Quantum Computing Architectures

Budapest University of Technology and Economics
2018 Fall

lecturers: Peter Makk, Andras Palyi

Lecture 1
Quantum bits

Lecture 1 is based on the book
Nielsen & Chuang: *Quantum Computation and Quantum Information*

Quantum Computing - what is it?

Quantum computing

From Wikipedia, the free encyclopedia

Quantum computing is computing using quantum-mechanical phenomena, such as superposition and entanglement.[1] A **quantum computer** is a device that performs quantum computing. Such a computer is different from binary digital electronic computers based on transistors. Whereas common digital computing requires that the data be encoded into binary digits (bits), each of which is always in one of two definite states (0 or 1), quantum computation uses quantum bits or qubits, which can be in superpositions of states. A quantum Turing machine is a theoretical model of such a computer, and is also known as the universal quantum computer. The field of quantum computing was initiated by the work of Paul Benioff[2] and Yuri Manin in 1980,[3] Richard Feynman in 1982,[4] and David Deutsch in 1985.[5]

Quantum Computing - why should anyone care?

QC could be useful

- algorithms solving computational problems can be slow or fast
- for example, prime factorization is a problem for which only slow classical algorithms are known
- prime factorization is important in information technology & security
- there is a fast quantum algorithm for prime factorization (Shor)

People are interested in QC

- many experimental research groups are trying to build and improve quantum computer prototypes
- private funding in quantum information technology increased a lot in the past few years (IBM, Google, Intel, Microsoft; Rigetti, Q-Ctrl, etc)

Quantum computers do exist

- prototype quantum computers that are available for anyone do exist, e.g., IBM Quantum Experience (small, noisy, not useful yet)

Schedule of this course

Szerda
augusztus 29. - Regisztrációs hét -
szeptember 5.
szeptember 12.
szeptember 19.
szeptember 26. TTK Dékáni szünet
október 3.
október 10.
október 17.
október 24.
október 31.
november 7.
november 14. TDK konferencia
november 21.
november 28.
december 5.

lecture 01 (today)

lecture 02

lecture 03

lecture 04

lecture 05

lecture 06

lecture 07

lecture 08

lecture 09

lecture 10

Introduction

Spin qubits
(electron spin)

Superconducting qubits
(transmon)

Course website on fizipedia.bme.hu

szócikk vitalap lapforrás laptörténet

Kvantumszámítógép-architektúrák

Quantum Computing Architectures

Course Information, 2018

- **Lecturers:** András Pályi, Péter Makk
- **Responsible lecturer:** András Pályi
- **Language:** English
- **Location:** building H, room H601
- **Time:** Wednesdays, 12:15-13:45
- **Schedule:** first lecture: Sep 5; no lecture on Sep 12 and Oct 10; last lecture: Dec 5.
- **Neptun Code:** BMETE15MF60
- **Credits:** 3
- **Exam:** The grade is based on an oral exam in the exam period. The emphasis is put on the level of understanding.

Syllabus

The building blocks of nowadays electronic devices have already reached a few tens on nanometers sizes, and further miniaturization requires the introduction of novel technologies. At such small length-scales the coherent behavior and the interaction of electrons, together with the atomic granularity of matter induce several striking phenomena, that are not observed at the macroscopic scale. The course gives an introduction to a broad set of nanoscale phenomena following the topics bellow:

- **1. Quantum bits**

Qubits, dynamics, measurement, polarization vector, composite systems, logical gates, circuits, algorithms.

- **2. Control of quantum systems.**

Hamiltonians, propagators, and quantum gates. Larmor precession, Rabi oscillations, dispersive resonator shift in the Jaynes-Cummings model, exchange interaction, virtual photon exchange.

- **3. Qubits based on the electron spin.**

Quantum dots, energy scales. Interactions: Zeeman, spin-orbit, hyperfine, electron-phonon, electron-electron.

Exam: oral exam including a short written test

test question examples:

1. List the three Pauli matrices. Determine their eigenvalues and normalized eigenstates.
2. What is the unitary matrix representing the Hadamard gate? What is the result of the Hadamard gate acting on the state $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$? What is the result of the Hadamard gate acting on the state $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$?
3. What is the unitary matrix representing a two-qubit $\sqrt{\mathrm{SWAP}}$ gate in the basis $|00\rangle, |01\rangle, |10\rangle, |11\rangle$? What is the result of the $\sqrt{\mathrm{SWAP}}$ gate acting on the state $\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$?
4. Determine the polarization vectors associated to the following three states: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$.
5. Let H be an N-dimensional time independent Hamiltonian, with known energy eigenvalues E_n and eigenstates ψ_n fulfilling $H\psi_n = E_n\psi_n$. Assume that the system is initialized in the state ψ_i at $t = 0$. Express the time evolution of this state, $\psi(t)$, using E_n, ψ_n and ψ_i.
6. Consider the single-qubit state $\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$. When we measure the qubit, what is the probability of measuring 1? What is the state of the qubit after the measurement?
7. Consider the two-qubit state $\frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|10\rangle + \frac{1}{\sqrt{3}}|01\rangle$. When we measure the first qubit, what is the probability of measuring 0? And that of measuring 1? What is the state of the system after measuring 0? And after measuring 1?

Classical bits, gates, circuits

truth tables

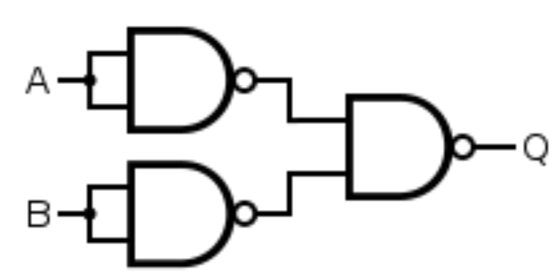
INPUT		OUTPUT
A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

INPUT		OUTPUT
A	B	A NAND B
0	0	1
0	1	1
1	0	1
1	1	0

INPUT		OUTPUT
A	B	A XOR B
0	0	0
0	1	1
1	0	1
1	1	0

- the value of a c-bit is 0 or 1
- *operations, gates:* a c-logical gate maps n c-bits to m c-bits; e.g., NOT, AND, OR, XOR.
- single-bit gate: $n = m = 1$
- there is only one non-trivial single-bit gate: NOT
- two-bit gate: $n = 2$, $m = 1$, e.g., AND, OR, XOR
- c-gates are not necessarily reversible: e.g., any $n > m$ gate is irreversible
- *c-circuit*: an arrangement of "wires" and gates
- *universal gate set*: a set of gates that allows to construct circuits for any algorithm
- exercise: construct a c-circuit that adds two single-bit numbers using only the NAND gate

OR can be built from NANDs:



Quantum bits

1. *quantum bit, qubit, q-bit, qbit*: two-level quantum system
2. state of a qubit: $|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$
3. α_0, α_1 are called *amplitudes*; they are complex numbers
4. $|0\rangle$ and $|1\rangle$ are the *qubit basis states*
5. normalization condition: $|\alpha_0|^2 + |\alpha_1|^2 = 1$
6. alternative notation (*vector notation* or *spinor notation*):

$$|0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \, |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \, \alpha_0 |0\rangle + \alpha_1 |1\rangle \equiv \alpha_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$$

7. realizations: electron spin, nuclear spins (e.g., H-1, C-13), superconducting circuits, etc.

Dynamics of a qubit

1. time-dependent Schrodinger equation: $\frac{\hbar}{i}\dot{\psi}(t) + H(t)\psi(t) = 0.$
2. for a qubit, $H(t)$ is a 2x2 Hermitian matrix
3. Hamiltonian can be expressed with Pauli matrices

$$H(t) = \sum_{j=0}^{3} c_j(t)\sigma_j$$

$$\sigma_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

4. dynamics for a time-independent Hamiltonian: $\psi(t) = \exp\left(-\frac{i}{\hbar}Ht\right)\psi(0) \equiv U(t)\psi(0)$
5. $U(t)$ is a unitary matrix, called the *propagator*
6. dynamics for a time-dependent Hamiltonian is also unitary: $\psi(t) = \mathcal{T}\exp\left(-\frac{i}{\hbar}\int_0^t dt' H(t')\right)\psi(0) \equiv U(t)\psi(0)$

Measurement (`readout') of a qubit

1. $|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$
2. the probability of measuring 0 is $P_0 = |\alpha_0|^2$
3. the probability of measuring 1 is $P_1 = |\alpha_1|^2 = 1 - P_0$
4. if the outcome of the measurement is 0, then the state changes to $|0\rangle$
5. if the outcome of the measurement is 1, then the state changes to $|1\rangle$

Geometrical representation of a qubit: the Bloch sphere

1. we can parametrize the qubit state with three angles, γ, θ, ϕ:

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle = e^{i\gamma}(\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

2. angle γ has no physical significance
3. the qubit state can be mapped to the surface of a unit sphere (*Bloch sphere*):

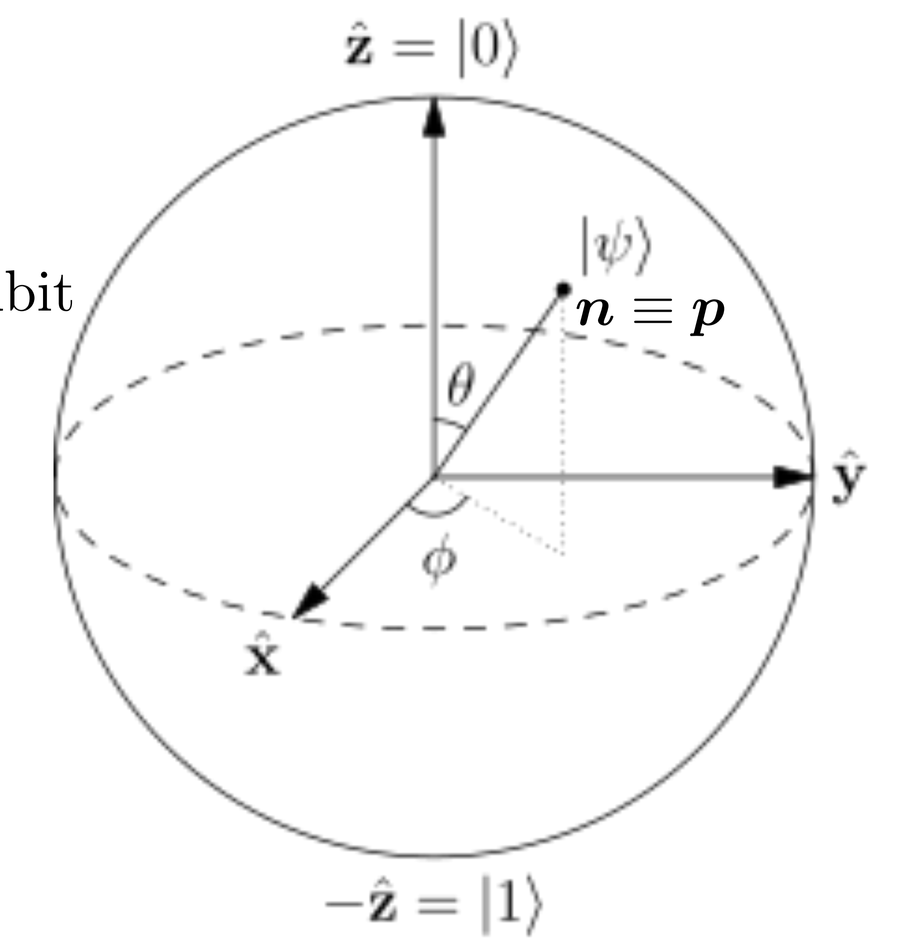
$$|\psi\rangle \mapsto (\theta, \phi) \mapsto \boldsymbol{n} = (\ \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta\ )$$

4. another mapping, seemingly different, but actually identical to $\boldsymbol{n}$:

$$\boldsymbol{p} = \langle\psi|\boldsymbol{\sigma}|\psi\rangle\,,$$

   where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$.

5. $\boldsymbol{n} \equiv \boldsymbol{p}$ is called the *Bloch vector* or the *polarization vector* of the qubit



More qubits

1. states of two qubits: $|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$
2. normalization condition: $\sum_{x\in\{0,1\}^2} |\alpha_x|^2 = 1$
3. a single-qubit state can be represented on the Bloch sphere; does not work for multiple-qubit states
4. measurement of one qubit: e.g., of the first one: $P_0 = |\alpha_{00}|^2 + |\alpha_{01}|^2$, and the post-measurement state after measuring 0 is

$$|\psi_{\mathrm{pm}}\rangle = \frac{\alpha_{00}|00\rangle + \alpha_{01}|01\rangle}{\sqrt{P_0}}$$

5. example for a two-qubit product state:

$$|\psi\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

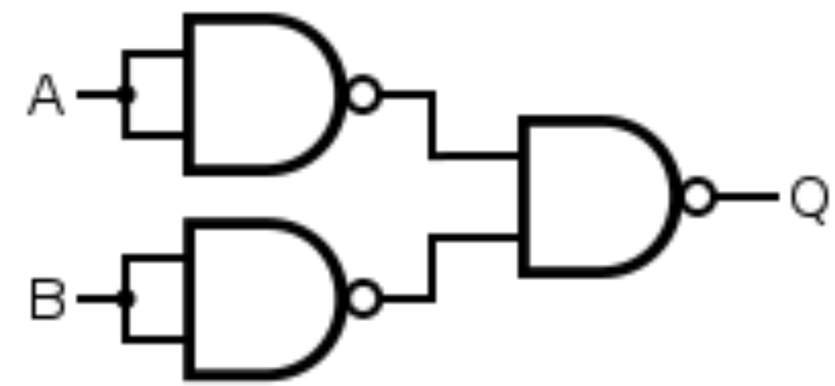
6. example for a two-qubit entangled state:

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

7. the state of n qubits is described by 2^n amplitudes

1-qubit quantum gates

c-circuit



1. q-circuit: an arrangement of "wires" and quantum gates
2. q-gates: unitary operations on a few qubits (reversible, unlike c-gates)
3. 1-qubit gate example: q-NOT (usually called the X *gate*):

$$|\psi_1\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \;\mapsto\; |\psi_2\rangle = \alpha\,|1\rangle + \beta\,|0\rangle$$

   matrix representation of this gate: $X \equiv \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

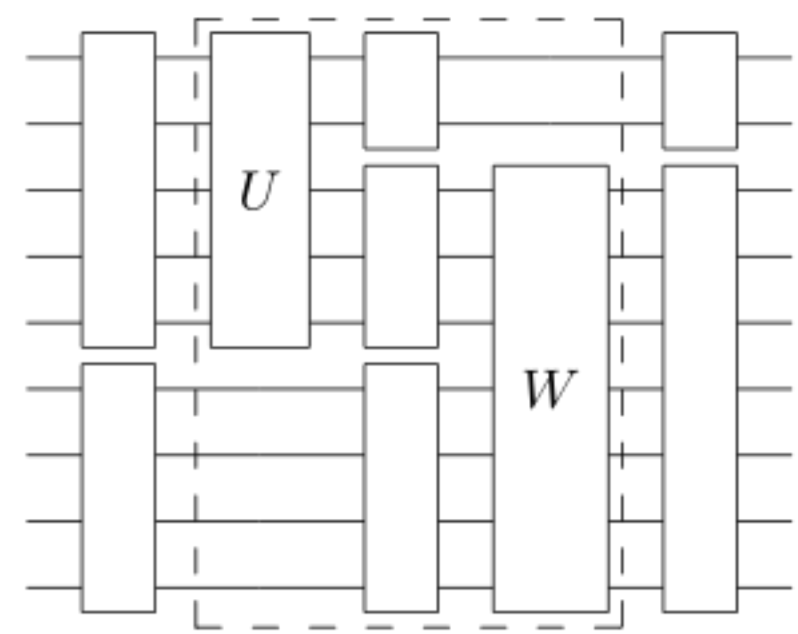
4. further 1-qubit gate examples:

$$\text{Z gate: } Z = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\text{Hadamard gate: } H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

5. each 1-qubit gate generates a bijective map of the Bloch sphere to itself
6. exercise: determine the transformations generated by 1-qubit gates listed above

q-circuit



2-qubit quantum gates

1. 2-qubit gate example: *controlled-NOT* or *CNOT*
   with the basis-state ordering $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$, it is represented by

$$U_{\mathrm{CNOT}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

   it could be represented by a 'classical' truth table

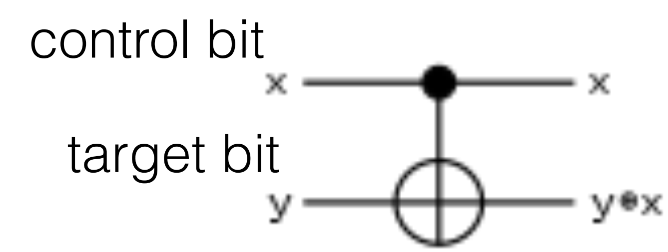
2. 2-qubit gate example: $\sqrt{\mathrm{SWAP}}$:

$$U_{\sqrt{\mathrm{SWAP}}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 \\ 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

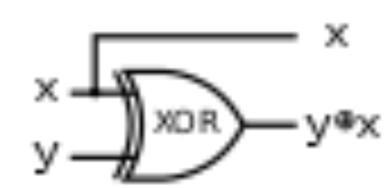
   cannot be represented by a classical truth table

3. 1-qubit gates together with CNOT form a unversal q-gate set

4. 1-qubit gates together with $\sqrt{\mathrm{SWAP}}$ form a universal q-gate set



input		output	
x	y	x	y+x
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$



input		output	
x	y	x	y+x
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Deutsch's problem

1. A simple oracle problem: $f : \{0,1\} \to \{0,1\}$ is an unknown function; i.e., it is one of the following 4 functions:

constant (value = 1)	constant (value = 0)
$0 \to 1$ $1 \to 1$	$0 \to 0$ $1 \to 0$
balanced (NOT)	balanced (*id.*)
$0 \to 1$ $1 \to 0$	$0 \to 0$ $1 \to 1$

2. task: figure out, by evaluating f a few times, whether f is constant or balanced

3. solution: one has to evaluate f twice, for input 0 and for input 1, and the results will tell if f is constant or balanced

4. a single evaluation of f is not sufficient to complete the task

A quantum version of Deutsch's problem

1. f is not necessarily bijective/unitary; how to implement it in a quantum-mechanical (unitary) fashion?
2. one way to make it unitary is to use an auxiliary qubit y:

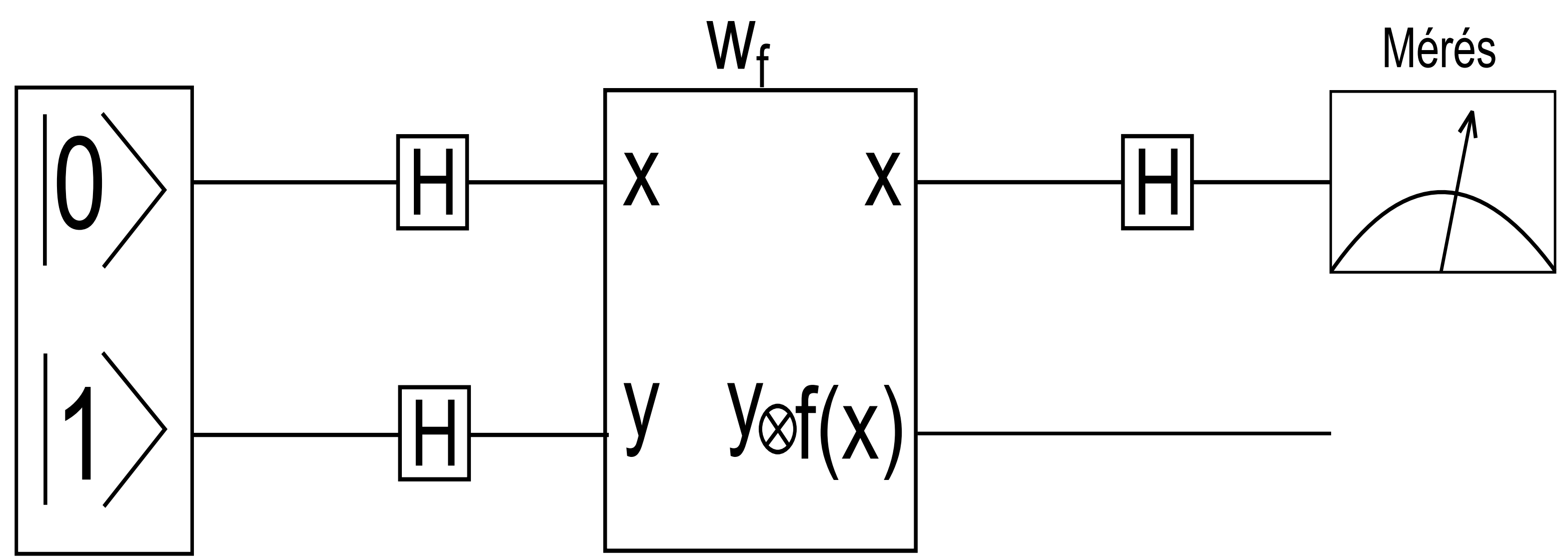
$$\begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{W_f} \begin{pmatrix} x \\ y \oplus f(x) \end{pmatrix}$$

   where $\oplus$ is the classical XOR

3. W_f is the unitary version of f, with truth table as follows:

$f = 0$	$f = 1$
$00 \to 00$	$00 \to 01$
$01 \to 01$	$01 \to 00$
$10 \to 10$	$10 \to 11$
$11 \to 11$	$11 \to 10$

$f = id.$	$f = NOT$
$00 \to 00$	$00 \to 01$
$01 \to 01$	$01 \to 00$
$10 \to 11$	$10 \to 10$
$11 \to 10$	$11 \to 11$



4. claim: the Deutsch algorithm decides whether f is constant or balanced by a *single evaluation* of W_f
5. if the measurement outcome for the upper qubit is 0, then f is constant, otherwise f is balanced
6. exercise: prove this

A generalization of Deutsch's problem

1. the unknown function is $f : \{0,1\}^n \to \{0,1\}$
2. we do not know f, but we know that it is either constant or balanced
3. task: determine whether f is constant or balanced, with the least possible evaluations of f
4. classically, the number of evaluations seems to scale exponentially with n
5. for example, it takes $2^{n-1}+1$ evaluations to find out the answer in the worst case
6. in the quantum version, the Deutsch-Jozsa algorithm performs the task with a *single evaluation*
7. if all measurement outcomes are zero, then f is constant, otherwise f is balanced

The unitary version of *f*

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \\ y \end{pmatrix} \xrightarrow{W_f} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \\ y \oplus f(x_1, x_2 \cdots x_N) \end{pmatrix}$$

Circuit of the Deutsch-Jozsa algorithm