Many-body problem 1. Exercises
(**Deadline**: 13. March 2017.)

1. Prove that the second quantized form of a single particle operator, $\sum_{j=1}^{N} V(r_j)$ for bosons is

$$H = \sum_{k,p} f_{k,p} a_k^+ a_p, \quad f_{k,p} = \int dr \psi_k^+(r) V(r) \psi_p(r), \tag{1}$$

where $[a_k, a_p^+] = \delta_{k,p}$, and the single particle eigenfunctions are $\psi_p(r)$'s. (30)

2. Express the lesser and greater Green's functions in terms of the spectral function for fermions and bosons! (15)

3. Go beyond the realm of linear response and determine the the second order correction to the expectation value of a physical observable, A, when a weak external field, $h_B(t)$ is applied by coupling it to operator B. Pay attention to the normalization of the wavefunction! (30)

(a) Write down the corresponding Schrödinger equation, and try to approximate its solution by a 2 step iteration!

(b) Normalize the wavefunction!

(c) Evaluate the expectation value of operator A to second order in the external field.

4. A spin-1/2 particle is placed in an external magnetic field as $\mathbf{B} = (B_1 \cos(\omega t), B_1 \sin(\omega t), B_0)$, where $B_0 \gg B_1$. (25)

(a) Treating the oscillating part of the Hamiltonian as the interaction, write down the Schrödinger equation in the interaction representation.

(b) Find the time evolution operator, $U(t) = T_t \exp\left[-i \int_0^t H_{int}(t')dt'\right]$ by solving the corresponding Schrödinger equation (hint: use Fourier transformation).

(c) If the particle starts out at time $t = 0$ from the eigenstate $S_z = -1/2$, what is the probability to find in in the very same state at time t later?