Problem Set 5 for Many-body Physics II. Fall 2018

1. (10 points) By performing the frequency sums, show that the bare polarizability ($-1\times$ the bare bubble) can be written as

$$\Pi^0(\mathbf{q},\nu_n) = -2\int \frac{d^3p}{(2\pi)^3}\frac{n^0(\mathbf{p}+\mathbf{q})-n^0(\mathbf{p})}{i\nu_n-(\epsilon^0_{\mathbf{p}+\mathbf{q}}-\epsilon^0_{\mathbf{p}})},$$

where $n^0(\dots)$ is either the Bose-Einstein or the Fermi-Dirac distribution function, and $\epsilon^0_{\mathbf{p}}$ is the energy of state $\mathbf{p}$ in the noninteracting system. Check the following symmetries:

$$\Pi^0(\mathbf{q},\nu_n) = \Pi^0(-\mathbf{q},\nu_n),$$
$$\Pi^0(\mathbf{q},\nu_n) = \Pi^0(\mathbf{q},-\nu_n).$$