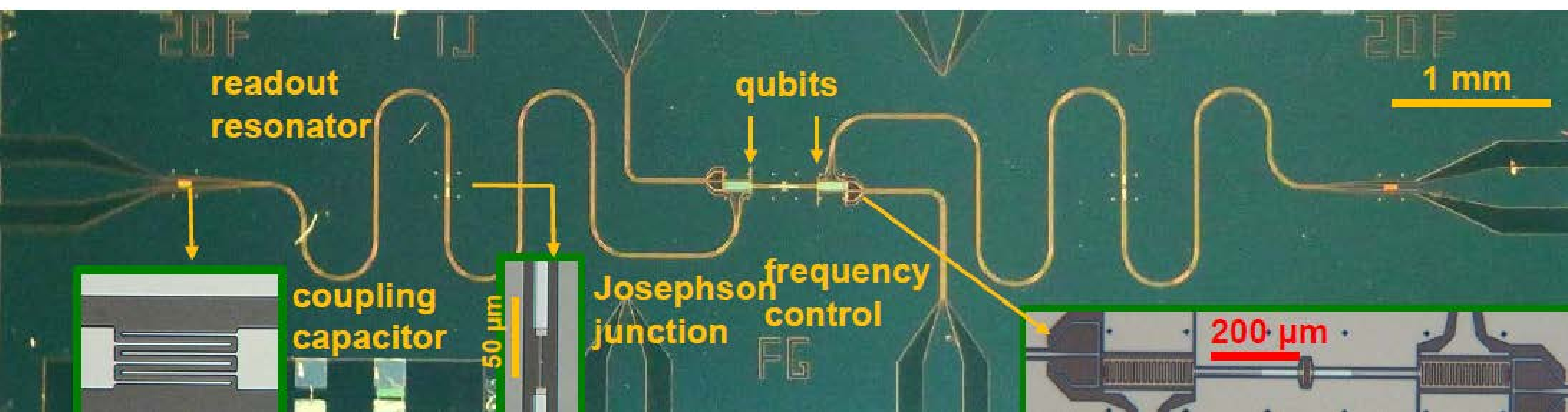
$$2\frac{E_{c,I}E_{c,II}}{E_{cc}}(\hat{N}_I - N_{g,I})(\hat{N}_{II} - N_{g,II})$$

...

$$\boxed{\hbar g(\sigma_I^{+}\sigma_{II}^{-} + \sigma_I^{-}\sigma_{II}^{+})}$$

$$\boxed{\hbar g = (2e)^2 \frac{C_c}{C_I C_{II}} \left|\langle 0_I|\hat{N}_I|1_I\rangle\langle 0_{II}|\hat{N}_{II}|1_{II}\rangle\right|}$$

A. Dewes et al., Phys. Rev. Lett. 108, 057002 (2012)



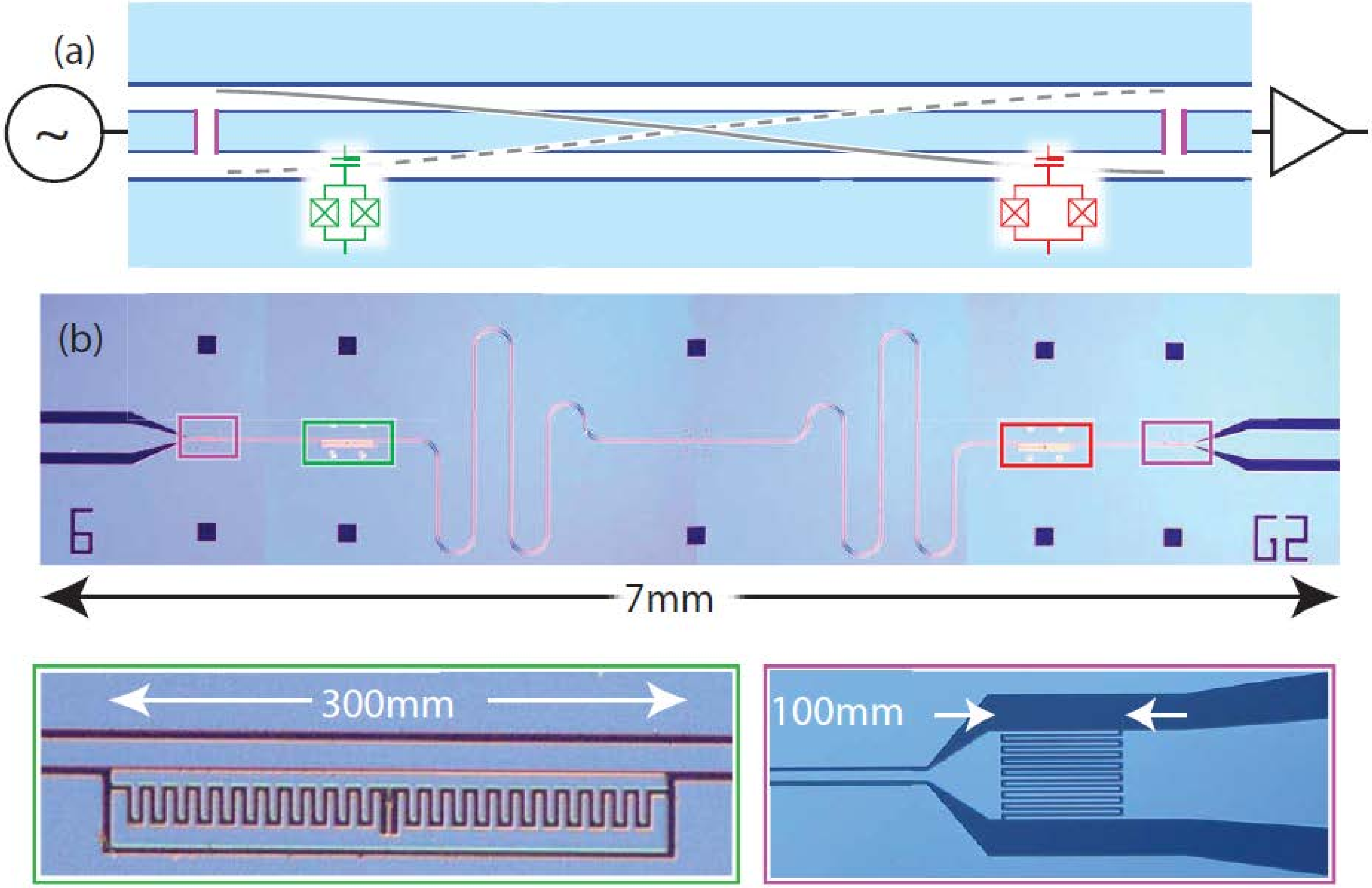
Coupling qubits
Quantum bus

Two qubits at opposite sides of the resonator
λ/2 mode

Different loop area – different EJ

Device parameters:

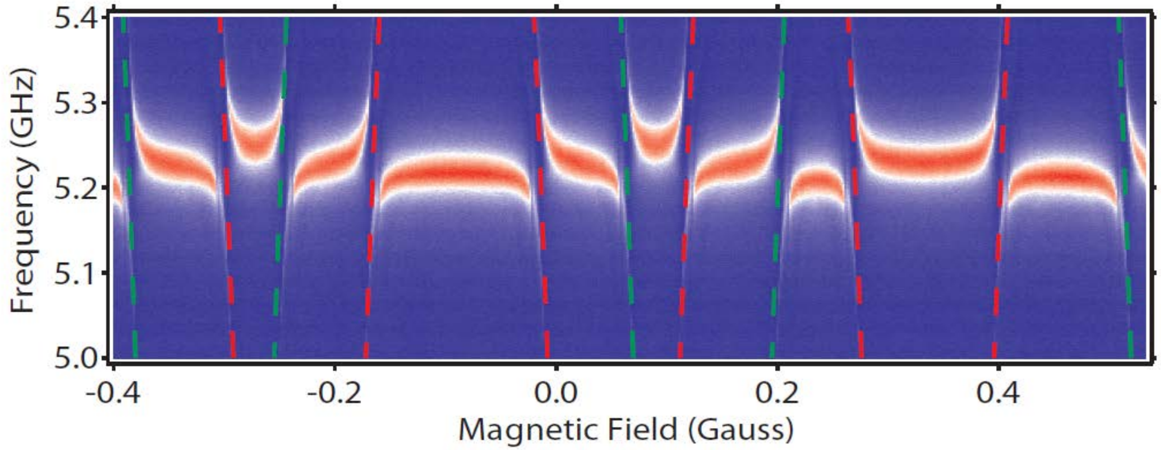
$$E_{C1}/h = 424\ \mathrm{MHz}$$
$$E_{C2}/h = 442\ \mathrm{MHz}$$
$$E_{J1}^{\max}/h = 14.9\ \mathrm{GHz}$$
$$E_{J2}^{\max}/h = 18.9\ \mathrm{GHz}$$



Resonator:

$$\omega_C/2\pi = 5.22\,\mathrm{GHz},\ \kappa/2\pi = 33\ \mathrm{MHz}$$

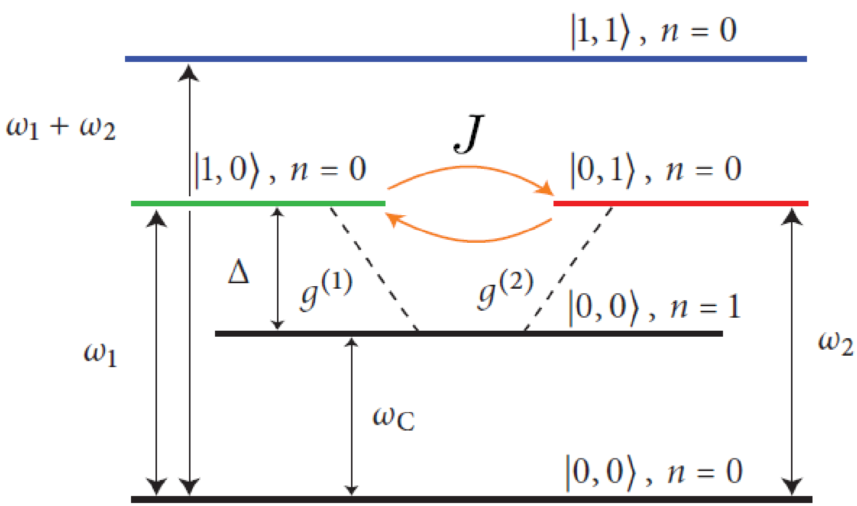
J. Majer et al., Nature 449, 443 (2007)
J. M. Chow Phd thesis



Single tone spectroscopy on resonator
Avoided crossings for both resonors suggest strong coupling – theory curve dashed lines
→ $g^{(1),(2)}/\pi$= 105 MHz

$$\hat{H} = \frac{1}{2}\hbar\omega_q^{(1)}\sigma_Z^1 + \frac{1}{2}\hbar\omega_q^{(2)}\sigma_Z^2 + \hbar\left(\omega_r + \chi^{(1)}\sigma_Z^{(1)} + \chi^{(2)}\sigma_Z^{(2)}\right)\hat{a}^\dagger\hat{a}$$
$$\boxed{+\hbar J\left(\sigma_-^{(1)}\sigma_+^{(2)} + \sigma_-^{(2)}\sigma_+^{(1)}\right)}$$

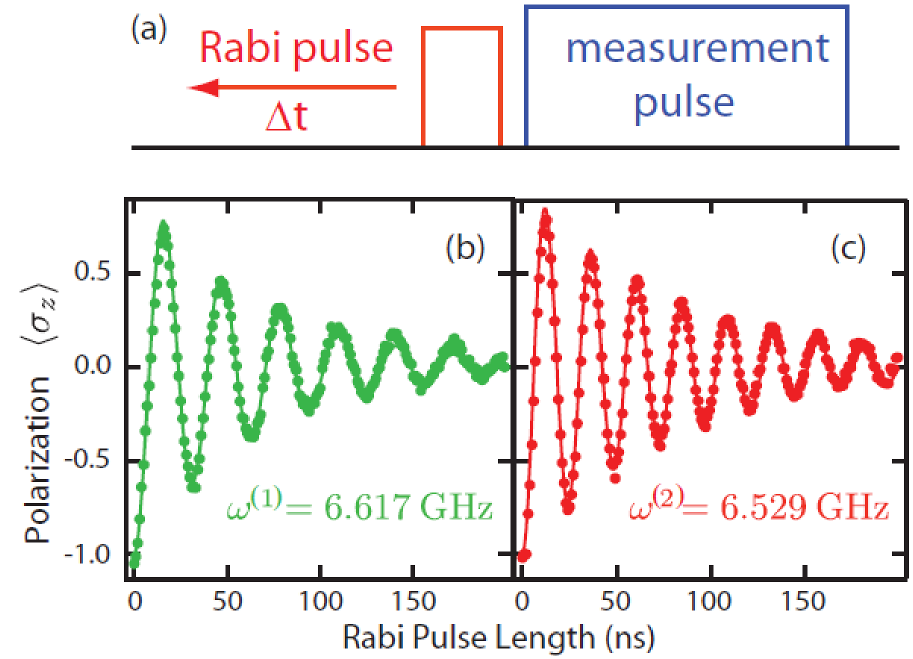
Lecture 2: Perturbation theory and in the rotating frame: 2 qubit + interaction term



$$J = \frac{g^{(1)}g^{(2)}}{2}\left(1/\Delta^{(1)} + 1/\Delta^{(2)}\right)$$
$$\left|\Delta^{(1),(2)}\right| = \left|\omega^{(1),(2)} - \omega_r\right| \gg g^{(1),(2)}$$

Virtual exchange of photons via the cavity with rate g1 and g2, if they are on resonce with each other (off to the cavity)

J. Majer et al., Nature 449, 443 (2007)
J. M. Chow Phd thesis



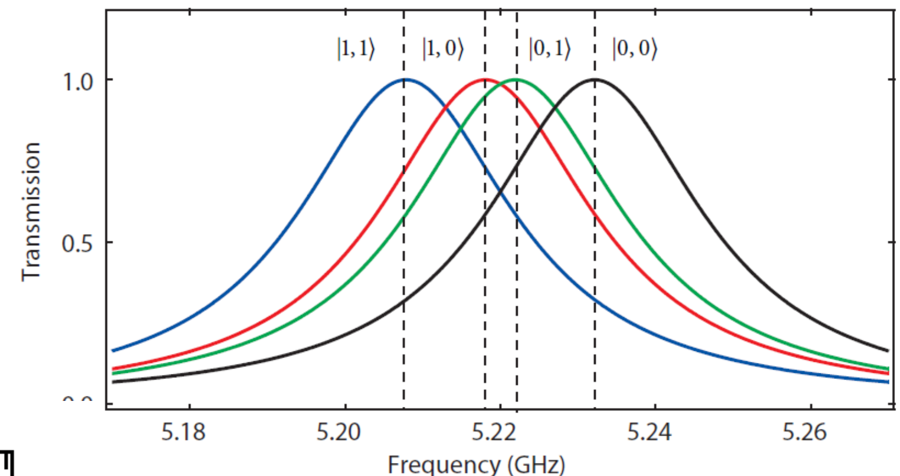
2 tone:

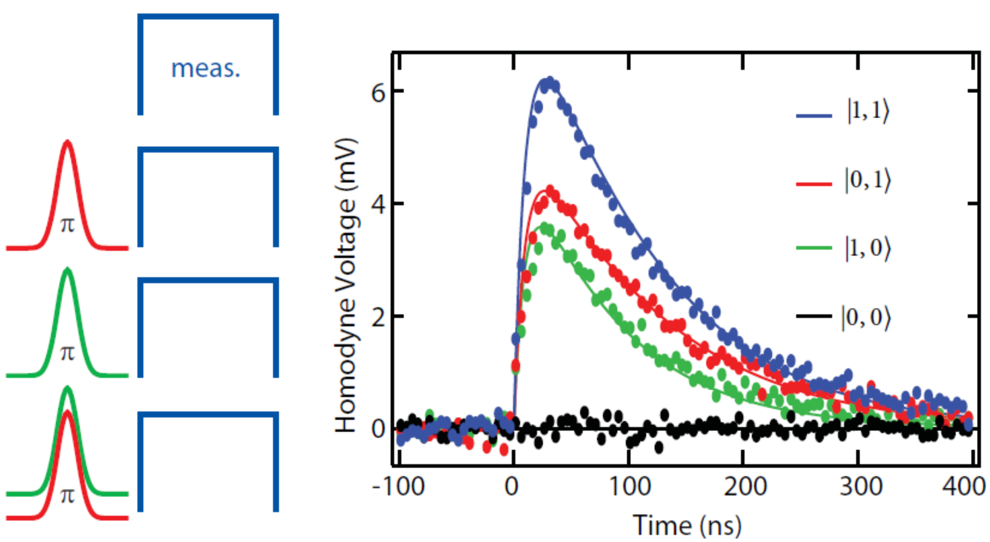
ω_S: qubit frequency (here continuous)

ω_{RF}: cavity frequency (here continuous)

The 2 qubits can be addressed separately if their frequncy is detuned

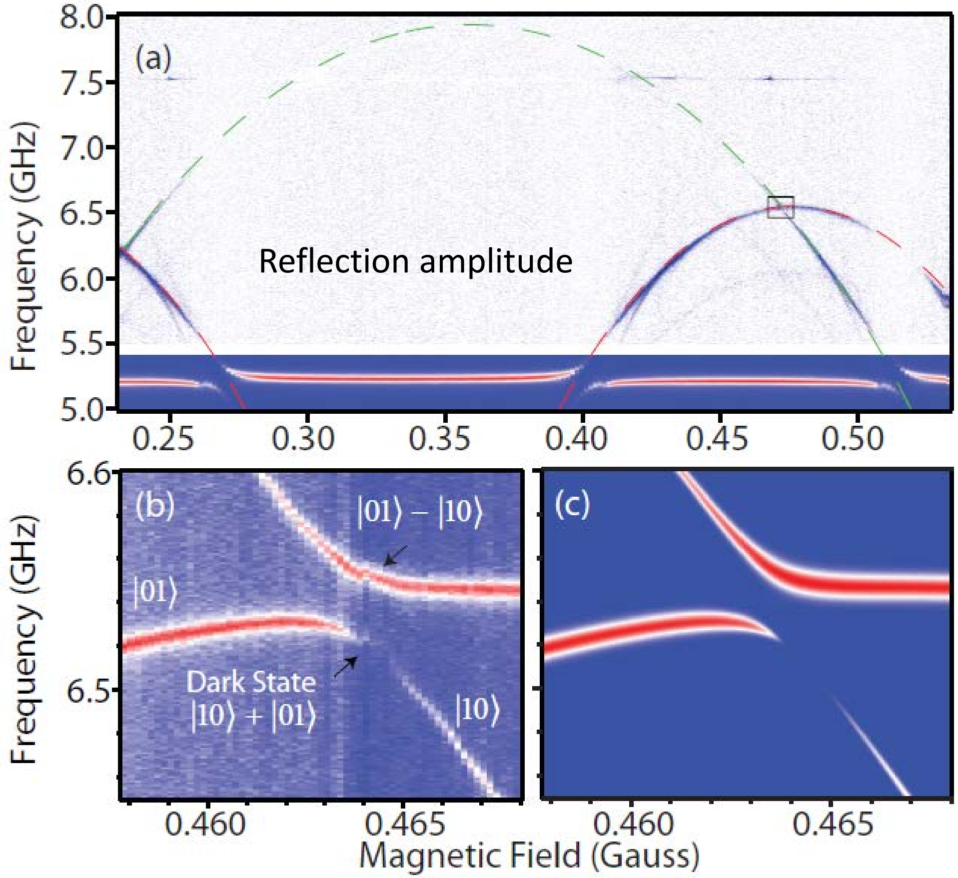
Separate characterization is possible





Dispersive readout: Due to the different parameters of the 2 qubit, all the states of the 2-qubit system can be read out with the cavity (different dispersive shift).

T1 and T2 can be measured.



2 tone:
ω_S: qubit frequency (here continuous)
ω_{RF}: cavity frequency (here continuous)

$$2J = 2g^{(1)}g^{(2)}/\Delta = 2\pi \cdot 26 \text{ MHz}$$

The qubit states hibridize with the cavity and also with each other if they are on resonance (dark state some interfence effect)

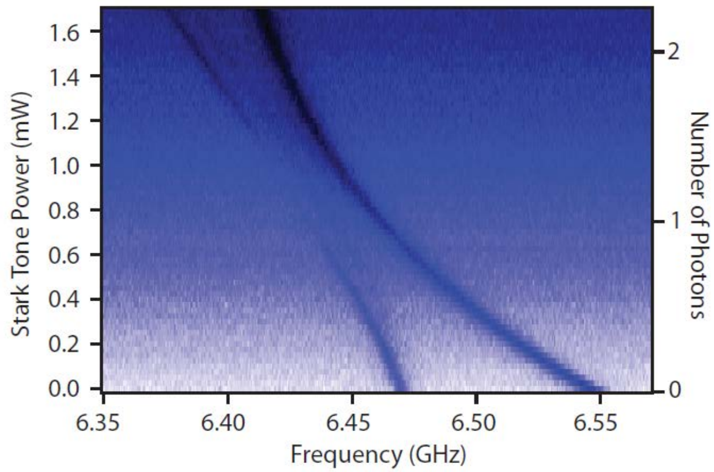
Global magnetic field – tunes both qubits.

J. Majer et al., Nature 449, 443 (2007)
J. M. Chow Phd thesis

$$\hat{H} = \frac{1}{2}\hbar\left(\omega_q^{(1)} + \chi^{(1)}\hat{a}^\dagger\hat{a}\right)\sigma_Z^1 + \frac{1}{2}\hbar\left(\omega_q^{(2)} + \chi^{(2)}\hat{a}^\dagger\hat{a}\right)\sigma_Z^2$$
$$+\hbar\omega_r\hat{a}^\dagger\hat{a} + \hbar J\left(\sigma_-^{(1)}\sigma_+^{(2)} + \sigma_-^{(2)}\sigma_+^{(1)}\right)$$

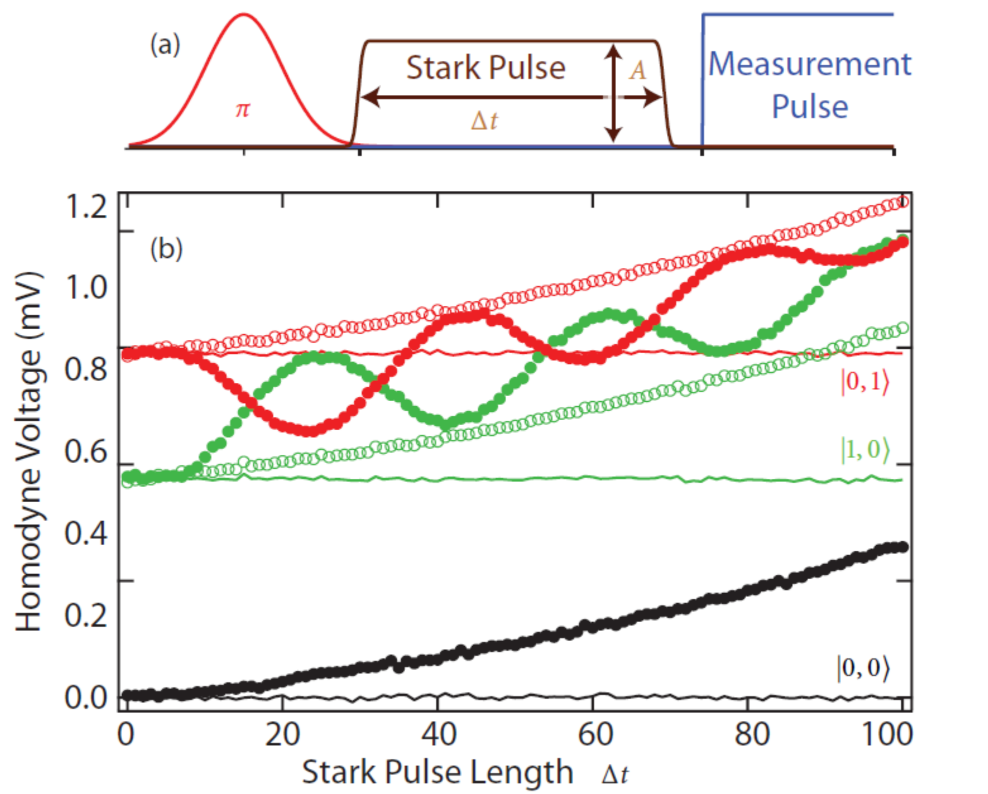
Flux biasing to avoided crossing
not fast enough
Use stark effect
2 qubits at 6.47 and 6.55 GHz -
close to resonance
Drive at 6.675 GHz

Avoided crossing using Stark effect



Size of the Stark shift depends on photon number and detuning

$$\chi = \frac{g^2}{\Delta}$$



Start at state 1,0 → pulse to the avoided crossing where the bonding and the antibonding states are the eigenstates.

The state evolves between 1,0 and 0,1. After Δt waiting time measurement of the state at the cavity frequency

SWAP operation

J. Majer et al., Nature 449, 443 (2007)
J. M. Chow Phd thesis

Density matrix

A single system is characterized by a wavefunction: $|\psi>=\sum_i a_i\,|\varphi_i>$ Superposition

Density matrix: $\rho=|\psi><\psi|$

Two systems are described on the direct product of the Hilbert spaces

$$|\psi>=\sum_{i,j} a_{i\,j}\,|\varphi_i>\otimes|\varphi_j>\in H_1\otimes H_2$$

Properties of the density matrix:

- Positive, hermetian, projector ($\rho^2=\rho$)
- $\mathrm{Tr}(\rho)=1$
- $\mathrm{Tr}(\rho A)=<A>$

If we switch on the interaction the two states can become **entangled**: $|\psi>\neq|\varphi_1>|\varphi_2>$
The state *can not* be written as a product of states from the two subsystems

Like the Bell singlet state: $|S>=\frac{1}{\sqrt{2}}(|\uparrow\downarrow>-|\downarrow\uparrow>)$ It can not be written as a product state

The density matrix for the singlet state:

$$\rho_S=\begin{array}{cc} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/2 & -1/2 & 0 \\ 0 & -1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{array}{c} |\downarrow\downarrow> \\ |\downarrow\uparrow> \\ |\uparrow\downarrow> \\ |\uparrow\uparrow> \end{array} \\ \begin{array}{cccc} |\downarrow\downarrow> & |\downarrow\uparrow> & |\uparrow\downarrow> & |\uparrow\uparrow> \end{array} & \end{array}$$

Pure system: it can be described with a single wave-function

Mixed system: cannot described by a single wave function, rather with probabilities

The system is in the state $|\psi_i>, i=1...N$ with a probability p_i

The density matrix can be still defined: $\rho = \sum_i p_i |\psi_i><\psi_i|$

In case of mixed states $Tr\rho^2 < 1$, for pure systems it is 1.

Mixed states can arise, when we investigate an entangled state on the subsystem.

We trace out for the second subsystem, qubit, and get the reduced density matrix for the first subsystem: $\rho_1 = Tr_2(\rho)$

For the spin-singlet case – using the definition of trace:

$\rho_1 = \frac{1}{2}(|\uparrow><\uparrow| + |\downarrow><\downarrow|) = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$ $Tr\rho_1^2 = 1/2 < 1$ Mixed state

The system cannot be described by one wave-function, the spin are either up or down (not absence of konwledge). It means from the state of the total system the state of subsystems can be derived, but this is not true reversely, in general.

Different as: $|\psi>= \frac{1}{\sqrt{2}}(|\uparrow> + |\downarrow>)$ $\rho = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} = \rho^2$ Pure state

Density matrix

Single qubit

$$\rho = \frac{1}{2}\sum_{k=0,x,y,z} r_k \sigma_k$$

r_k is the polarization vector

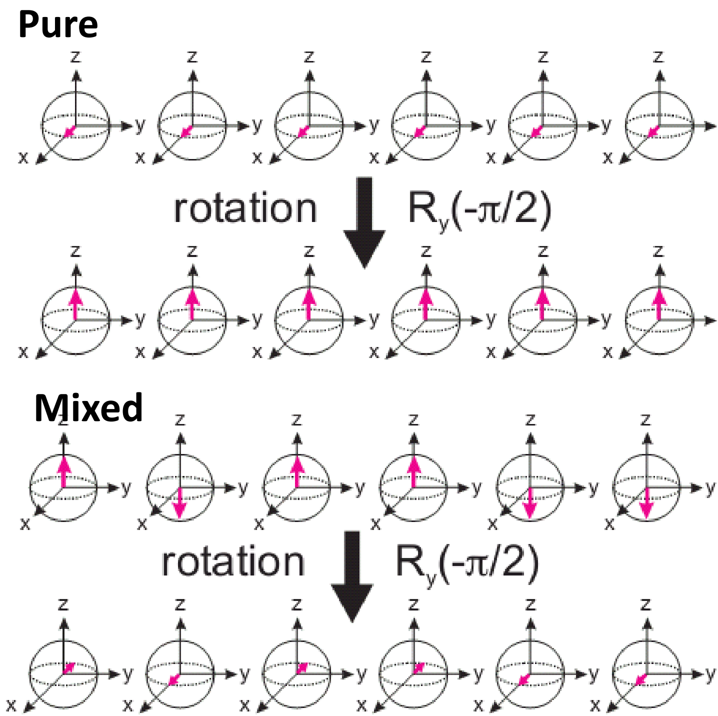
$$r = Tr(\rho\overline{\sigma}) = \langle \overline{\sigma} \rangle$$

$$Tr(\rho^2) = (1 + r^2)/2$$

Pure state |r|=1
Mixed |r|<1
(not on the Bloch-sphere)

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} = \frac{1}{2}\sum_{k=0,x,y,z} r_k \sigma_k$$

$$r_0 = \rho_{00} + \rho_{11} = 1$$
$$r_x = \rho_{01} + \rho_{10}$$
$$r_y = i(\rho_{01} - \rho_{10})$$
$$r_z = \rho_{00} - \rho_{11}$$



Idea: by projective measurements reconstruct the density matrix
Usually measurements are only possible in one direction (e.g. In z-direction).

$$P_0 = Tr(\rho\,|0><0|) = \frac{1}{2}(\sigma_0 + \sigma_z) = \frac{1}{2}(1 + r_z) = \rho_{00}$$

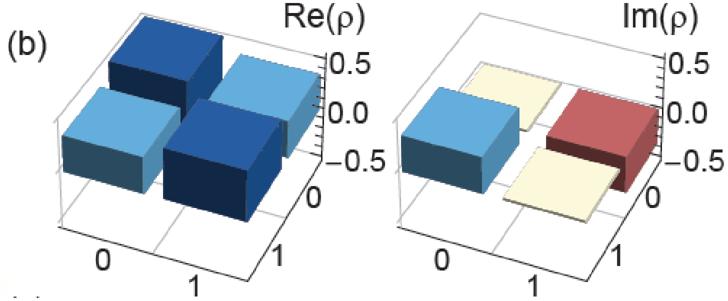
Other components, eg. r_x comes from the rotation around y by π/2 so, that ***x*** and ***z*** are interchanged – than measurement in z-basis

State Fidelity - how well the state was prepared (F [0,1]):

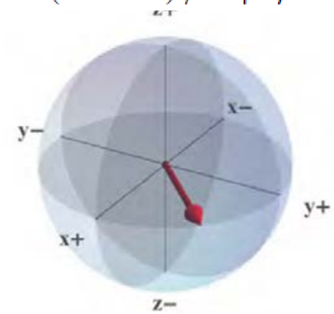
$$F = \mathrm{Tr}[\rho\,|\psi\rangle\langle\psi|]$$

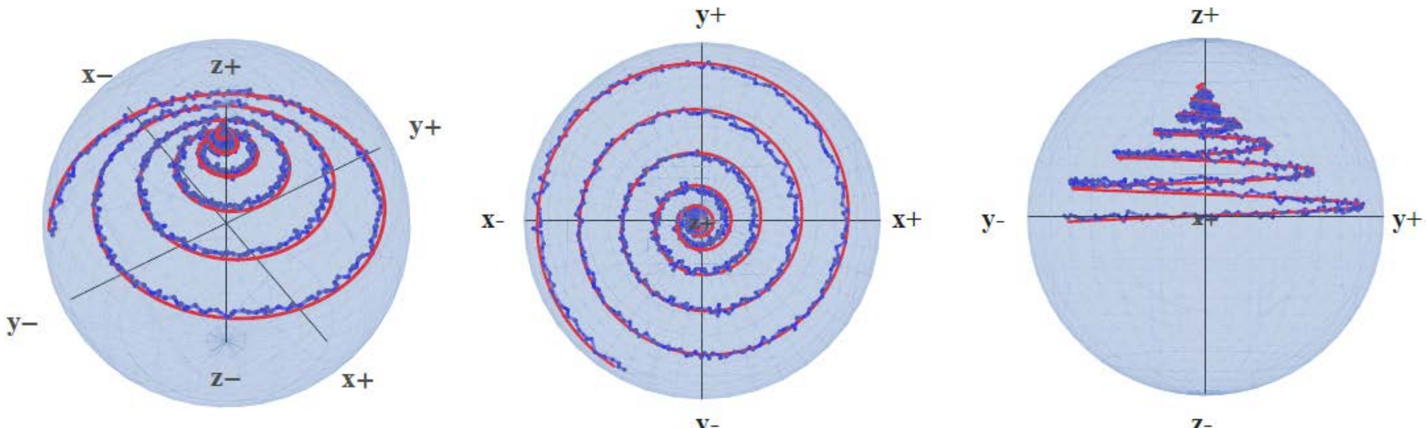
Included readout error, preparation, initialization error (and many measurements are needed to reach this limit – quantum measurements, and not statistical error should dominate).

If the readout is perfect, initialization is perfect can be used to characterize gate operation (***gate fidelity***) – calculating F with different initial states on the Bloch sphere and using the same gate operation. If the gate operation is unknown, can be reconstructed: ***process tomography.***



$$|\psi\rangle = 1/\sqrt{2}\,|0\rangle + (1+i)/2\,|1\rangle$$





Not on the Bloch sphere –mixed state

Example: Follow evolution of state
Start: in the equator with finite detuning (precess even in rotating frame)
Time dependence decreasing radius – *dephasing*
Z component build up – *relaxation*

Time evolution – Bloch equations from magnetism

$$\frac{dM_{x,y}}{dt} = \gamma[\mu \times B_0]_{x,y} - \frac{M_{x,y}}{T_2}$$

$$\omega_L = \gamma H_0$$

$$\frac{dM_z}{dt} = \gamma[\mu \times B_0]_z - \frac{M_0 - M_z}{T_1}$$

$$\frac{d\rho_{00}}{dt} = -\frac{d\rho_{11}}{dt} = -\Gamma_{\uparrow}\rho_{00} + \Gamma_{\downarrow}\rho_{11}$$

$$\frac{d\rho_{01}}{dt} = -iH_0\rho_{01} + \frac{\rho_{01}}{T_2}$$

$$\Gamma_{\uparrow} + \Gamma_{\downarrow} = \frac{1}{T_1}$$

$$\frac{d\rho_{00}}{dt} = -\frac{d\rho_{11}}{dt} = -\Gamma_{\uparrow}\rho_{00} + \Gamma_{\downarrow}\rho_{11}$$

T_1 enters in diagonal (relaxation)
T_2 (which includes T1) in the offdiagonal
(decoherence and relaxation)

$$\frac{d\rho_{01}}{dt} = -iH_0\rho_{01} + \frac{\rho_{01}}{T_2}$$

E.g. decoherence from flux noise, fluctuations in the magnetic field

$$|\Psi\rangle = \alpha|1\rangle + \beta|2\rangle \qquad \rho = \begin{pmatrix} \alpha^2 & \alpha^*\beta \\ \beta^*\alpha & \beta^2 \end{pmatrix}$$

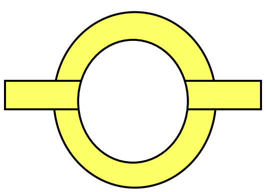
$$R_z(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

With Gaussian noise, with standard deviation α:

$$\rho_j = \int_{-\infty}^{\infty} R_z(\phi)|\psi_j><\psi_j|R_z^+(\phi)p(\phi)d\phi = \begin{pmatrix} |a|^2 & ab^*e^{-\alpha} \\ a^*be^{-\alpha} & |b|^2 \end{pmatrix} \qquad p(\phi) = (4\pi\alpha)^{-\frac{1}{2}}e^{-\frac{\phi^2}{4\alpha}}$$

decoherence!

Decoherence – offdiagonal elements



$$|\Psi\rangle = \alpha|1\rangle + \beta|2\rangle \qquad \rho = \begin{pmatrix} \alpha^2 & \alpha^*\beta \\ \beta^*\alpha & \beta^2 \end{pmatrix}$$

Transmission

$$Tr(\rho T) = \langle\Psi|\mathbf{T}|\Psi\rangle = |\alpha|^2\langle 1|\mathbf{T}|1\rangle + |\beta|^2\langle 2|\mathbf{T}|2\rangle + \underbrace{\alpha^*\beta\langle 1|\mathbf{T}|2\rangle + \beta^*\alpha\langle 2|\mathbf{T}|1\rangle}$$

Interference: The offdiagonal elements of density matrix describe the ability to interfere
The dissapperance of these elements show ***decoherence***

Tomography
2 qubit states

ρ: 4 x 4 matrix - 15 independent elements
Characterization: 15 measurements need

	Pre-rotation	Measurement operator
M_{01}	$I \otimes I$	$+\beta_{ZI}ZI + \beta_{IZ}IZ + \beta_{ZZ}ZZ$
M_{02}	$R_x^\pi \otimes I$	$-\beta_{ZI}ZI + \beta_{IZ}IZ - \beta_{ZZ}ZZ$
M_{03}	$I \otimes R_x^\pi$	$+\beta_{ZI}ZI - \beta_{IZ}IZ - \beta_{ZZ}ZZ$
M_{04}	$R_x^{\pi/2} \otimes I$	$+\beta_{ZI}YI + \beta_{IZ}IZ + \beta_{ZZ}YZ$
M_{05}	$R_x^{\pi/2} \otimes R_x^{\pi/2}$	$+\beta_{ZI}YI + \beta_{IZ}IY + \beta_{ZZ}YY$
M_{06}	$R_x^{\pi/2} \otimes R_y^{\pi/2}$	$+\beta_{ZI}YI - \beta_{IZ}IX - \beta_{ZZ}YX$
M_{07}	$R_x^{\pi/2} \otimes R_x^\pi$	$+\beta_{ZI}YI - \beta_{IZ}IZ - \beta_{ZZ}YZ$
M_{08}	$R_y^{\pi/2} \otimes I$	$-\beta_{ZI}XI + \beta_{IZ}IZ - \beta_{ZZ}XZ$
M_{09}	$R_y^{\pi/2} \otimes R_x^{\pi/2}$	$-\beta_{ZI}XI + \beta_{IZ}IY - \beta_{ZZ}XY$
M_{10}	$R_y^{\pi/2} \otimes R_y^{\pi/2}$	$-\beta_{ZI}XI - \beta_{IZ}IX + \beta_{ZZ}XX$
M_{11}	$R_y^{\pi/2} \otimes R_x^\pi$	$-\beta_{ZI}XI - \beta_{IZ}IZ + \beta_{ZZ}XZ$
M_{12}	$I \otimes R_x^{\pi/2}$	$+\beta_{ZI}ZI + \beta_{IZ}IY + \beta_{ZZ}ZY$
M_{13}	$R_x^\pi \otimes R_x^{\pi/2}$	$-\beta_{ZI}ZI + \beta_{IZ}IY - \beta_{ZZ}ZY$
M_{14}	$I \otimes R_y^{\pi/2}$	$+\beta_{ZI}ZI - \beta_{IZ}IX - \beta_{ZZ}ZX$
M_{15}	$R_x^\pi \otimes R_y^{\pi/2}$	$-\beta_{ZI}ZI - \beta_{IZ}IX + \beta_{ZZ}ZX$

A possible set of measurements to obtain the density matrix
β: non ideality factors taking into account non-ideality of the readout – should be taken as 1 for simplicity

Measurement – measure the population of |0,0> state

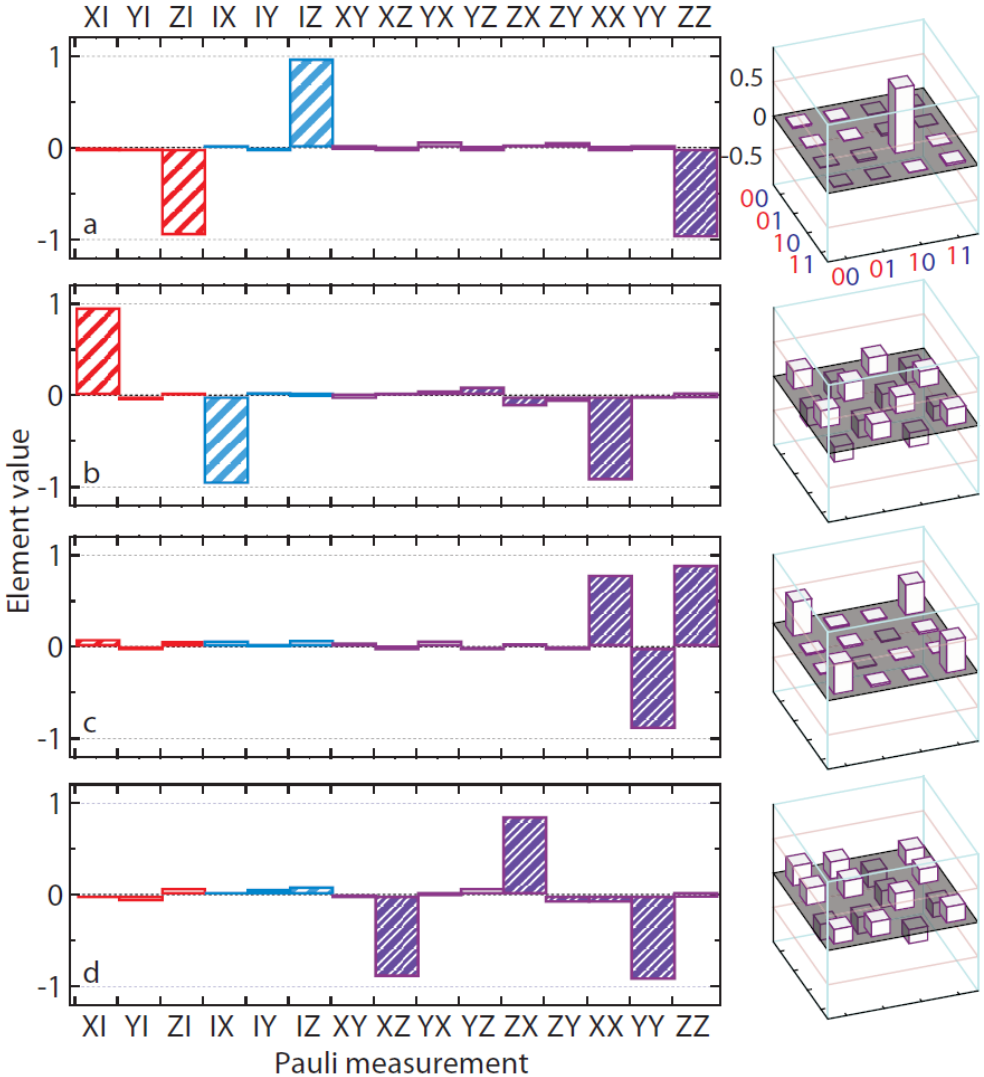
Pre-rotation: operation done before measurement

Measurement operator: the density matrix of rotated state to be measured

Note: *I* matrix has to added to obtain ρ density matrix (more precisely 4 ρ)

E.g. M01: |0,0><0,0|;
M02: |1,0><1,0|
But M04 contains combinations of the elements of the density matrix

From all these measurements the elements of the density matrix can be calculated



Pauli representation of the final state:
Polarization vector for 2-qubit state

E.g. |10> state
The density matrix can be written as
1/4 (II-ZI+IZ-ZZ)
Separable state

|00>+|11>: highly entangled state

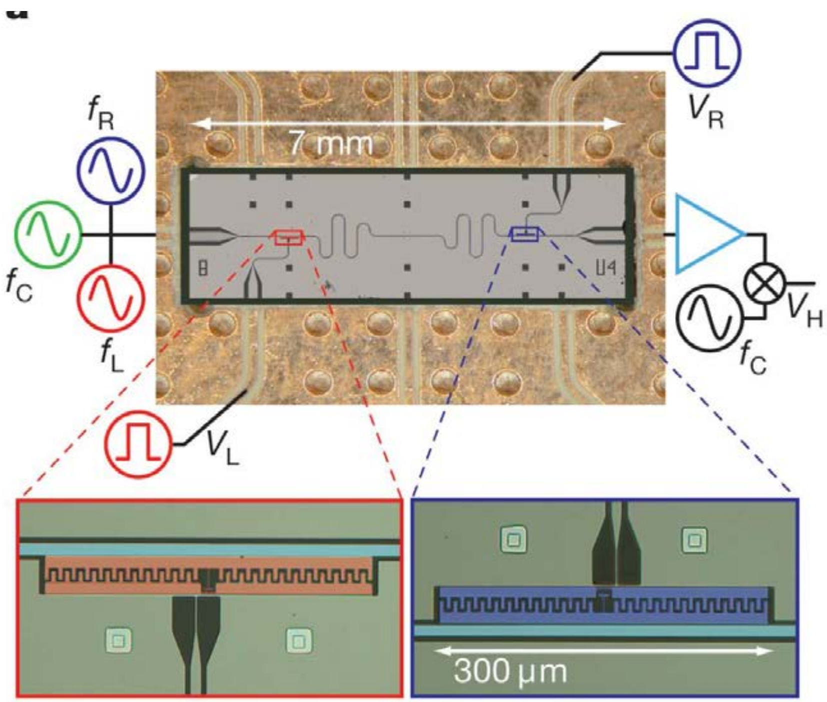
3 sectors

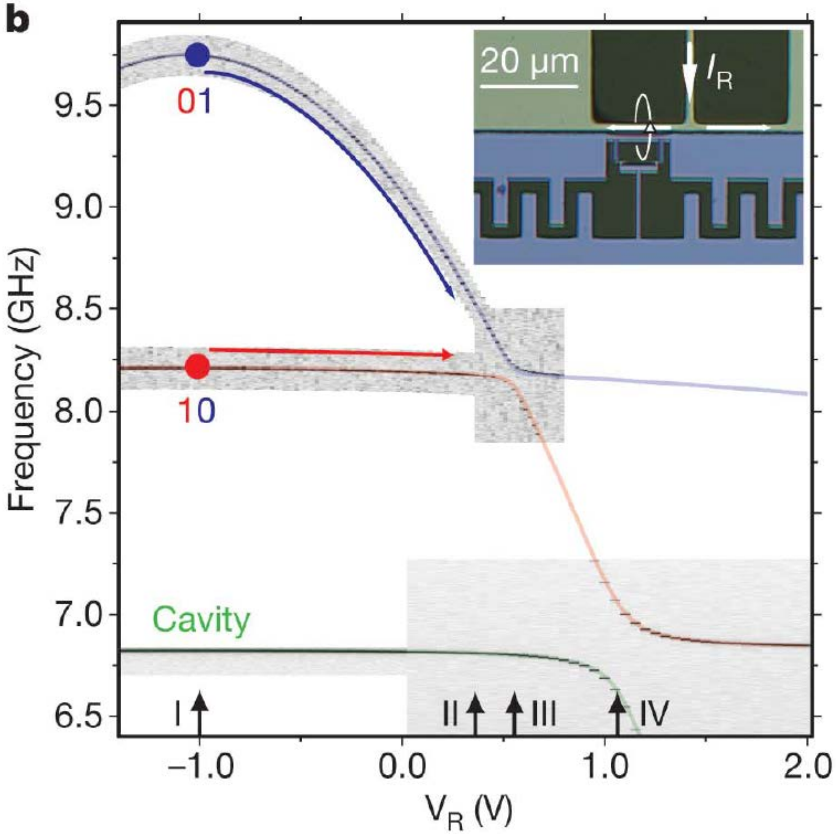
- Left qubit* I, I*Right qubit, Qubit-Qubit correlation

Claim:
For pure states weight ~ 1 in all 3 sector
For entangled: most weight in qubit-qubit correlation

Coupling qubits
Coupling transmons

Individual flux lines for controlling the qubit itself (not via Stark shift)





Point I

Far detuning – effectively decoupled states L and R qubit can be addressed separately

Computational states

00 - GS

10 - L excited

01 - R excited

11 - both excited

μs lifetimes of individual qubits

Point IV

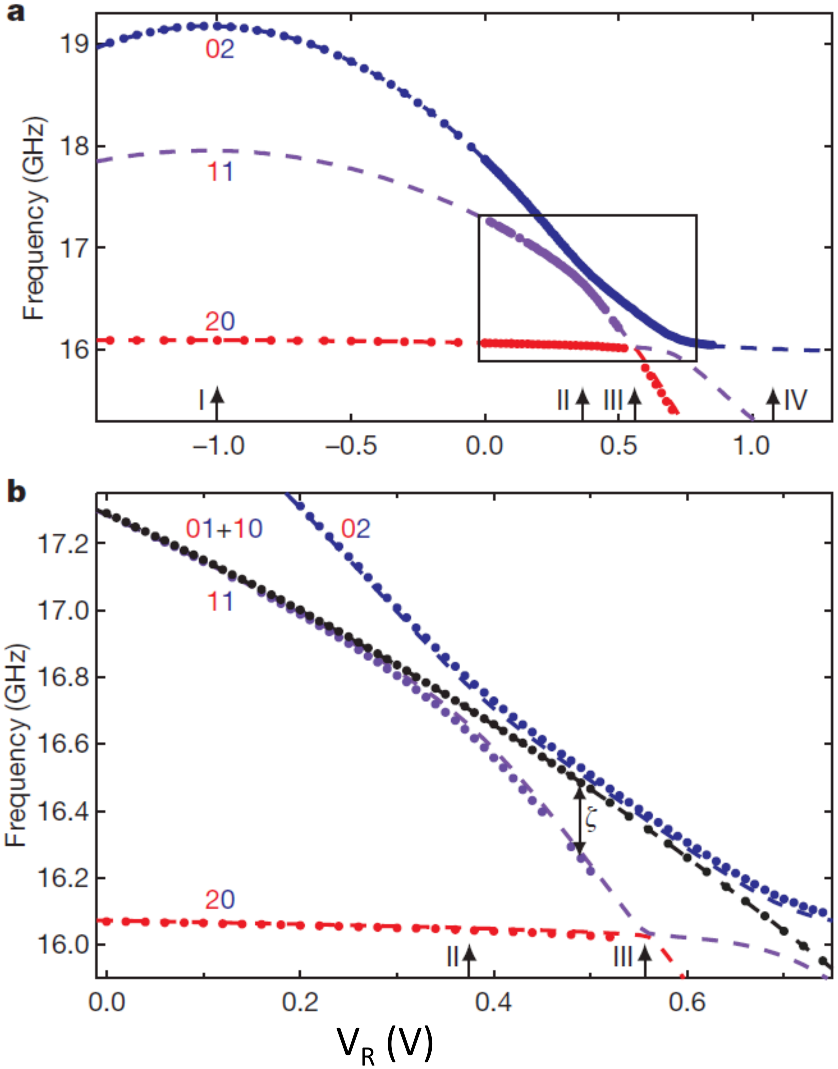
Cavity – qubit strong coupling

Point III

Qubit-qubit coupling via cavity (2nd order perturbation as seen previously)

Point II

Point of operation



Point II

Point of operation
Transmon: higher levels can also play a role
02 state also becomes important
Should cross with 11 at point II, however avoided crossing is seen.
f_{11} should be $f_{10} + f_{01}$, but lowered with $\zeta/2\pi$

c-Phase gate can be implemented with this

$$\zeta \sigma_z^1 \otimes \sigma_z^2$$

Usually small interaction, however using second levels can be enhanced, when becomes close to resonant

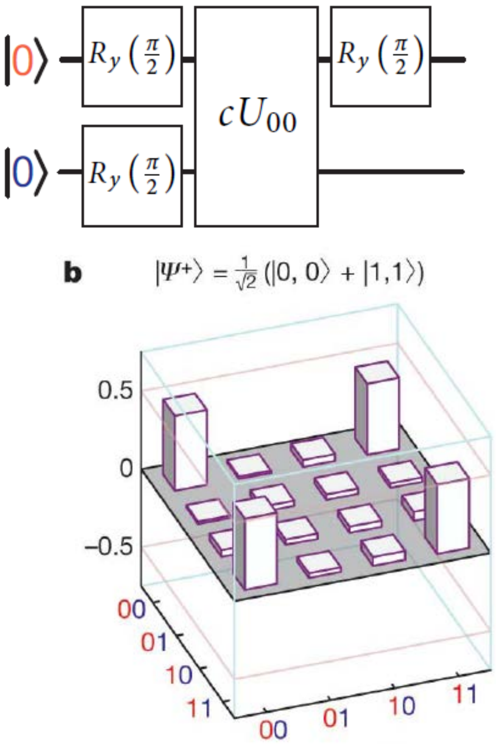
$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\phi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\phi_{10}} & 0 \\ 0 & 0 & 0 & e^{i\phi_{11}} \end{pmatrix} \qquad \theta_z^{ij} = \int \delta\omega_{ij}(t)\,dt$$

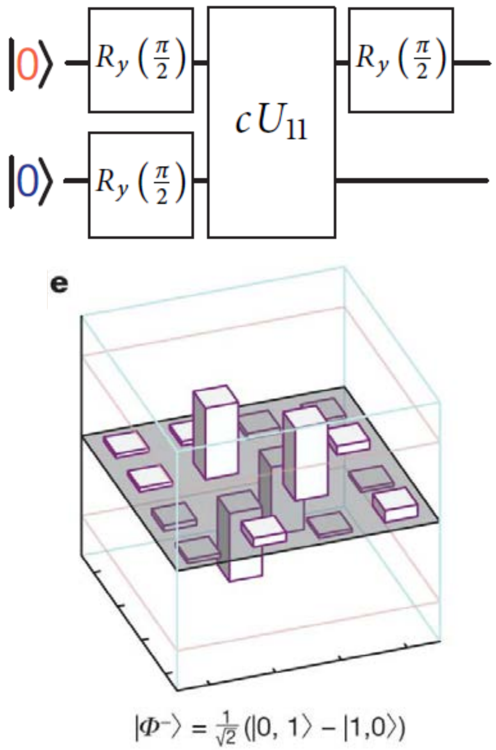
$$\int_0^{t_f} \zeta(t)\,dt = (2n+1)\pi \qquad \theta_z^{01} = \theta_z^{10} = 0$$

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Phase gate
Adjust single qubit phase gates – adiabatic pulses are fine
Measure ζ by spectroscopy or by Ramsey of L for 10 and 11

Entanglement with C-Phase





F~0.91

F~0.87

Re part

Different C-phase gates – tuning the sign of Φ_{01} and Φ_{10}

Imaginary part of density matrix is small (≤0.05)

Tomography:

Measure the elements of the density matrix, using 00 measurements and single qubit rotations

$$F(\rho,\psi) = \langle\psi|\rho|\psi\rangle$$

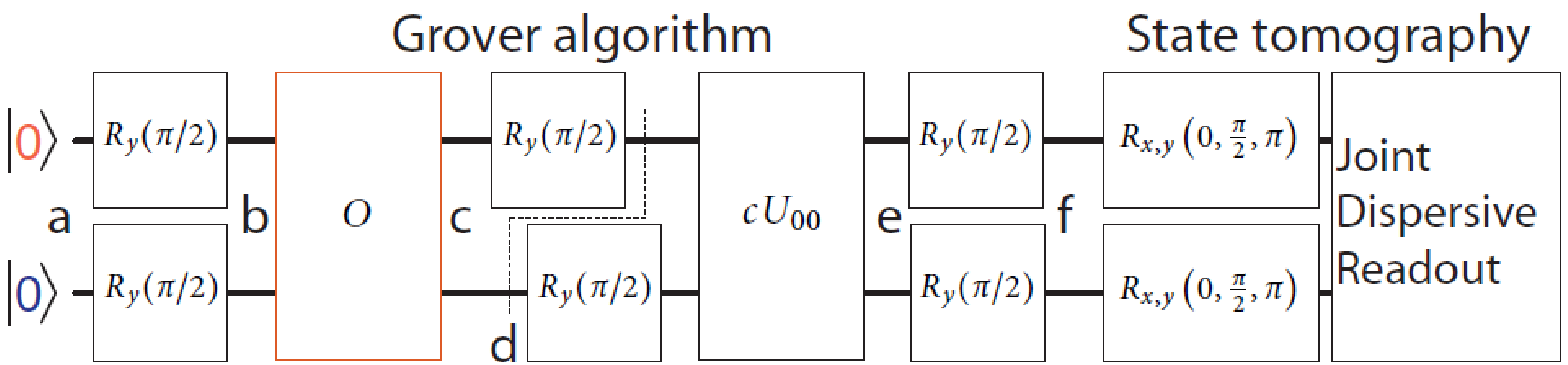
Grover search algorithm

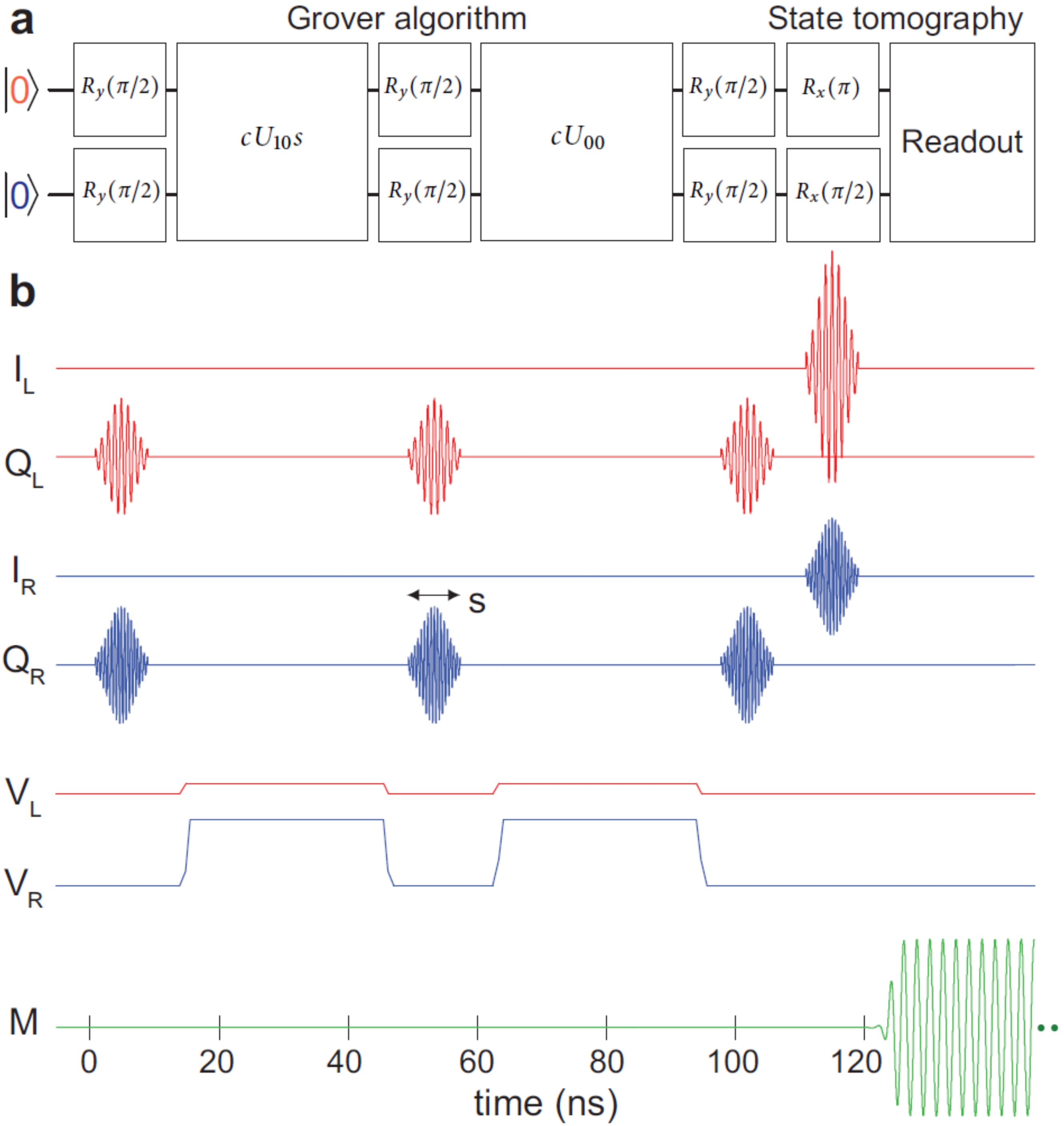
<u>Motivation: find a given name in an unordered list of N</u>
Classical: ~N trial
Quantum ~ sqrt(N)

Grover: $N=2^n$, can be represented with basis states: e.g. N=4: 00,01,10,11
Oracle: O operator: recognizes the solution

$$O|x\rangle = (-1)^{f(x)}|x\rangle$$

state is marked – still has to be read out
For more than 4 states, iteration of these operations are needed



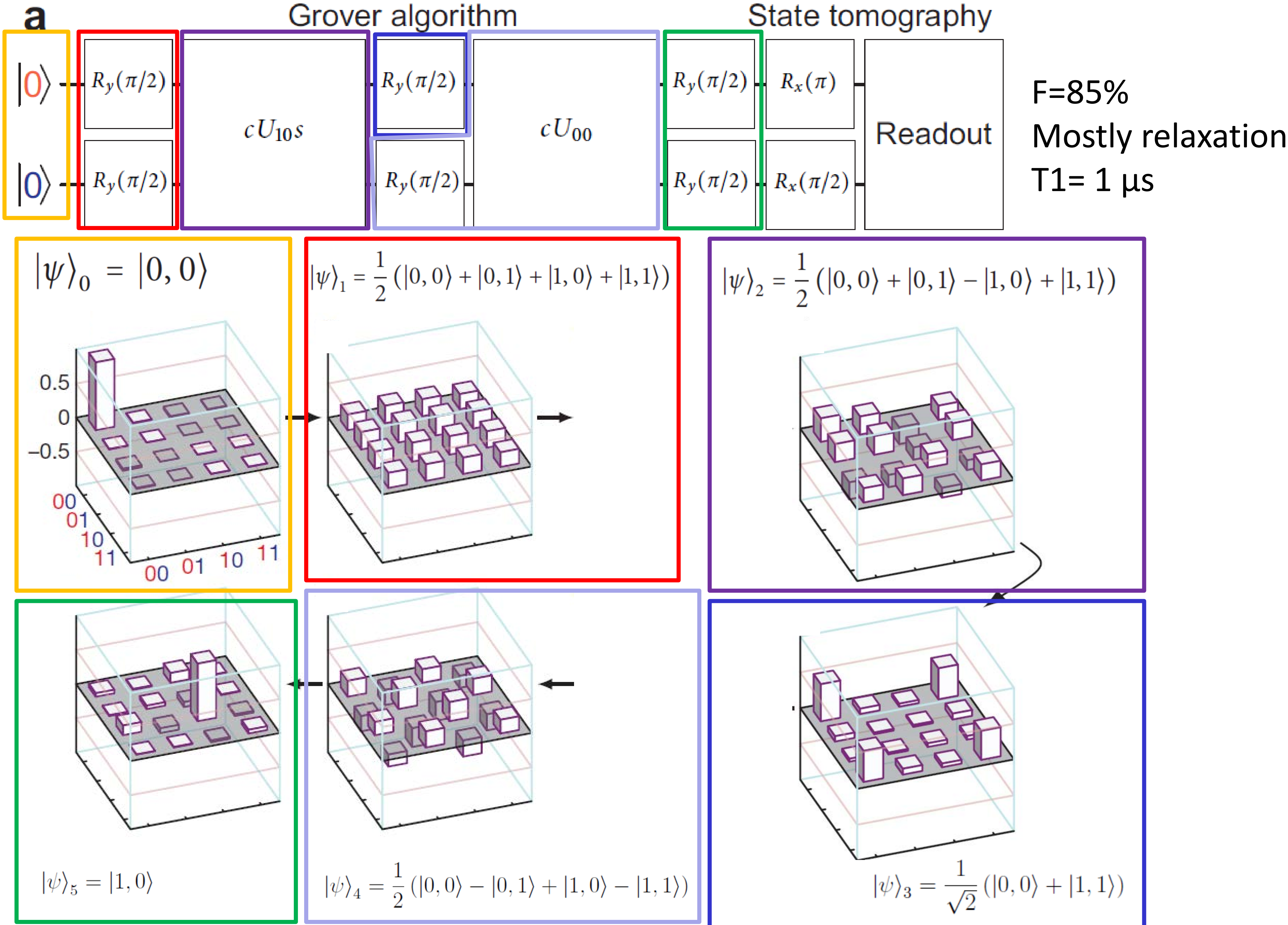


<u>Realization of Gorver algorithm</u> searching for 10

Pulse sequency (I, Q – 90 degrees phase shift) These are the single qubit rotations

Phase gates to make the qubits interact

Readout tone


a
Grover algorithm
State tomography
$|0\rangle$
$|0\rangle$
$R_y(\pi/2)$
$R_y(\pi/2)$
cU_{10s}
$R_y(\pi/2)$
$R_y(\pi/2)$
cU_{00}
$R_y(\pi/2)$
$R_y(\pi/2)$
$R_x(\pi)$
$R_x(\pi/2)$
Readout
F=85%
Mostly relaxation
T1= 1 μs
$|\psi\rangle_0 = |0,0\rangle$
0.5
0
−0.5
00
01
10
11
00 01 10 11
$|\psi\rangle_1 = \frac{1}{2}(|0,0\rangle + |0,1\rangle + |1,0\rangle + |1,1\rangle)$
$|\psi\rangle_2 = \frac{1}{2}(|0,0\rangle + |0,1\rangle - |1,0\rangle + |1,1\rangle)$
$|\psi\rangle_5 = |1,0\rangle$
$|\psi\rangle_4 = \frac{1}{2}(|0,0\rangle - |0,1\rangle + |1,0\rangle - |1,1\rangle)$
$|\psi\rangle_3 = \frac{1}{\sqrt{2}}(|0,0\rangle + |1,1\rangle)$