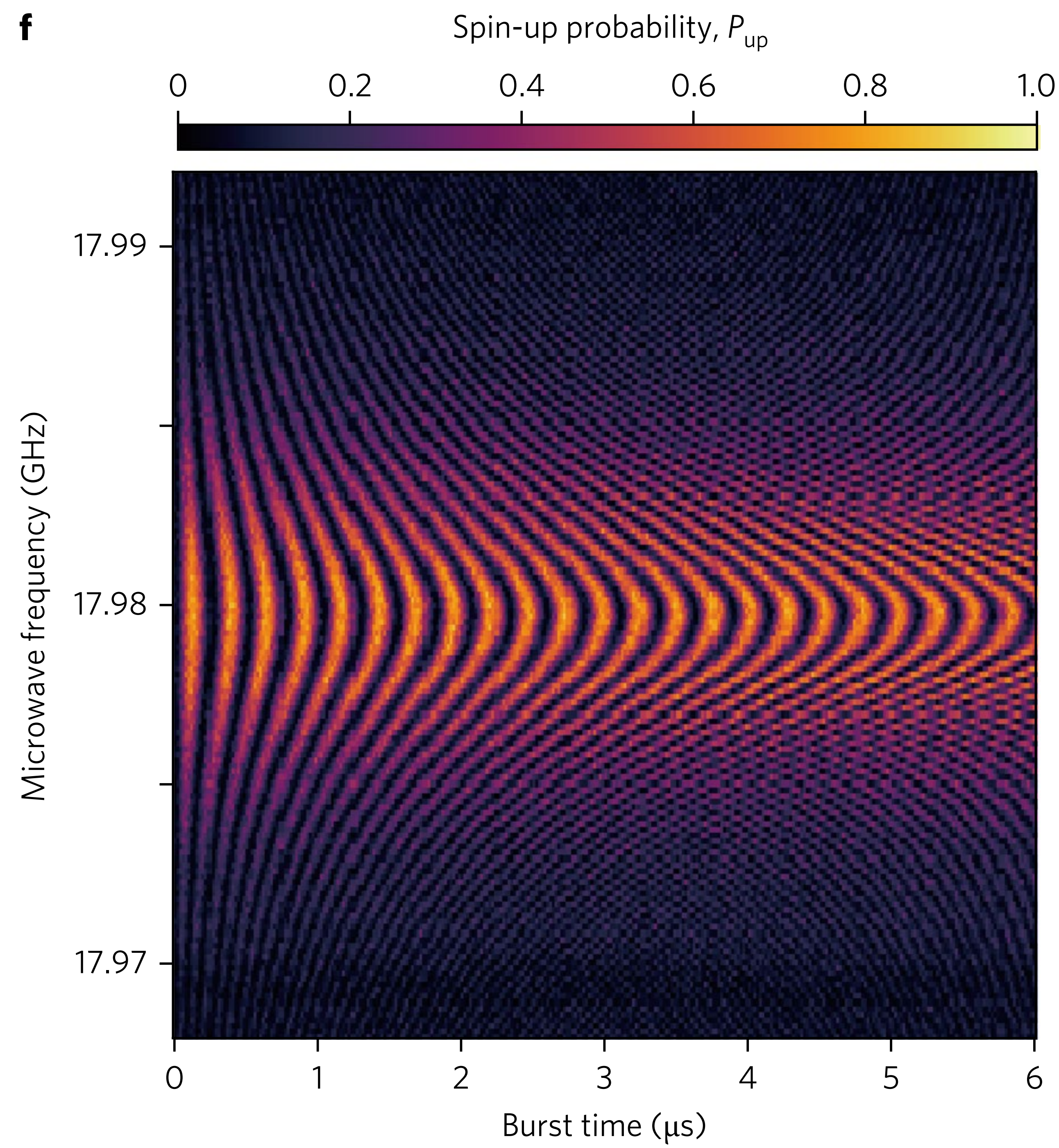
Quantum Computing Architectures

Budapest University of Technology and Economics
2018 Fall

Lecture 4

Coherent control of electron spins



Schedule of this course

Szerda		
augusztus 29. - Regisztrációs hét -		
szeptember 5.	lecture 01	**Introduction**
szeptember 12.		
szeptember 19.	lecture 02	
szeptember 26. TTK Dékáni szünet		
október 3.	lecture 03	**Spin qubits** (electron spin)
október 10.		
október 17.	lecture 04 (today)	
október 24.	lecture 05	
október 31.	lecture 06	**Superconducting qubits** (transmon)
november 7.	lecture 07	
november 14. TDK konferencia		
november 21.	lecture 08	
november 28.	lecture 09	
december 5.	lecture 10	

(Spin) Qubit Checklist

1. make a few qubits
2. initialize
3. control (1-qubit gate, 2-qubit gate)
4. read out
5. understand and reduce information loss

today

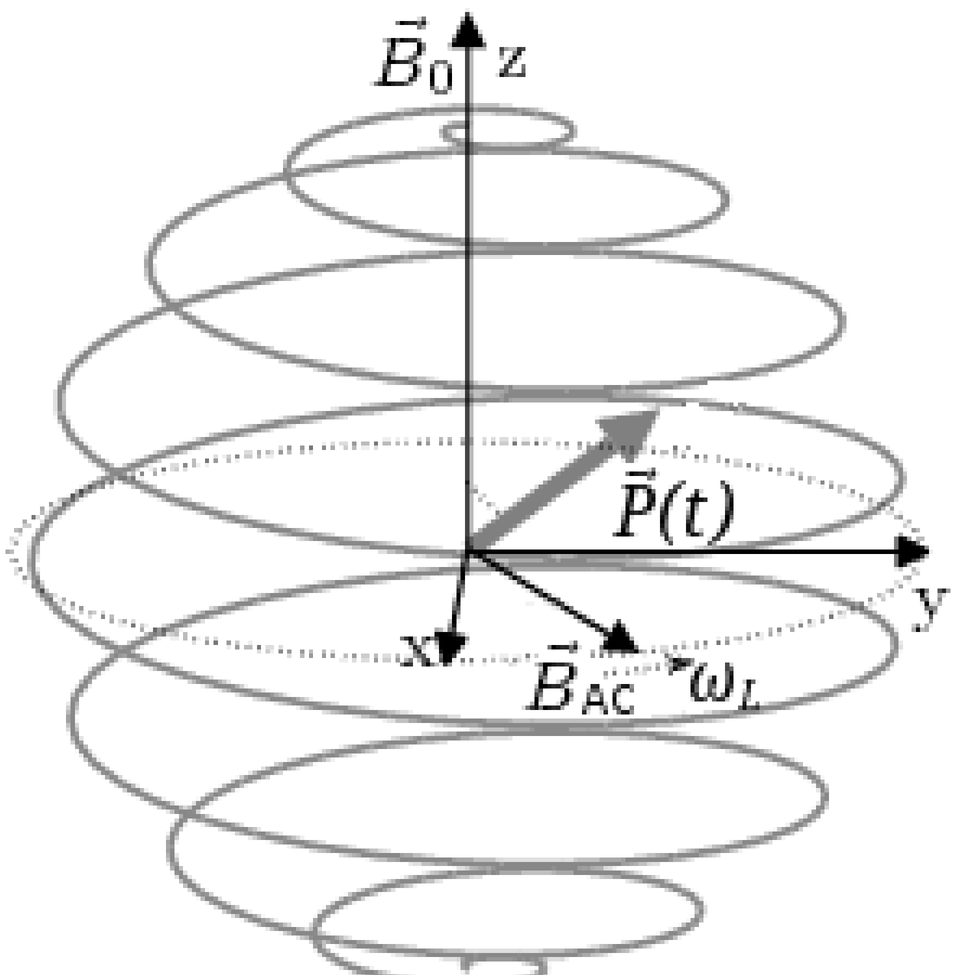
lecture 3

review papers: Hanson et al., Rev. Mod. Phys. (2007), Zwanenburg et al., Rev. Mod. Phys. (2013)

this is a slide from lecture 2

Spin resonance (linear drive)

$$H(t) = \frac{1}{2} g\mu_B B_0 \sigma_z + \frac{1}{2} g\mu_B B_{\text{ac}} \sigma_x \cos \omega t$$



weak driving:

$$\Omega \ll \omega_L$$

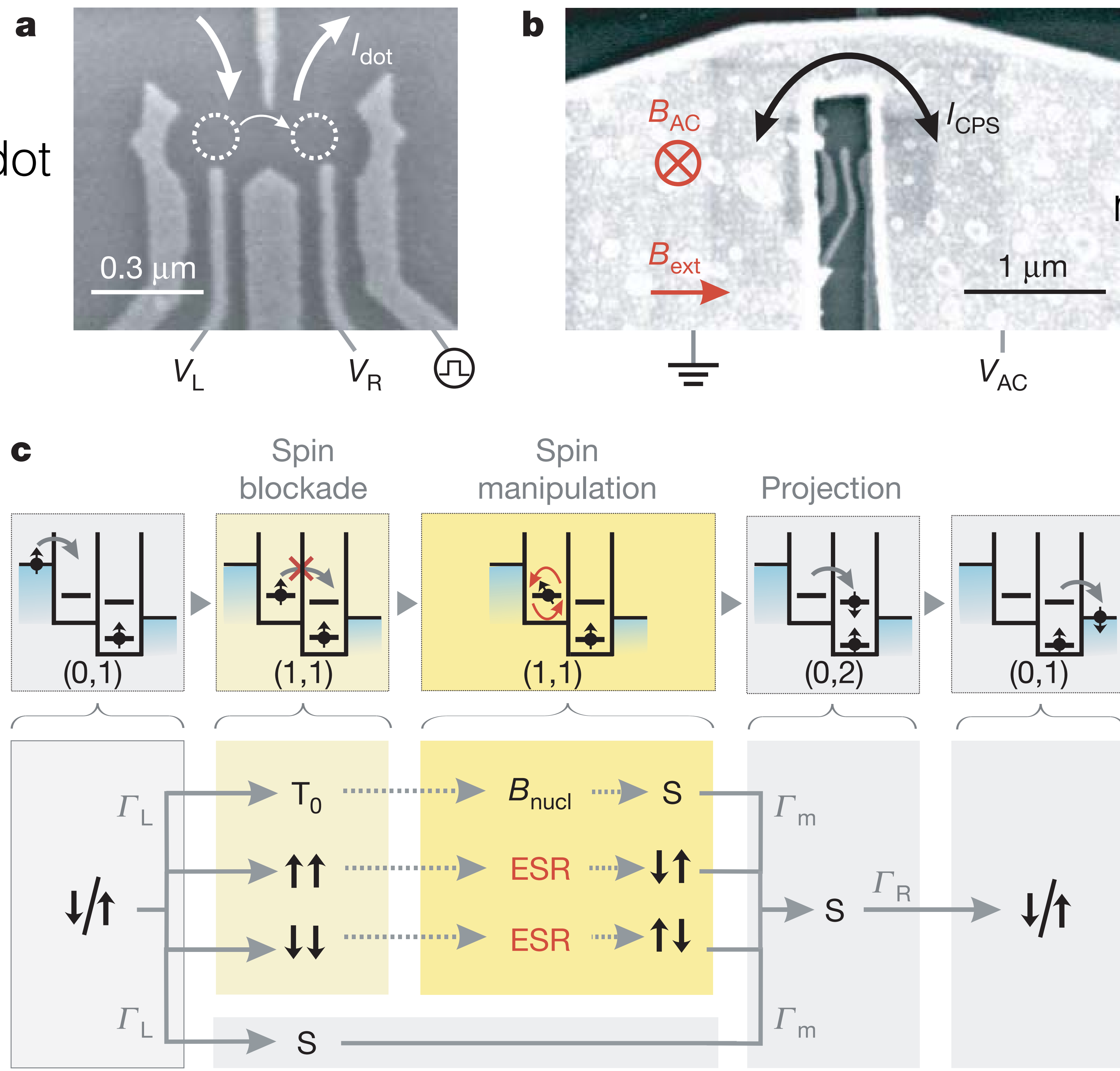
for weak driving, the qubit dynamics is approximately the same as with rotating drive

most experiments use linear drive (simpler)

Demonstration of single-electron spin resonance

double quantum dot (DQD)

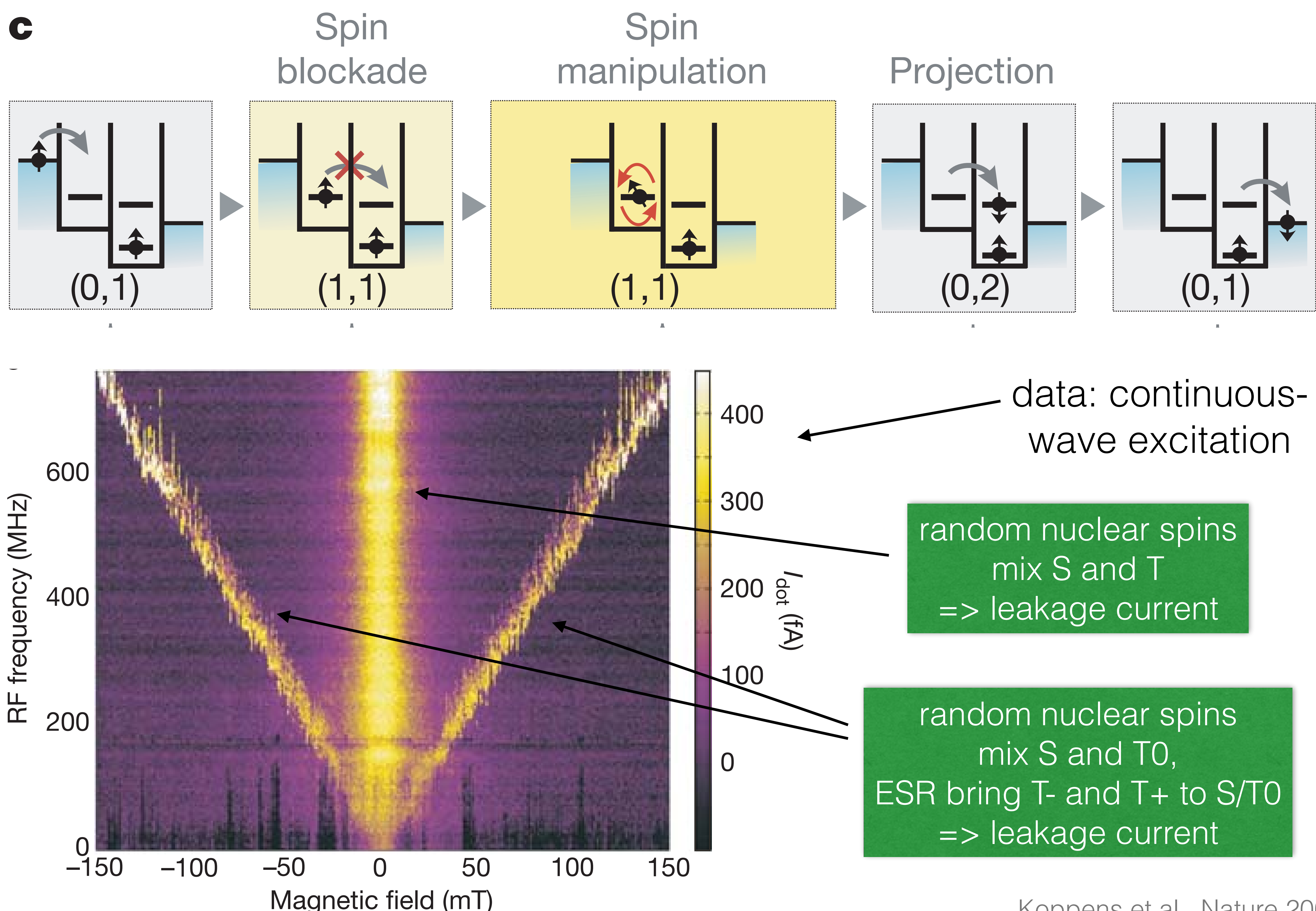
DQD covered by a wire to make ac B-field



DC transport through the DQD (Pauli blockade) is used for readout

Koppens et al., Nature 2006

Electron spin dynamics is revealed by increased current



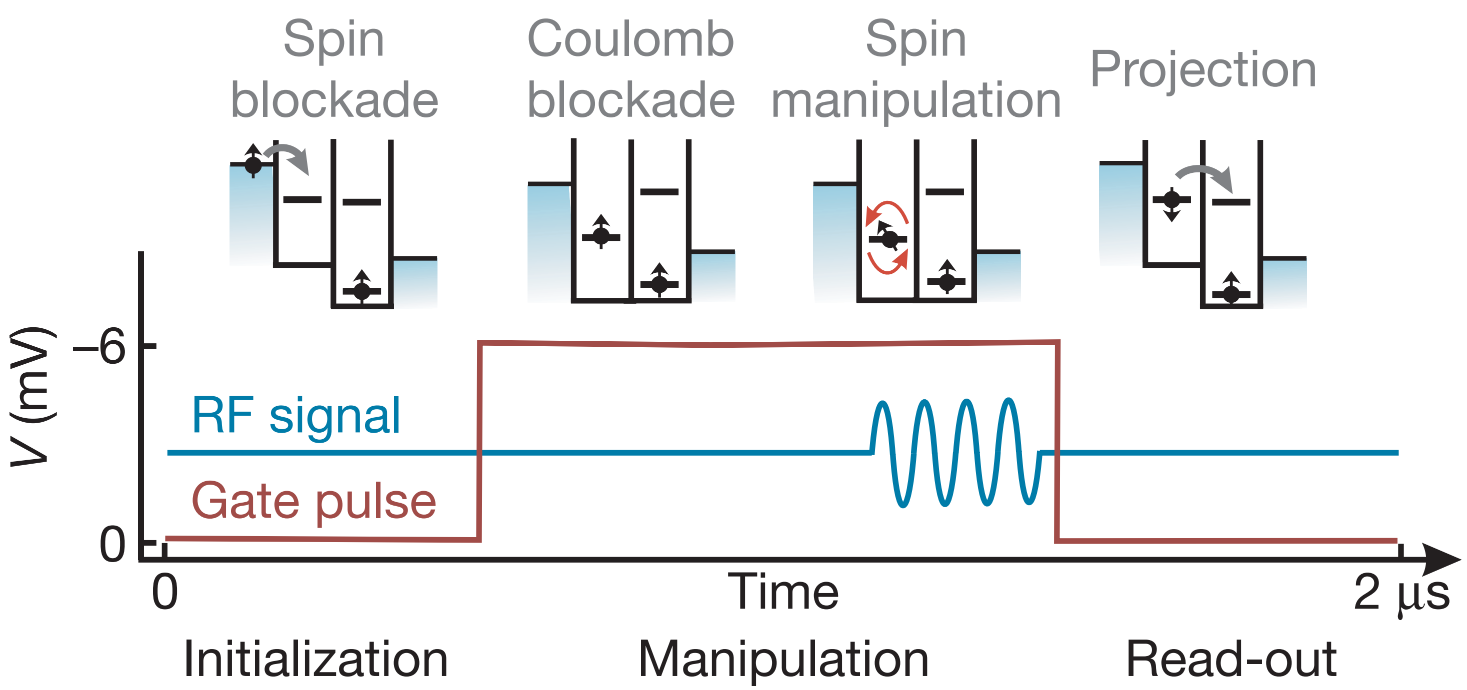
data: continuous-wave excitation

random nuclear spins mix S and T => leakage current

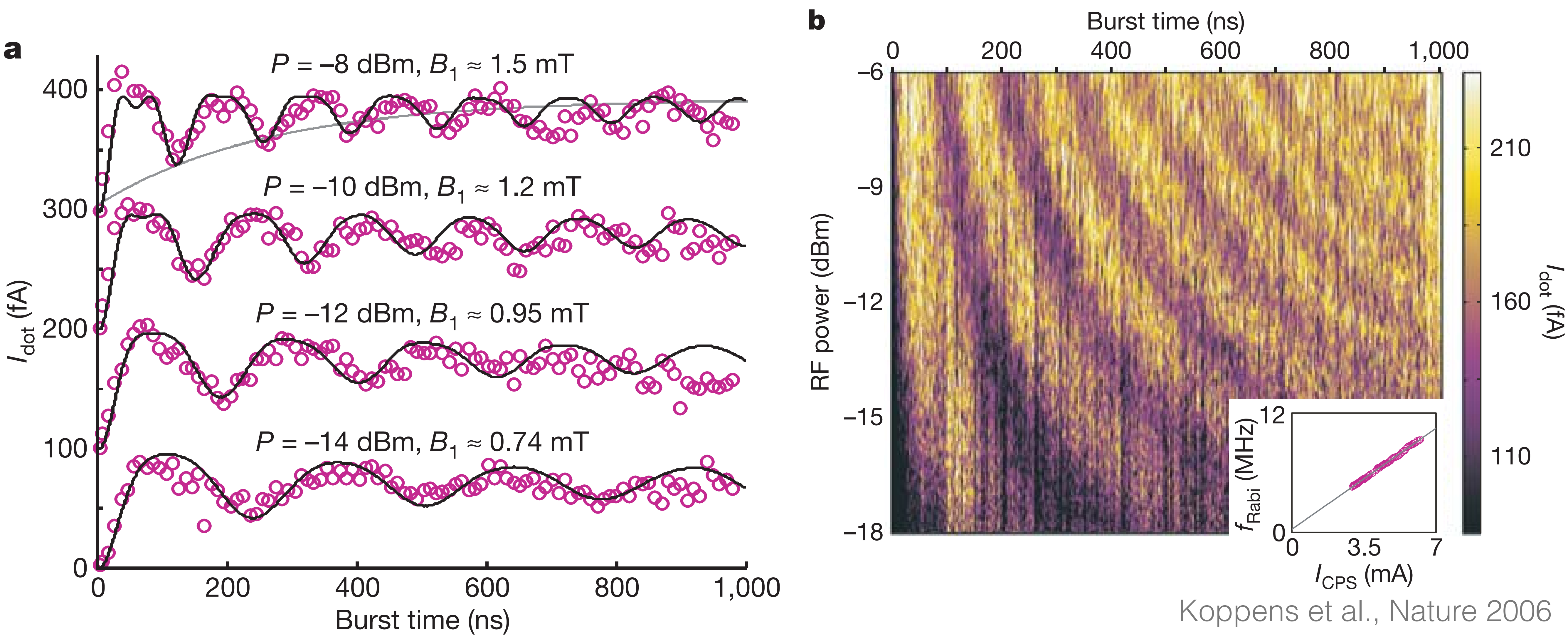
random nuclear spins mix S and T0, ESR bring T- and T+ to S/T0 => leakage current

Koppens et al., Nature 2006

Time-resolved spin dynamics: Rabi oscillations

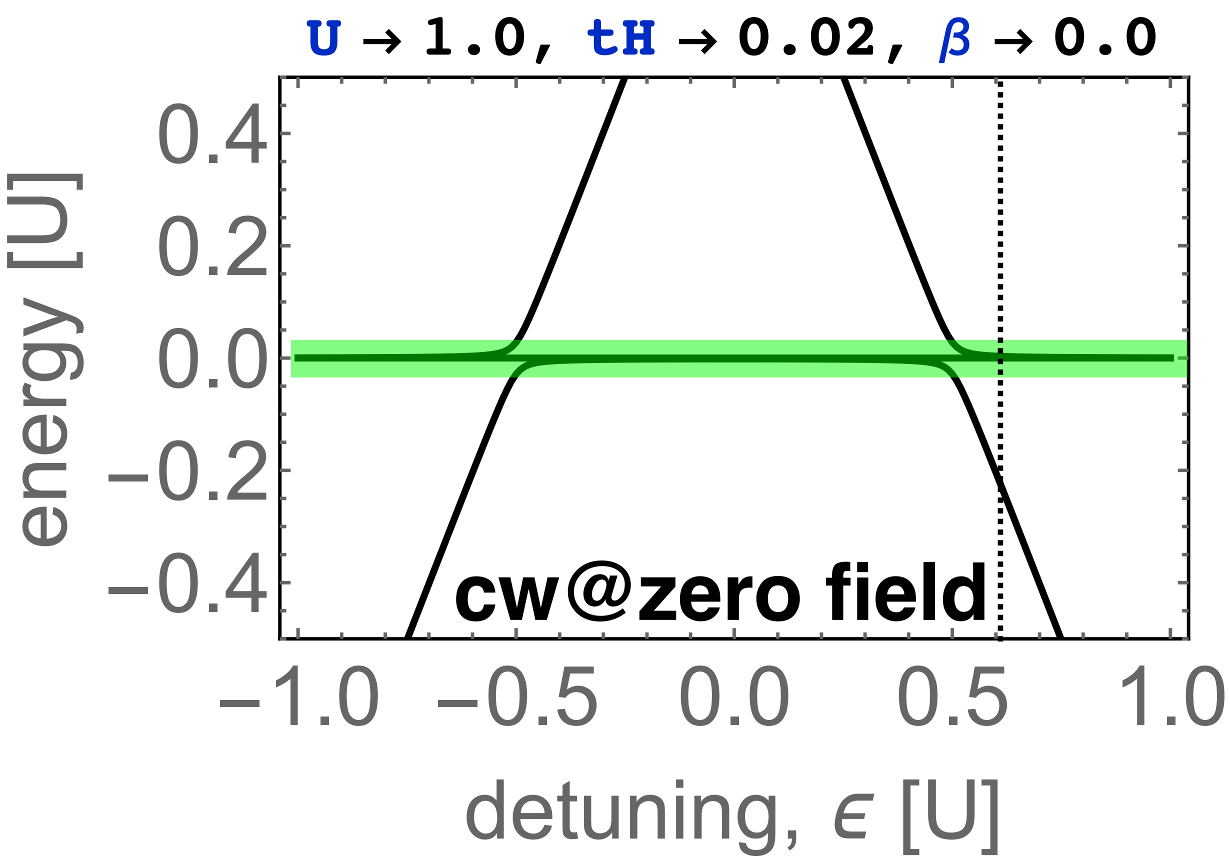


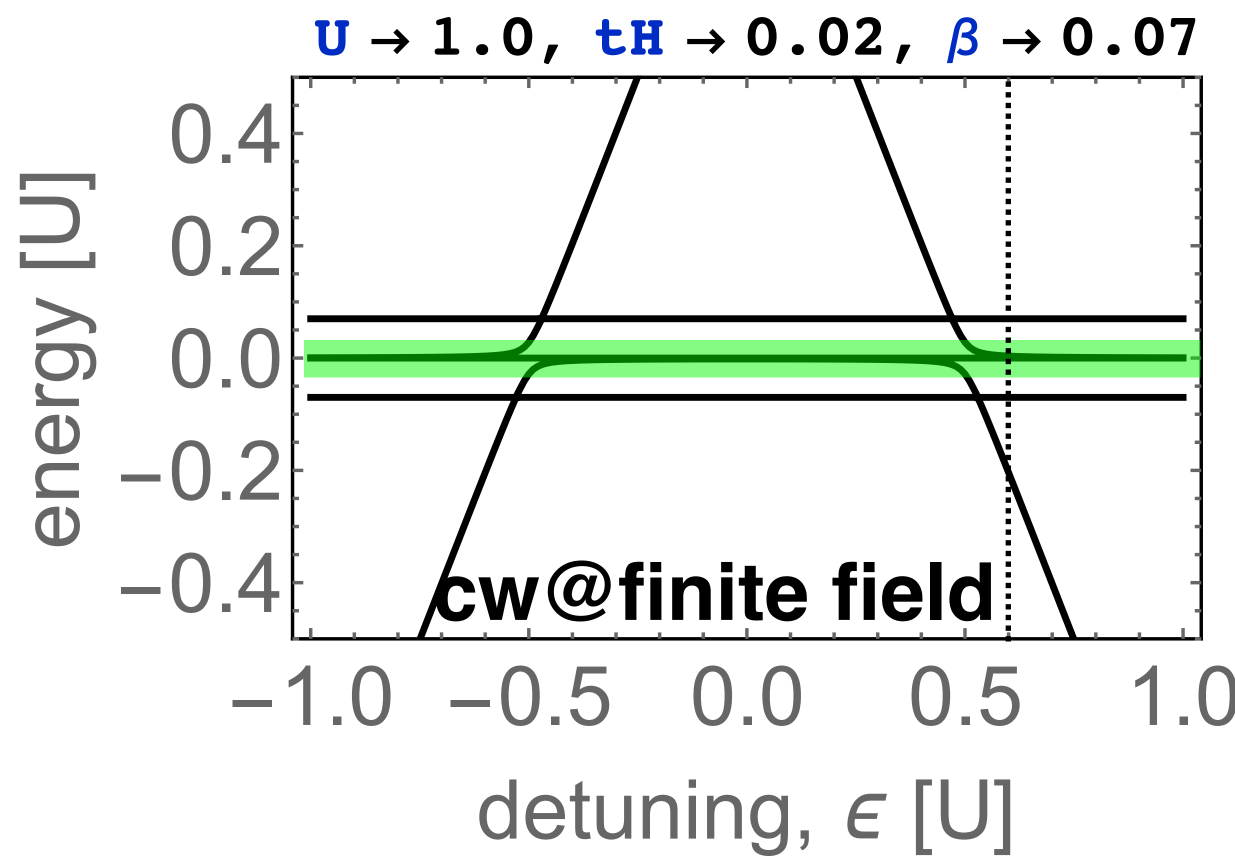
Exercise: estimate the maximum dc current achievable from this cycle



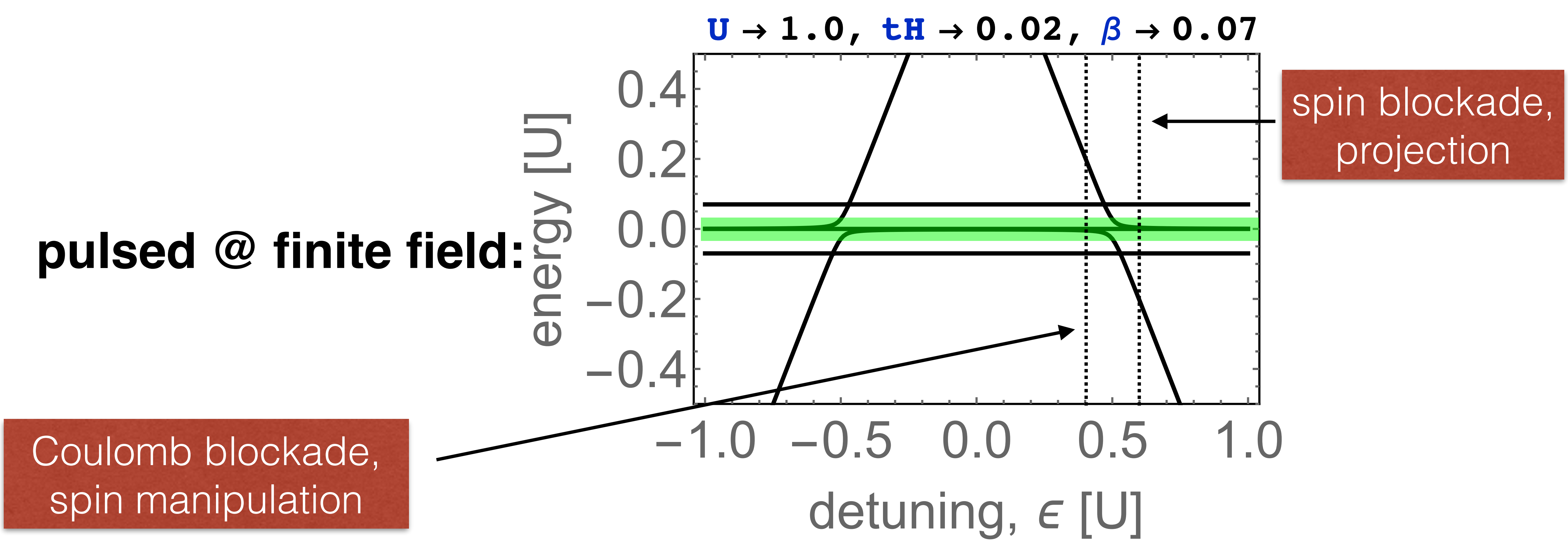
Koppens et al., Nature 2006

Continuous-wave and pulsed experiments





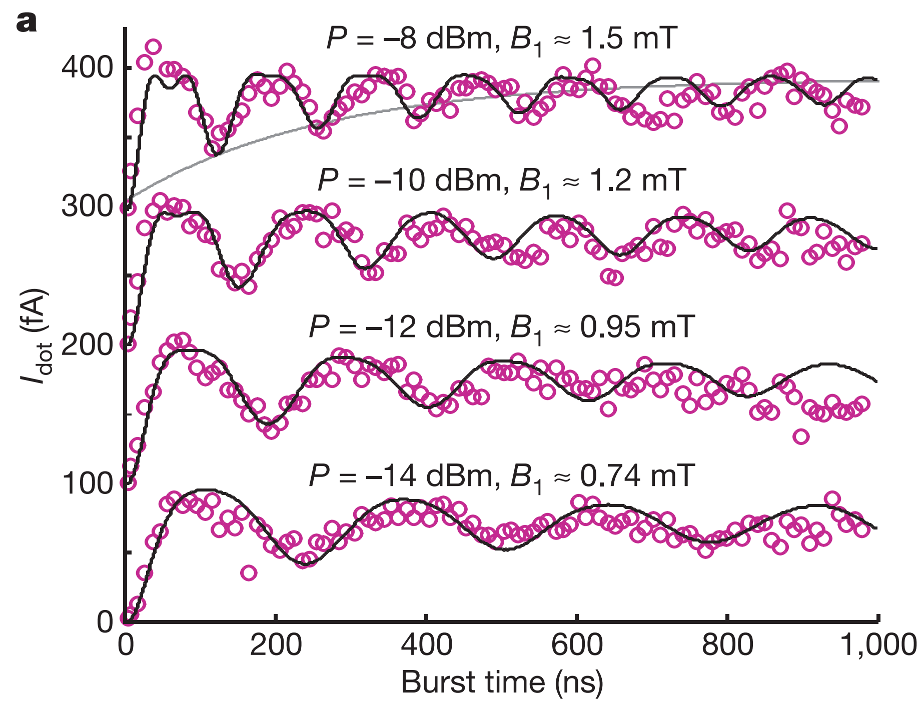
pulsed @ finite field:

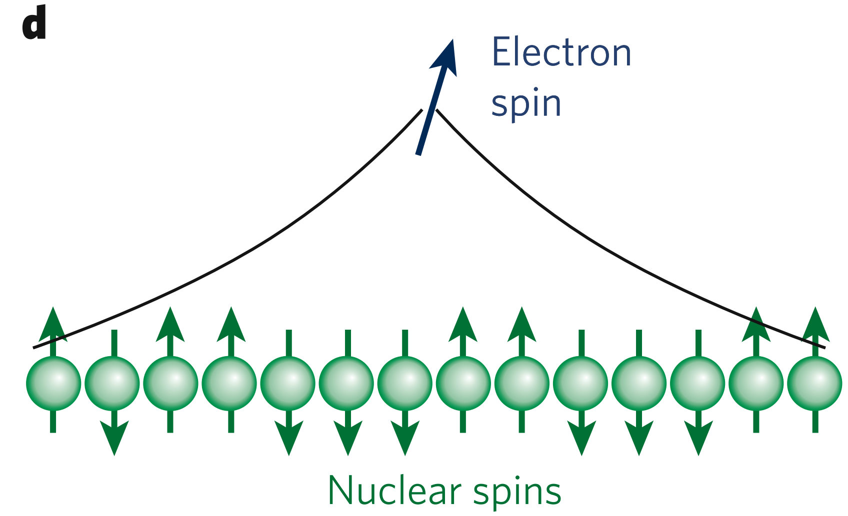


Quality of control has improved at lot since 2006

Koppens et al., Nature **2006**

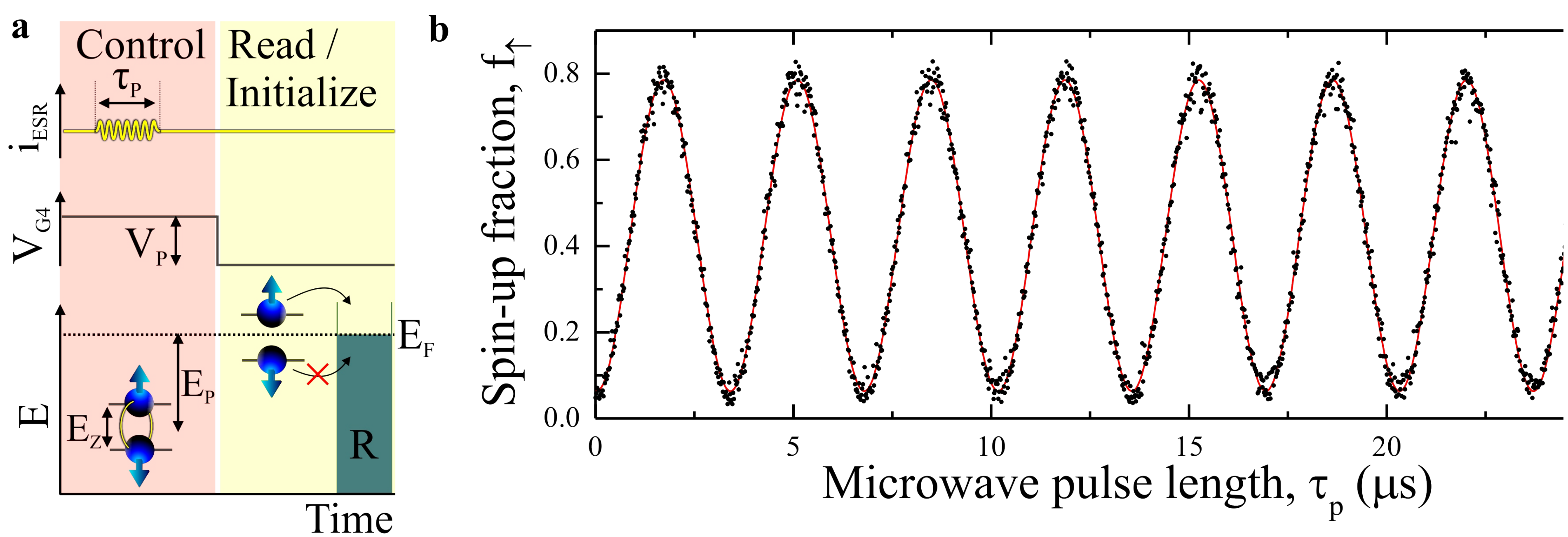
GaAs:
random nuclear spins
=> uncontrolled B-field
component





Silicon: only 5% has nuclear spins (Si-29, spin-1/2)
isotopic purification => 0.08% Si-29 content

Veldhorst et al.,
Nat. Nanotech. **2014**



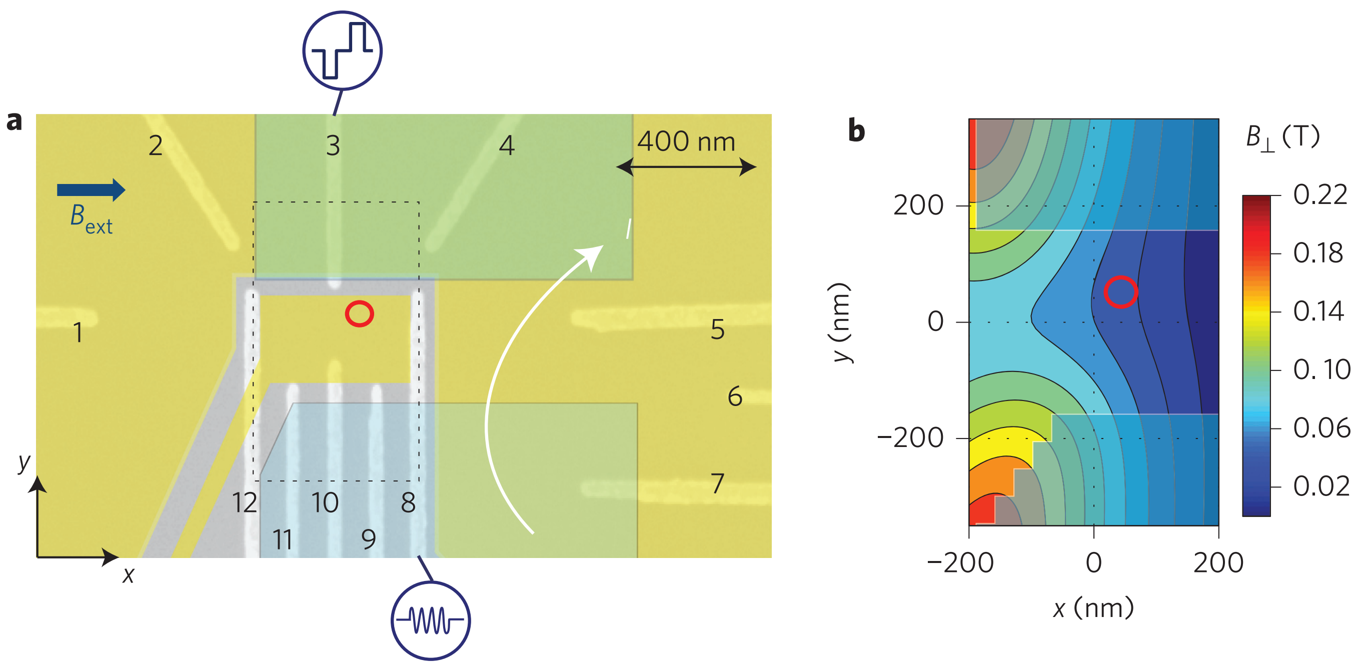
Spin control with an ac electric field

EDSR = `electrically driven spin resonance' or `electric dipole spin resonance'

Interaction between spin and electric field mediated by:

inhomogeneous magnetic field

(created by `micromagnet'; mu-EDSR)

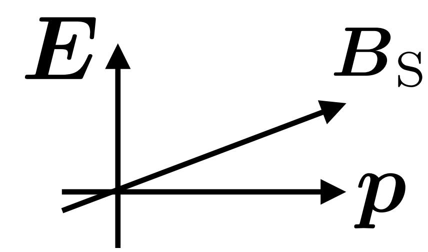


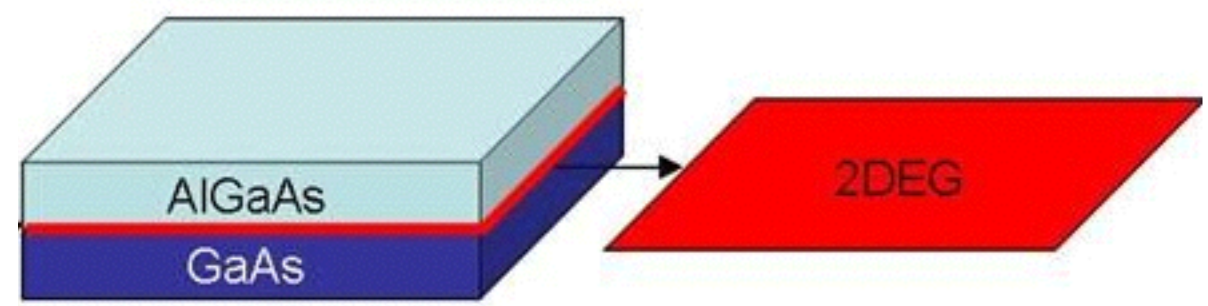
theory: Tokura et al., PRL 2006
experiment: Pioro-Ladriere, Nat. Phys. 2008
Kawakami et al., Nat. Nanotech. 2014
Yoneda et al. Nat. Nanotech. 2018

spin-orbit interaction

(SOI-EDSR)

$$\boldsymbol{B}_{\mathrm{SOI}} \propto \boldsymbol{E} \times \boldsymbol{p}$$

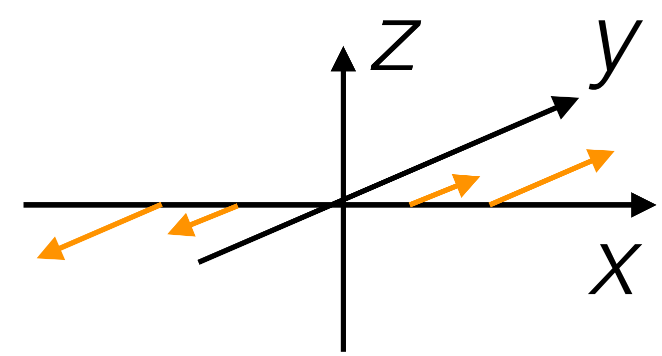




theory: Golovach et al., PRB 2006
experiment: Nowack et al., Science 2007
Kawakami et al., Nat. Nanotech. 2014
Yoneda et al. Nat. Nanotech. 2018

Semiclassical minimal model of mu-EDSR

$$\boldsymbol{B} = \begin{pmatrix} 0 \\ \beta x \\ B_0 \end{pmatrix}$$



$$H = H_{\text{osc}} + H_{\text{hom}} + H_{\text{inh}} + H_E(t)$$

$$H_{\text{osc}} = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2} m\omega_0^2 (x^2 + y^2)$$

$$H_{\text{hom}} = \frac{1}{2} g^* \mu_B B_0 \sigma_z$$

$$H_{\text{inh}} = \frac{1}{2} g^* \mu_B \beta x \sigma_y$$

$$H_E(t) = exE(t) = exE_0 \sin(\omega t)$$

Electronic potential along x :

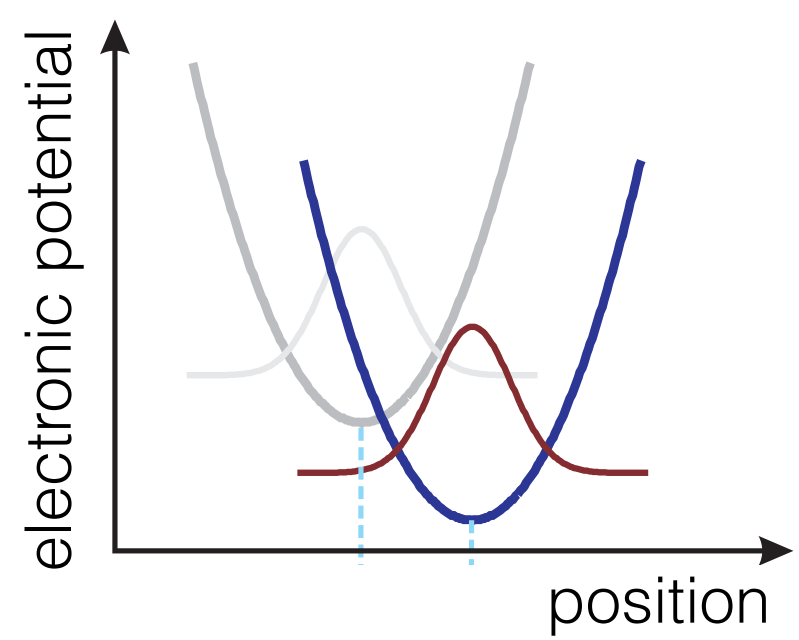
$$V(x) = \frac{1}{2} m\omega_0^2 (x + x_0(t))^2 - \ldots$$

where $x_0(t) = \dfrac{eE_0 \sin \omega t}{m\omega_0^2}$

Electron follows potential minimum.

Excitation resonant with Zeeman splitting:

$$\hbar\omega = \Delta E_Z \equiv \hbar\omega_L \approx g^* \mu_B B_0 \ll \hbar\omega_0$$



Semiclassical minimal model of mu-EDSR

$$H_{\rm inh} = \frac{1}{2} g^* \mu_B \beta x \sigma_y \mapsto \frac{1}{2} g^* \mu_B \beta x_0(t) \sigma_y = \frac{1}{2} g^* \mu_B \frac{\beta e E_0}{m \omega_0^2} \sin(\omega t) \sigma_y$$

We have seen (resonant rotating drive):

Resonance condition: $\omega = \frac{g^* \mu_B B_0}{\hbar}$

$$H(t) = \frac{1}{2} g \mu_B B_0 \sigma_z + \frac{1}{2} g \mu_B B_{\rm ac} \left(\sigma_x \cos \omega t + \sigma_y \sin \omega t \right)$$

$$\rightarrow P_e(t) = \sin^2 \left(\frac{1}{2} \Omega t \right)$$

with Rabi frequency $\Omega = g \mu_B B_{\rm ac} / \hbar$

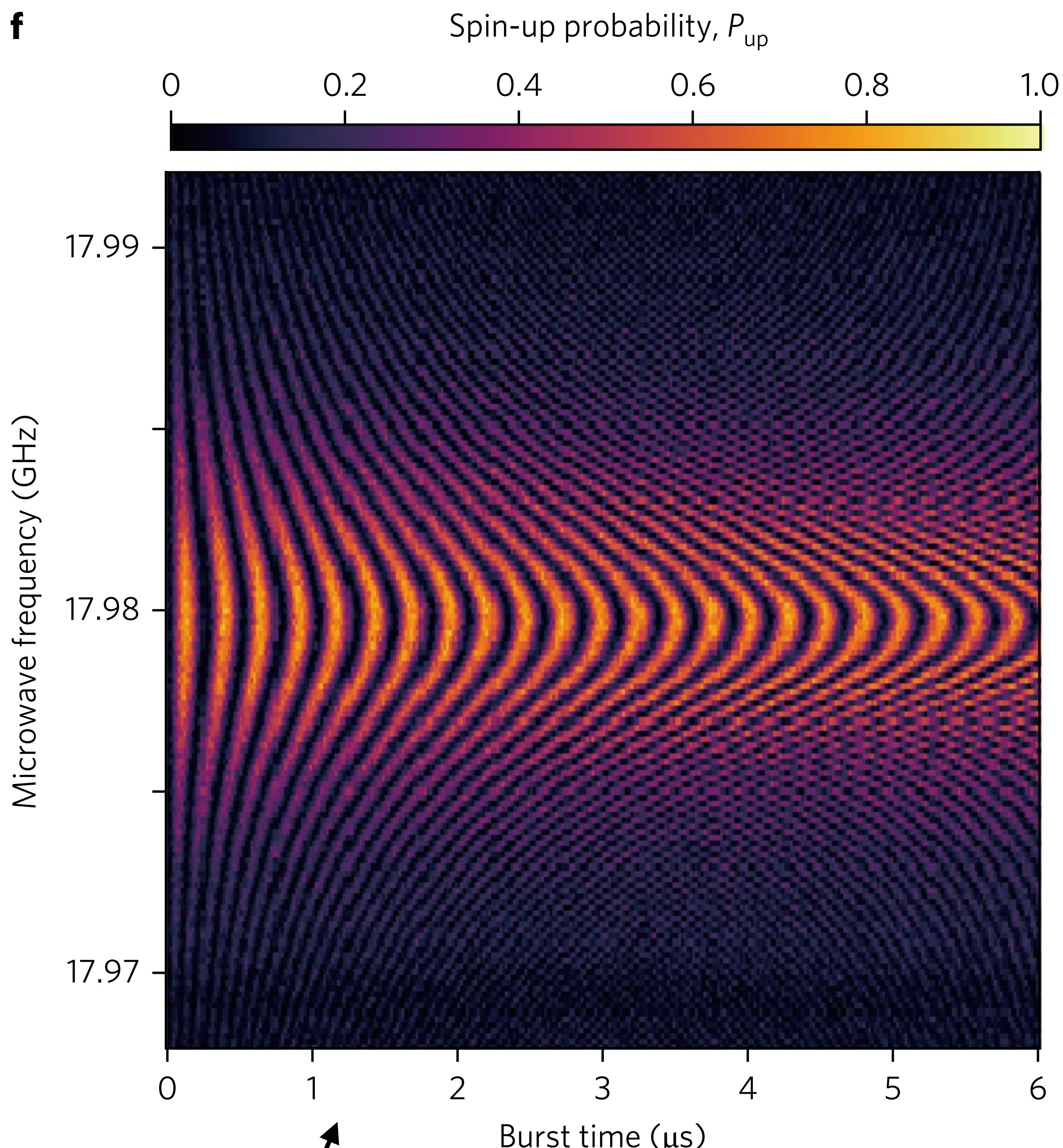
We have seen (resonant linearly polarized weak drive):

$$H(t) = \frac{1}{2} g \mu_B B_0 \sigma_z + \frac{1}{2} g \mu_B B_{\rm ac} \sigma_x \cos \omega t \rightarrow P_e(t) = \sin^2 \left(\frac{1}{2} \Omega t \right)$$

EDSR Rabi frequency: $\Omega_{\rm EDSR} = \frac{g^* \mu_B \beta e E_0}{2 m \hbar \omega_0^2}$

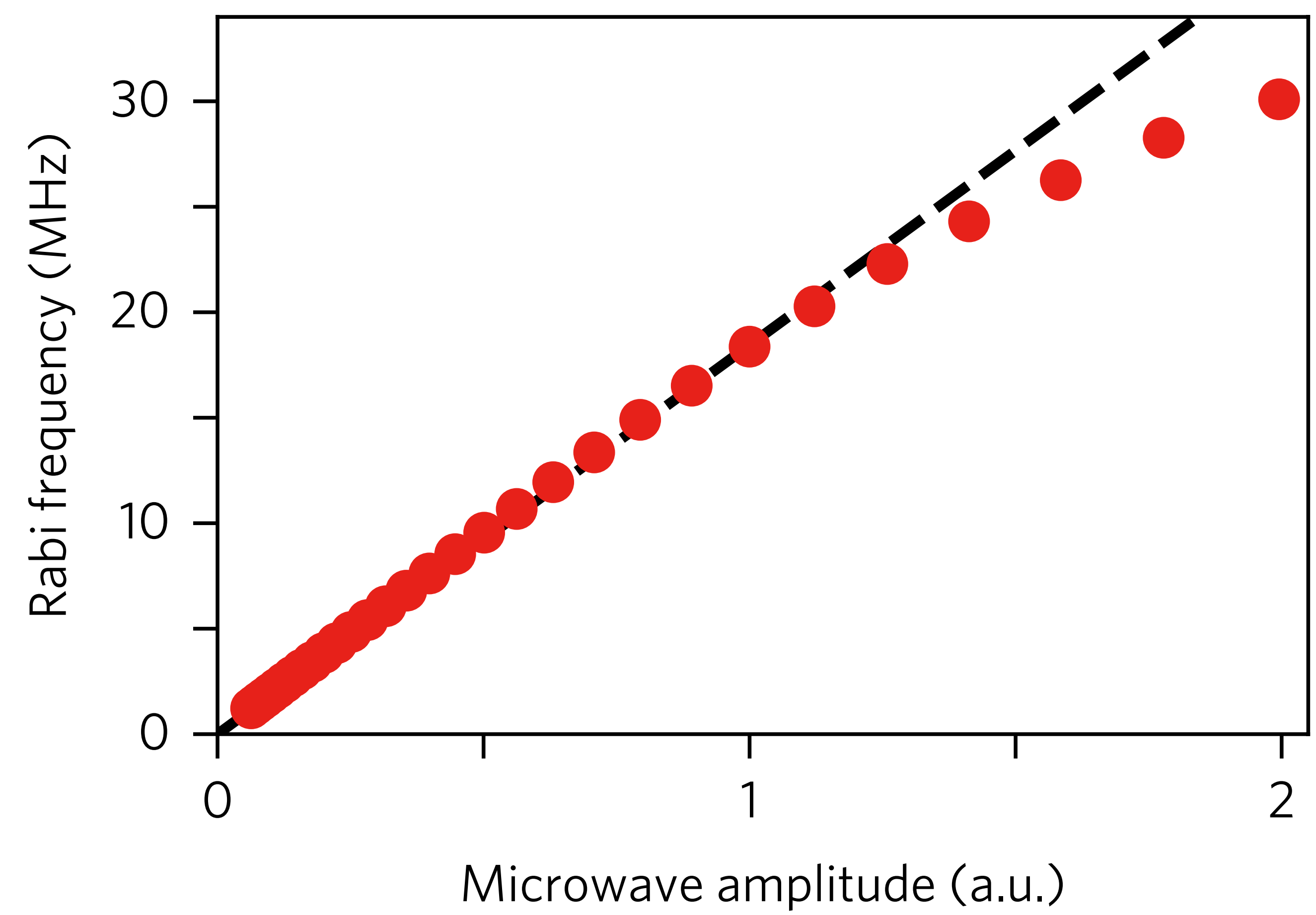
with Rabi frequency $\Omega = \frac{1}{2} g \mu_B B_{\rm ac} / \hbar$

A very recent mu-EDSR experiment in silicon



EDSR Rabi frequency:

$$\Omega_{\mathrm{EDSR}} = \frac{g^* \mu_B \beta e E_0}{2 m \hbar \omega_0^2}$$



$$P_e(t) = P_{\max}(\delta) \sin^2\left(\tfrac{1}{2}\sqrt{\Omega^2 + \delta^2 t}\right) = \frac{\Omega^2}{\Omega^2 + \delta^2} \sin^2\left(\tfrac{1}{2}\sqrt{\Omega^2 + \delta^2 t}\right)$$

An EDSR experiment without a micromagnet

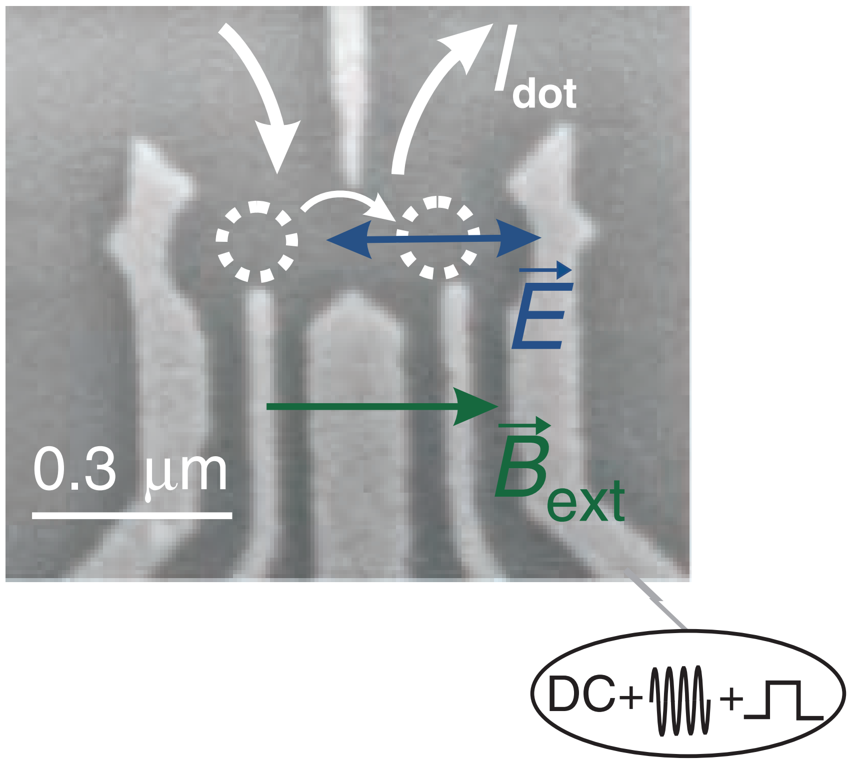
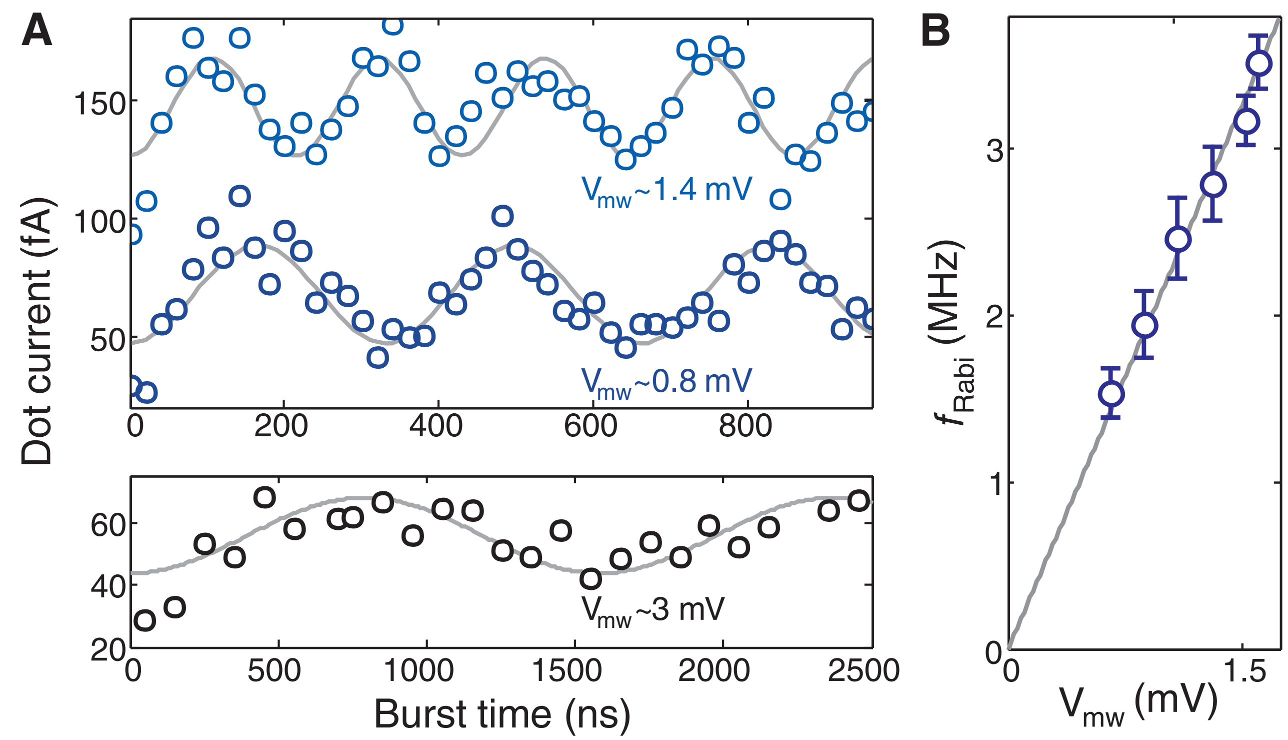


Fig. 3. (**A**) Rabi oscillations at 15.2 GHz (blue, average over five sweeps) and 2.6 GHz (black, average over six sweeps). The two oscillations at 15.2 GHz are measured at different amplitudes of the microwaves V_{mw}, leading to different Rabi frequencies. (**B**) Linear dependence of the Rabi frequency on applied microwave amplitude measured at f_{ac} = 14 GHz.



- No micromagnet, no microwave antenna
- Spin rabi oscillations are seen
- Stronger excitation => faster Rabi oscillations
- Smaller B-field => slower Rabi oscillations

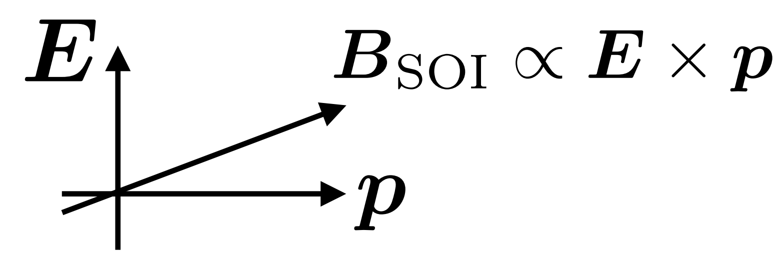
Claim: spin dynamics is caused by spin-orbit interaction

Golovach et al., PRB 2006

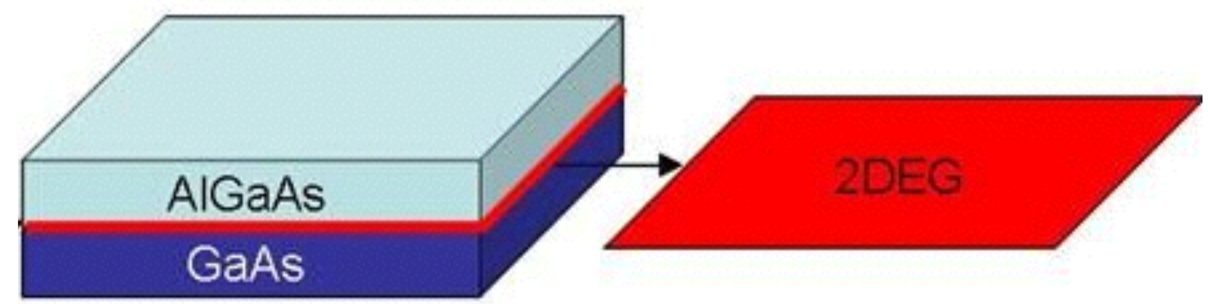
Spin-orbit-induced electrically driven spin resonance

$$H = H_{\mathrm{osc}} + H_{\mathrm{hom}} + H_{\mathrm{SOI}} + H_E(t)$$

$$H_{\mathrm{osc}} = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2} m\omega_0^2 (x^2 + y^2)$$



$$H_{\mathrm{hom}} = \frac{1}{2} g^* \mu_B B_0 \sigma_z$$



$$H_{\mathrm{SOI}} = \alpha(\sigma_x p_y - \sigma_y p_x)$$

Rashba spin-orbit interaction

$$H_E(t) = exE(t) = exE_0 \sin(\omega t)$$

Excitation resonant with Zeeman splitting:

$$\hbar\omega = \Delta E_Z \equiv \hbar\omega_L \approx g^* \mu_B B_0 \ll \hbar\omega_0$$

Golovach et al., PRB 2006

Spin-orbit-induced electrically driven spin resonance

Tool #1: creation/annihilation operators

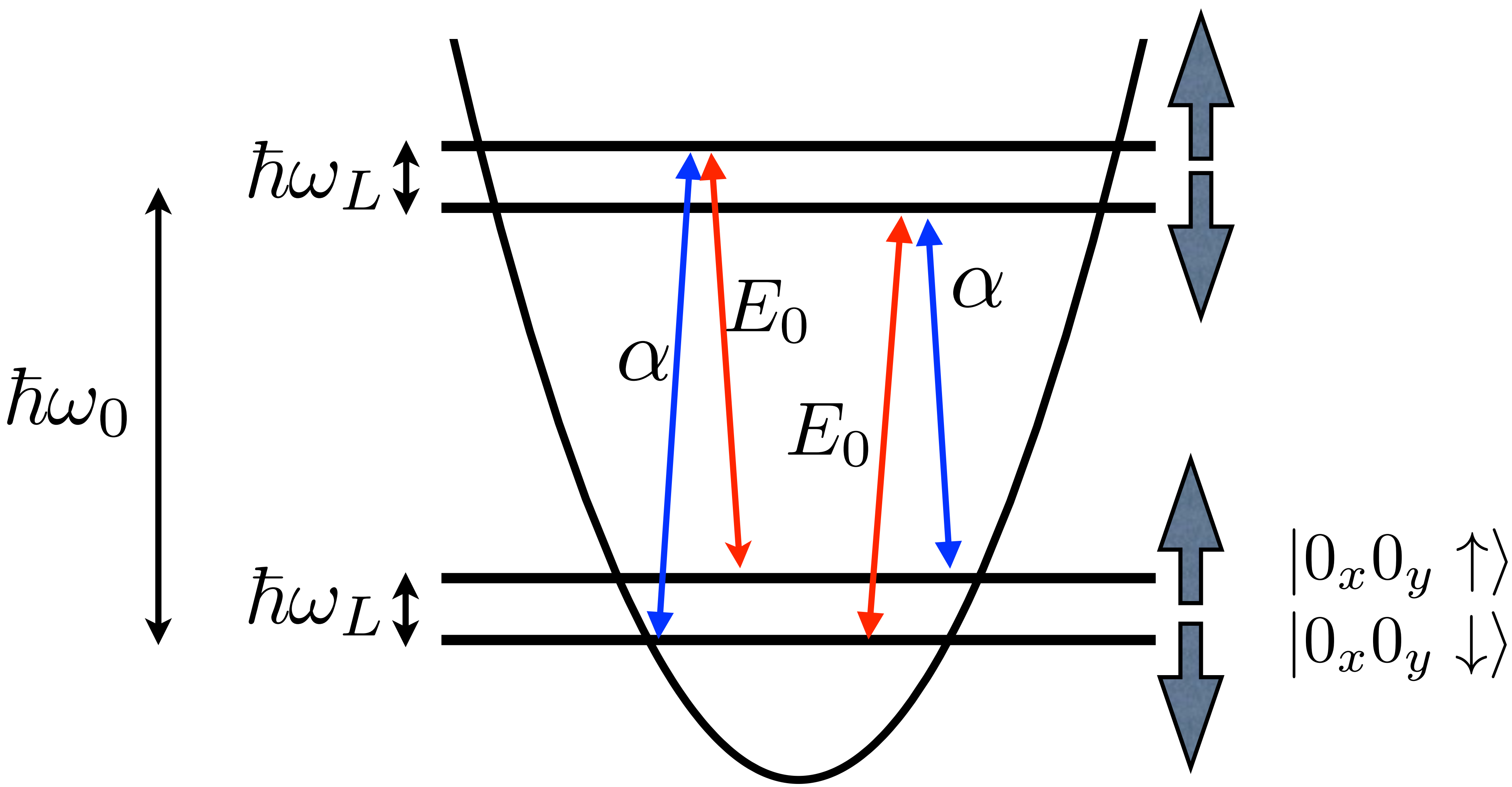
$$x = \frac{\ell}{\sqrt{2}}(a + a^\dagger),\ p = \frac{\hbar}{\sqrt{2}i\ell}(a - a^\dagger),\quad \text{with oscillator length } \ell = \sqrt{\frac{\hbar}{m\omega_0}}$$

Tool #2: first-order perturbation theory

$$H = H_0 + H_1,\ \text{and } H_0 \left|n\right\rangle = E_n^{(0)} \left|n\right\rangle;\ \text{then } \left|\overline{n}\right\rangle \approx \left|n\right\rangle + \sum_{m\neq n} \left|m\right\rangle \frac{\langle m|H_1|n\rangle}{E_n^{(0)} - E_m^{(0)}}$$

Spin-orbit-induced electrically driven spin resonance

Consider perturbative limit: $\hbar\omega, \hbar\omega_L, \frac{\hbar\alpha}{\ell}, eE_0\ell \ll \hbar\omega_0$



$$H_0 = H_{\mathrm{osc}} + H_{\mathrm{hom}}$$

$$H_1 = H_{\mathrm{SOI}}$$

qubit basis states dressed by SOI:

$$|\overline{0_x0_y\uparrow}\rangle \approx |0_x0_y\uparrow\rangle + \sum\ldots$$

$$|\overline{0_x0_y\downarrow}\rangle \approx |0_x0_y\downarrow\rangle + \sum\ldots$$

Golovach et al., PRB 2006

Spin-orbit-induced electrically driven spin resonance

qubit basis states dressed by SOI:

$$|\overline{0_x 0_y \uparrow}\rangle \approx |0_x 0_y \uparrow\rangle + \sum \ldots$$
$$|\overline{0_x 0_y \downarrow}\rangle \approx |0_x 0_y \downarrow\rangle + \sum \ldots$$

Slow drive: $\omega \ll \omega_0$

$\mapsto$ no leakage from qubit subspace

$\mapsto$ effective qubit Hamiltonian

$$H_{E,\mathrm{q}}(t) = P H_E(t) P$$

with $P = |\overline{0_x 0_y \uparrow}\rangle \langle \overline{0_x 0_y \uparrow}| + |\overline{0_x 0_y \downarrow}\rangle \langle \overline{0_x 0_y \downarrow}|$

$$H_{E,\mathrm{q}} \approx \frac{eE_0 \alpha \omega_L}{\omega_0^2} \bar{\sigma}_x \sin(\omega t), \text{ with } \bar{\sigma}_x = \left(|\overline{0_x 0_y \uparrow}\rangle \langle \overline{0_x 0_y \downarrow}| + h.c. \right)$$

EDSR Rabi frequency

$$\Omega_{\mathrm{EDSR}} = \frac{eE_0 \alpha \omega_L}{\hbar \omega_0^2}$$

Indeed, $\Omega_{\mathrm{EDSR}} \sim B$.

Summary

1. `Initialization' and `readout' with Pauli-blockade in a double quantum dot
2. Continuous-wave and pulsed detection of ESR and EDSR
3. Electron Spin Resonance with a wire providing ac B-field
4. Electrically Driven Spin Resonance with a micromagnet
5. Electrically Driven Spin Resonance due to spin-orbit interaction

Potential extensions

1. Potential advantages of EDSR over ESR (simpler design, selective addressing, lower power)
2. Electrically driven nuclear-spin resonance
3. Two-qubit gates for the single-electron spin qubit (see Lecture 2)
4. Single-qubit gates for the S-T0 qubit
5. Two-qubit gates for the S-T0 qubit (capacitive, exchange-based)