Theoretical Nanophysics, Lecture 7. 2019/03/27

Lect. 6 cont.

VI. Current noise spectral density of a mesoscopic conductor

⋮

③ Results:

spin

Ⓐ Landauer-Büttiker formula: $\langle \hat{I}(t) \rangle = (2\times)\frac{e^2}{h}\tau V$

$$\rightarrow G = (2\times)\frac{e^2}{h}\tau$$

Ⓑ Noise:

$$S(\omega) = \frac{e^2}{h}\left\{ 2\tau^2 \frac{\hbar\omega}{1-e^{-\beta\hbar\omega}} + \right.$$

$$\left. + \tau(1-\tau)\left[\frac{\hbar\omega + eV}{1-e^{-\beta(\hbar\omega+eV)}} + \frac{\hbar\omega - eV}{1-e^{-\beta(\hbar\omega-eV)}} \right] \right\}$$

(B/1) Zero-frequency noise at $T=0$:

$$S(\omega=0; T=0) = (2\times)\, e \frac{e^2}{h} V \tau(1-\tau)$$

$$\rightarrow \text{Fano factor } F = \frac{\tau(1-\tau)}{\tau} = 1-\tau \quad \begin{cases} \tau \ll 1 \rightarrow F \approx 1 \\ \tau = 1 \rightarrow F = 0 \end{cases}$$

(B/2) Zero-frequency noise at finite T → Fig 1.

$S(\omega=0; V, T, \tau)$ → two-parameter fit (T, τ) to data →
→ primary thermometer

(B/3) $S(\omega; T; V=0) = (2\times)\, 2\frac{e^2}{h}\tau \frac{\hbar\omega}{1-e^{-\beta\hbar\omega}}$ ⟶ Fig 2.

(i) this is not white noise (ii) even symmetrized version is not white

Fig 1: Zero-frequency noise measurement provides a primary thermometer

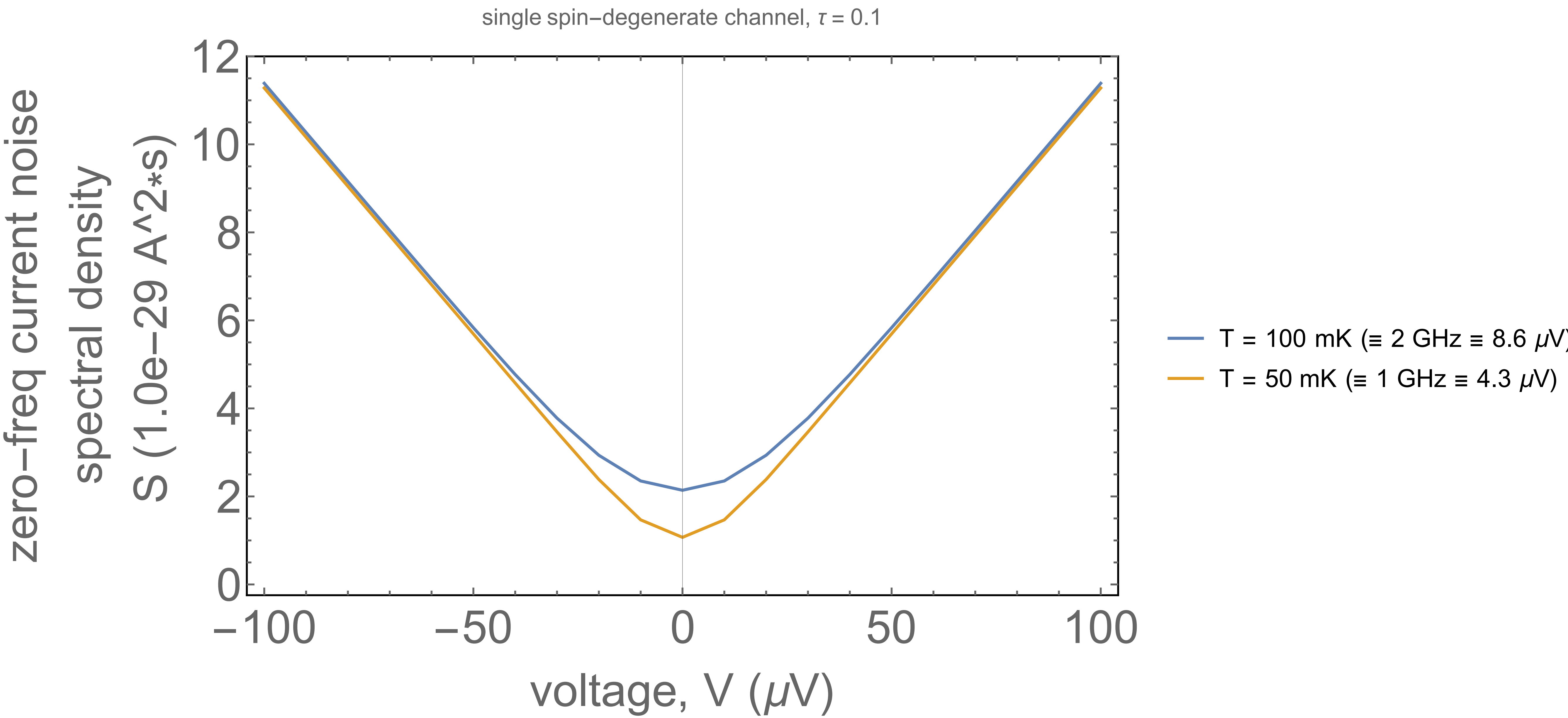
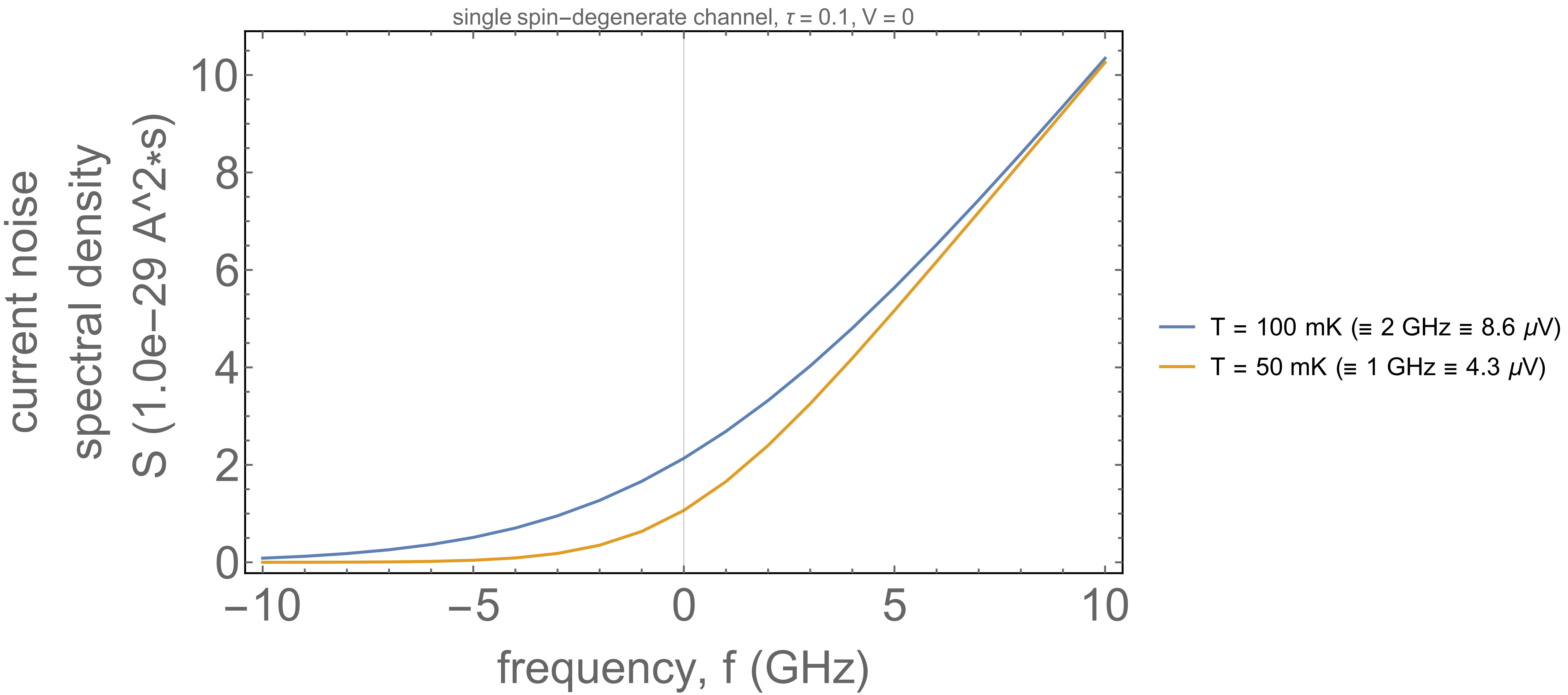


Fig 2: Quantum noise is definitely not white



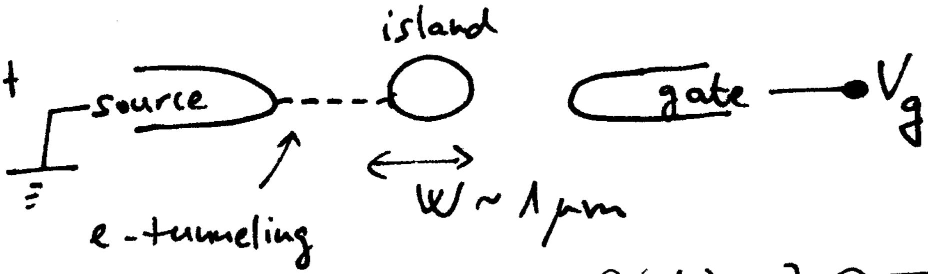
Electronic circuits with a nano-sized island

Theoretical Nanophysics
BME, 2019 Spring
Lectures 7-8, 2019/03/27, 04/03

Yuli Nazarov and Yaroslav Blanter
Quantum Transport (Introduction to Nanoscience)
Cambridge University Press, 2009
chapter 3.1, 3.2

Lec. 7 (I) Electrical circuits with a metallic nano-island

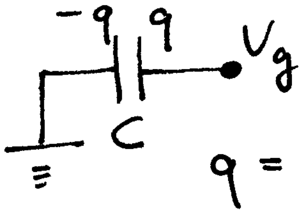
(1) 'Capacitor' circuit



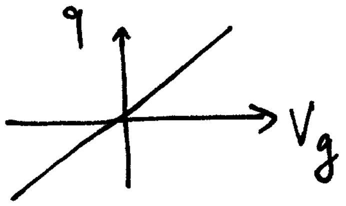
$Q(V_g) = ?$ @ $T=0$

island charge

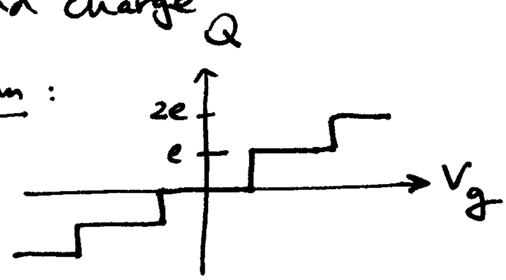
classical capacitor: linear



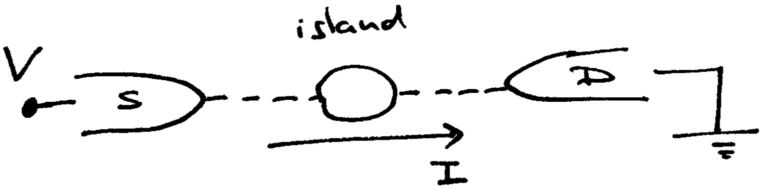
$q = CV_g$



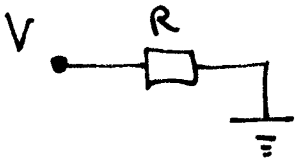
claim :



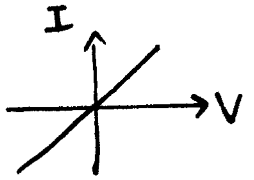
(2) 'Resistor' circuit



classical resistor: linear

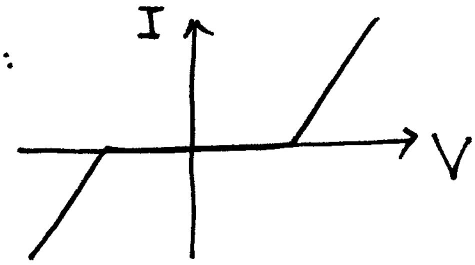


$I(V) = G \cdot V = \frac{V}{R}$

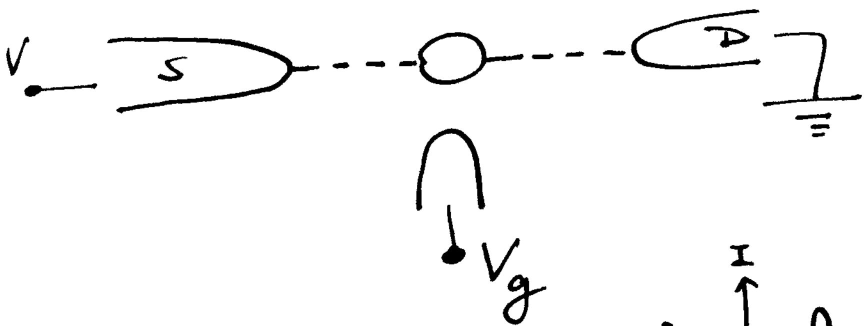


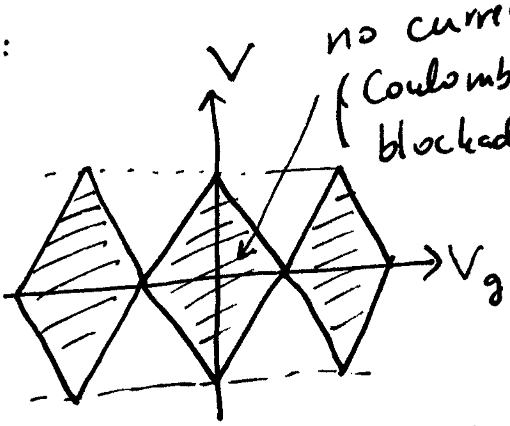
$I(V) = ?$

claim :



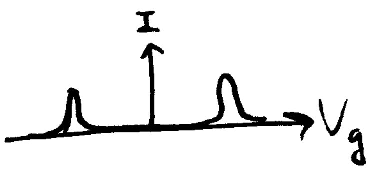
(3) 'Sigle-electron transistor' circuit :





no current (Coulomb blockade)

for low bias (V) :



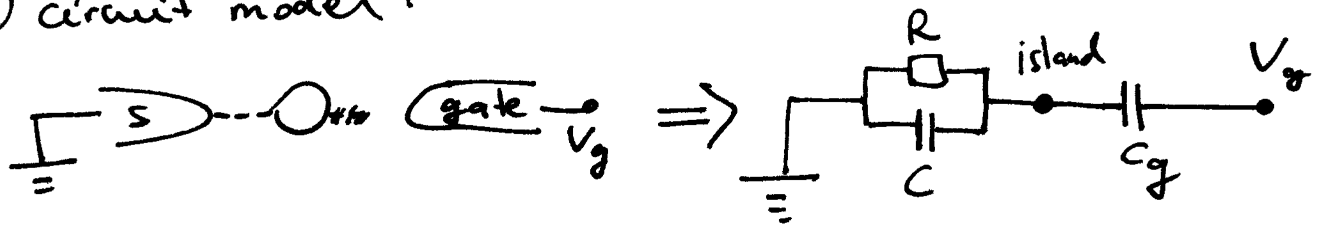
similar to field-effect transistor.

II Capacitor setup

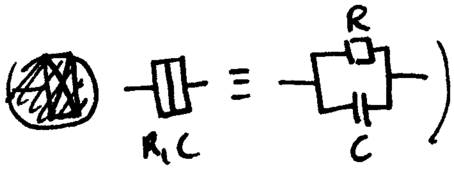
1. $Q(V_g) = ?$ at thermal equilibrium, $T=0$

strategy: (i) disregard size quantization of kinetic energy
(ii) use classical electrostatics
(iii) assume N_e excess electrons on island
calculate total energy $E(N_e)$
ground state occupation: N_e^{GS} minimizing $E(N_e)$

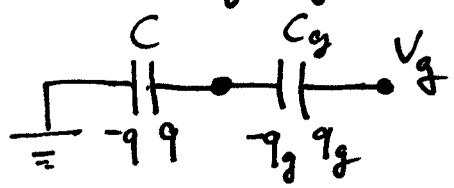
2. circuit model:



3. Reference case for energy: $V_g = 0$, $q = q_g = 0$, $E = 0$

(shorthand: 

)



4. finite V_g: $E = \dfrac{q^2}{2C} + \dfrac{q_g^2}{2C_g} - V_g q_g$

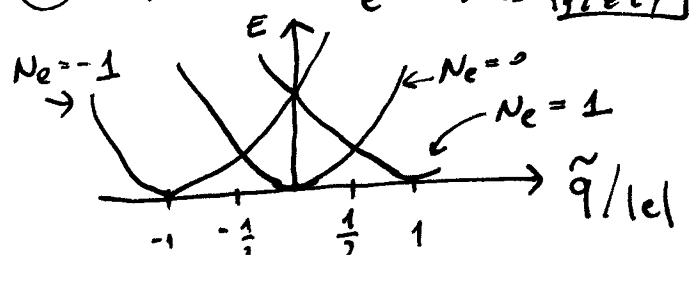
5. determine q, q_g from (i) $q - q_g = -|e| N_e$
(ii) $qC + q_g C_g = V_g$

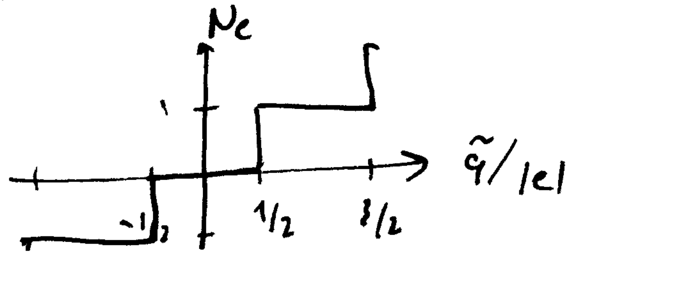
6. $$E = E_C\left(N_e - \frac{\tilde q}{|e|}\right)^2 \underbrace{- \frac{\tilde q^2}{2C_g}}$$

$E_C := \dfrac{e^2}{2(C+C_g)}$, $\tilde q := C_g V_g$

'charging energy' 'effective gate charge'

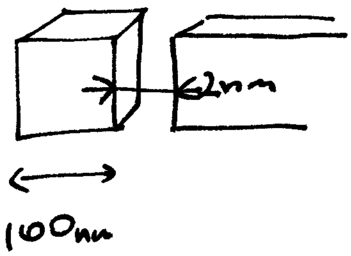
7. determine N_e^{GS}: this ~~factor~~ term is irrelevant.





(8) $T \approx 0$ approx: when does it make sense? If $T \lesssim 10K$

example:

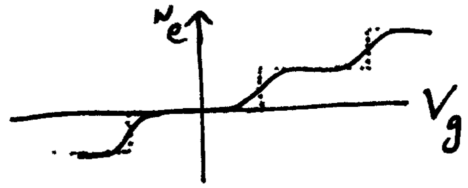


... $C = \frac{\varepsilon_0 A}{d} \approx 44\,aF \to E_C \approx 0.9\,meV$

$E_C \approx k_B \cdot (10K)$

(9) Description for $T > 0$: Boltzmann weights for excited states

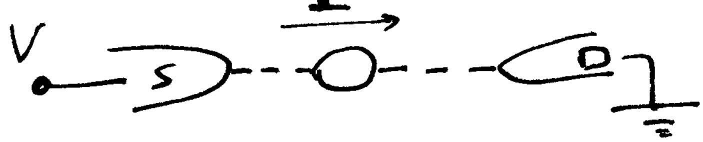
$\to$



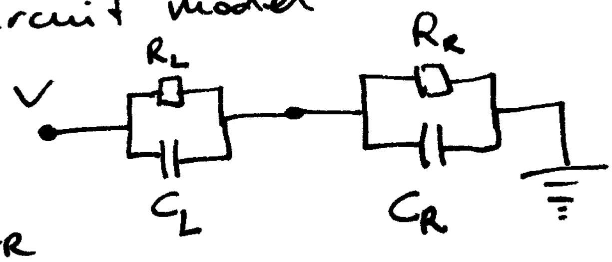
'smeared steps'

(III) Resistor setup

(1) $I(V) = ?$ @ $T=0$



$\to$ circuit model:

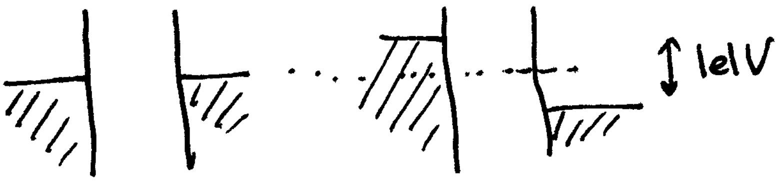


(2) $E(N_e) = E_C N_e^2$ $\leftarrow$ assume symmetric bias and $C_L = C_R$

$\uparrow$

charging energy, $E_C = \frac{e^2}{2C_\Sigma}$ $\qquad C_\Sigma = C_R + C_L$

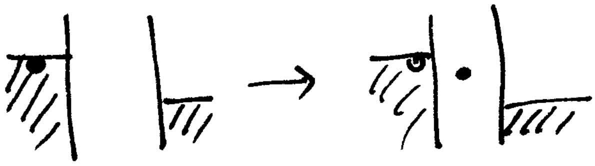
(3) example: GS: $N_e = 0$, low voltage



$V=0$

conditions for current flow:

(A)



energetically allowed

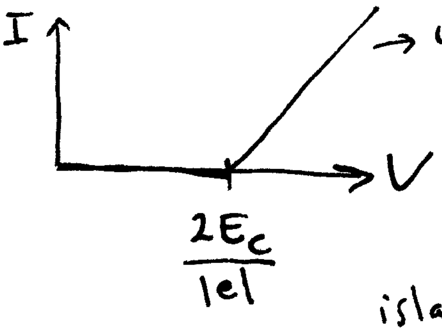
(B)

—— " ——

Ⓐ $\frac{1}{2}|e|V + E_0 \geqslant E_1$ Ⓑ $E_1 \geqslant E_0 + (-\frac{1}{2}|e|V)$

$\hookrightarrow |e|V \geqslant 2E_c$ $\hookrightarrow |e|V \geqslant -2E_c$

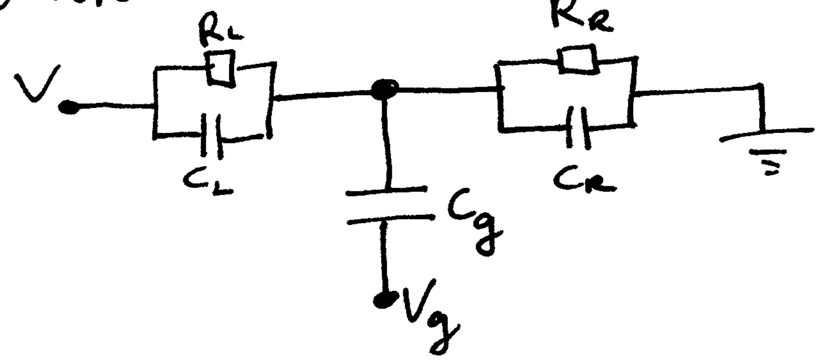
$\hookrightarrow$ this is the relevant condition



→ why ~linear? ↓ island is metallic

IV Single-electron transistor setup

① circuit model:



~~② $E = E_c\left(N_e - \frac{\tilde{q}}{|e|}\right)^2$ $\frac{e^2}{2C_\Sigma} \leftarrow C_L + C_R + C_g$~~

② assume symmetric bias and $C_L = C_R$

then $E(N_e) = E_c\left(N_e - \frac{\tilde{q}}{|e|}\right)^2$

$\frac{e^2}{2C_\Sigma} \leftarrow C_L + C_R + C_g = 2C_L + C_g$

③ consider $N_e = 0$ as the ground state, $V_g > 0$ $\left(V_g \in [0, \frac{e}{2C_g}]\right)$

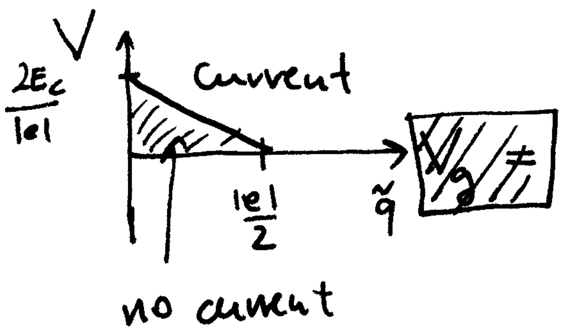
conditions for current flow: Ⓐ, Ⓑ above in III / ③ $\left(\tilde{q} \in [0, |e|/2]\right)$

Ⓐ $\frac{1}{2}|e|V + E_0 \geqslant E_1$

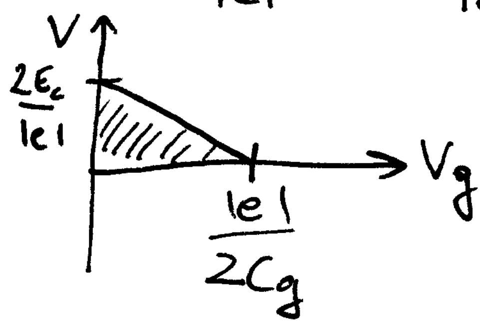
Ⓑ $E_1 \geqslant E_0 - \frac{1}{2}|e|V$

(4) $\frac{1}{2}|e|V + E_c \frac{\tilde{q}^2}{e^2} \geq E_c\left(1-\frac{\tilde{q}}{|e|}\right)^2$

$$\frac{1}{2}|e|V \geq E_c\left(1-\frac{2\tilde{q}}{|e|}\right) \rightarrow V \geq \frac{2E_c}{|e|}\left(1-\frac{2\tilde{q}}{|e|}\right)$$



$\rightarrow$



(5) similar analysis yields figure (Coulomb diamonds) for generic V, V_g in I/(3)

(6) 'application': $I(V,V_g)$ reveals C_L, C_R, C_g capacitances.

(V) Discussion

(1) size quantization of kinetic energy neglected → correct?

recall $E_c \approx 1\,\mathrm{meV}$

nr of electronic levels in 1meV energy window?

~~$N_{levels} = \frac{1}{\rho(\varepsilon_F)} = \frac{1}{\tilde{\rho}(\varepsilon_F) W^3} = \frac{1}{\ }$~~

~~Density of states~~ ~~DoS per unit volume~~

Sólyom Ch. 16.2.4

$\sim \frac{\pi}{2a}$ $\sim 5\,\text{Å}$

$$N_{levels} = (1\,\mathrm{meV})\cdot\tilde{\rho}(\varepsilon_F) = (1\,\mathrm{meV})\cdot\rho(\varepsilon_F)\cdot W^3 = (1\,\mathrm{meV})\cdot\frac{m_e k_F}{\hbar^2\pi^2} \approx 4200 \gg 1.$$

density of state

DoS per unit volume

$(100\,\mathrm{nm})^3$

Theoretical Nanophysics, Lecture 8. 2019/04/03

Lec 7: Electrical circuits with a nano-island

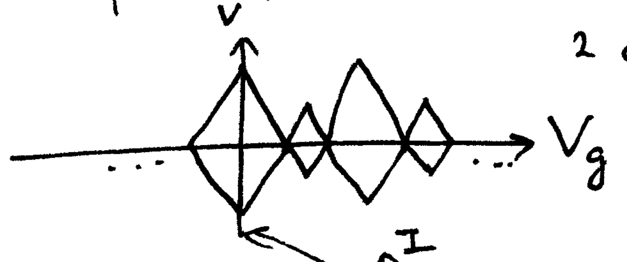
I ... V Discussion

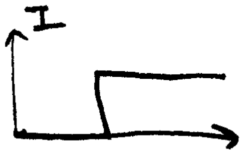
② typically k_F (semiconductor) $\ll k_F$ (metal)

$\rightarrow$ in semiconductor islands (quantum dots), $\Delta E \approx E_C \approx 1\,\mathrm{meV}$ is possible.

③ consequence for SET setup ↓

2 different Coulomb diamonds

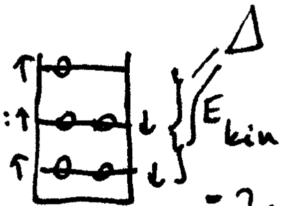




step instead of 

④ 'proof':

$$E(N_e) = E_C\left(N_e - \frac{\tilde{q}}{|e|}\right)^2 + E_{kin}(N_e)$$

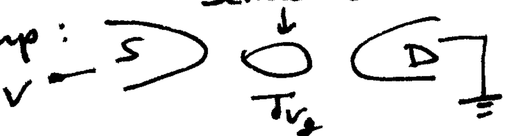
estimate: 

$E_{kin}(5) = 2\cdot\Delta + 2\cdot(2\Delta) + 3\Delta = 9\Delta$

+ see VI below.

VI Current (and counting statistics) from Fermi's Golden Rule

① goal: to derive a master equation ~~and~~ for current (and c.s.)

② setup: 

spin neglected; assume large Δ, E_C.

$T = 0$

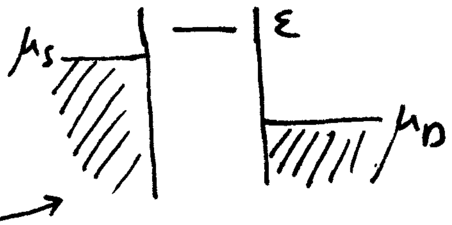
③ model (spinless Anderson model)

$$H = H_S + H_{tS} + H_i + H_{tD} + H_D$$

$$H_S = \sum_{\underline{k}} \varepsilon_{\underline{k}} c^{+}_{S\underline{k}} c_{S\underline{k}}, \quad H_{tS} = t \sum_{\underline{k}} c^{+}_{S\underline{k}} d + h.c.$$

$$H_i = \varepsilon d^{+} d, \quad H_{tD} = t \sum_{\underline{k}} c^{+}_{D\underline{k}} d + h.c., \quad H_D = \sum_{\underline{k}} \varepsilon_{\underline{k}} c^{+}_{D\underline{k}} c_{D\underline{k}}$$

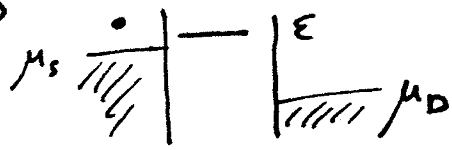
(4) Source is filled up to μ_S
drain ——— " ——— μ_D



(5) Think pert. theory: $H_0 = H_S + H_i + H_D$, $H_1 = H_{tS} + H_{tD}$

H_0 eigenstates: $|\mu_S, 0, \mu_D\rangle$, $c^{+}_{S\underline{k}} |\mu_S, 0, \mu_D\rangle, \dots$

$|\mu_S, 1, \mu_D\rangle, \dots$

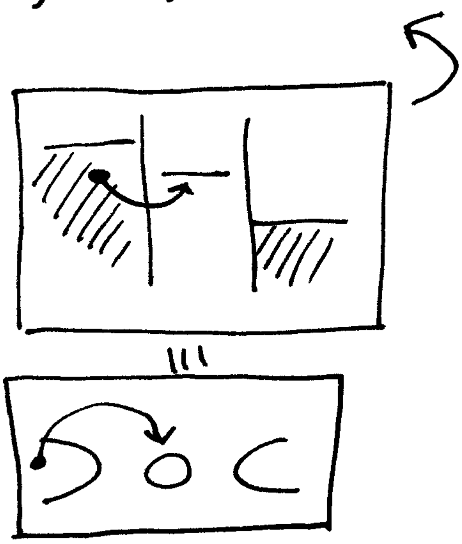


(6) Fermi's Golden Rule, second-order: transition rate b/w i and f:

$$\Gamma_{f \leftarrow i} = \frac{2\pi}{\hbar} |\langle f | H_1 | i \rangle|^2 \delta(\varepsilon_f - \varepsilon_i)$$

(7) Here: $$\Gamma^{S}_{1 \leftarrow 0} = \frac{2\pi}{\hbar} \sum_{\underline{k}} |\langle \mu_S, 1, \mu_D | c^{+}_{S\underline{k}} H_1 | \mu_S, 0, \mu_D \rangle|^2 \delta(\varepsilon - \varepsilon_{\underline{k}})$$

similar for $\Gamma^{D}_{1 \leftarrow 0}$, $\Gamma^{S}_{0 \leftarrow 1}$, $\Gamma^{D}_{0 \leftarrow 1}$



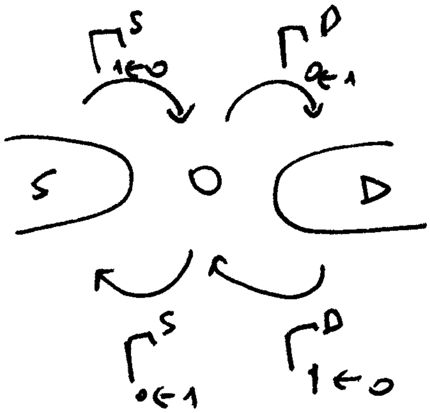
(8) dot described by occupation prob:

$P_1(t)$ and $P_0(t) = 1 - P_1(t)$

(9) 'simple' master eq.

$$\partial_t P_0(t) = -(\Gamma^{S}_{1 \leftarrow 0} + \Gamma^{D}_{1 \leftarrow 0}) P_0(t) + (\Gamma^{S}_{0 \leftarrow 1} + \Gamma^{D}_{0 \leftarrow 1}) P_1(t)$$

$$\partial_t P_1(t) = -(\Gamma^{S}_{0 \leftarrow 1} + \Gamma^{D}_{0 \leftarrow 1}) P_1(t) + (\Gamma^{S}_{1 \leftarrow 0} + \Gamma^{D}_{1 \leftarrow 0}) P_0(t)$$



$$\partial_t \underline{P}(t) = \underline{\underline{M}}\,\underline{P}(t)$$

(10) total probability is conserved, as

$$\sum_i M_{ij} = 0, \text{ hence } \partial_t \sum_i P_i(t) = 0.$$

(11) Electric current (between i and D): $I = -|e|\left[P_1(t)\,\Gamma^D_{0\leftarrow 1} + P_0(t)\,\Gamma^D_{1\leftarrow 0}\right]$

(12) Example: $\varepsilon = 0$, $\mu_D = -1\,\text{meV}$, $\mu_S = +1\,\text{meV}$, $T = 0$

$$\Gamma^S_{1\leftarrow 0} \neq 0, \quad \Gamma^D_{0\leftarrow 1} \neq 0, \quad \Gamma^S_{0\leftarrow 1} = \Gamma^D_{1\leftarrow 0} = 0.$$

$$\left.\begin{array}{l}\text{symmetric case } \left(\Gamma^S_{1\leftarrow 0} = \Gamma^D_{0\leftarrow 1} \equiv \Gamma\right) \\ \text{steady state } (\partial_t P_i = 0)\end{array}\right\} \rightarrow P_0 = P_1 = \frac{1}{2}$$

$$\downarrow$$

$$\boxed{I = -|e|\frac{\Gamma}{2}}$$

(13) Rates:

$$\Gamma^S_{1\leftarrow 0} = \frac{2\pi |t|^2 \rho(\varepsilon)}{\hbar}\,\Theta(\mu_S - \varepsilon) \quad \text{(Heaviside fn.)}$$

$$\Gamma^S_{0\leftarrow 1} = \frac{2\pi |t|^2 \rho(\varepsilon)}{\hbar}\,\Theta(\varepsilon - \mu_S)$$

$$\Gamma^D_{1\leftarrow 0} = \frac{2\pi |t|^2 \rho(\varepsilon)}{\hbar}\,\Theta(\varepsilon - \mu_D)$$

$$\Gamma^D_{0\leftarrow 1} = \frac{2\pi |t|^2 \rho(\varepsilon)}{\hbar}\,\Theta(\mu_D - \varepsilon)$$

$$\rho(\varepsilon) = \sum_k \delta(\varepsilon - \varepsilon_k)$$

$\uparrow$ density of states in leads S, D.

(14) example calculation:

$$\Gamma^{S}_{1\leftarrow 0} = \frac{2\pi}{\hbar}\sum_{\underline{k}} \left|\langle \mu_S, 1, \mu_D | c^{+}_{S\underline{k}} H_1 | \mu_S, 0, \mu_D\rangle\right|^2 \delta(\underbrace{(\overset{\varepsilon_f}{\varepsilon - \varepsilon_{\underline{k}}})}_{} - \overset{\varepsilon_i}{0})$$

$$= \frac{2\pi}{\hbar}\sum_{\underline{k}} \left|\langle \mu_S, 1, \mu_D | c^{+}_{S\underline{k}}\, t \sum_{\underline{k}'} d^{+} c_{S\underline{k}'} | \mu_S, 0, \mu_D\rangle\right|^2 \delta(\varepsilon - \varepsilon_{\underline{k}})$$

$$= \frac{2\pi}{\hbar}\sum_{\underline{k}} |t|^2 \delta(\varepsilon - \varepsilon_{\underline{k}})\,\Theta(\mu_S - \varepsilon_k) = \frac{2\pi}{\hbar}|t|^2\,\Theta(\mu_S - \varepsilon)\underbrace{\sum_{\underline{k}}\delta(\varepsilon - \varepsilon_{\underline{k}})}_{\rho(\varepsilon)}$$

(15) exercise: relation between Anderson-model parameters and params of classical electrostatics model?

(16) Generalizations:

(i) N-resolved master eq. for counting statistics

(ii) finite T $\longrightarrow$ Θ-s turn into f-s (Fermi-Dirac)

(iii) add spin $\longrightarrow$ Anderson model

(iv) 'arbitrary' molecules between S and D

(v) quantum master eq for phase-coherent effects

(vi) cotunneling, e.g., from higher-order FGR.