A1

The charged π particles (π mesons or pions) (π_+ or π_-) are unstable particles that decay with half time $t_{1/2} = 0.7836 \cdot 10^{-8} s$ in the reference frame where they rest. We have produced a ray of π-mesons, where the velocity of the particles is $0.8c$.

(a) What half-time do we measure for the mesons?

(b) Let's assume we drive the π-mesons trough a tunnel of length $d = 36m$. What fraction of the particles decay in the tunnel?

(c) What result would we get if we used non-relativistic approximation?

(d) What is the length of the tunnel in the π-mesons' frame of reference?

(e) (*) How long does it take (from the particles point of view) to cross the tunnel?

(f) (*) What fraction of them decay in the tunnel, if we calculate in the reference frame of π-mesons?

(Questions denoted by * are just for practice. They will not appear in small tests.)

A2

Consider the following space-time transformation:

$$\Lambda^{\mu}_{\nu} = \begin{pmatrix} 2 & 0 & \sqrt{3} & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

(a) There is a 4-vector: $b_\mu = (5, 4, 0, 3)$. Transform this vector using the above transformation and write down the b'_μ transformed coordinates!

(b) Determine the Minkowski length-square of the original vector b_μ.

(c) Determine the Minkowski length-square of the transformed vector b'_μ, and show that it remained invariant.

(d) Show in general that the transformation is a Lorentz transformation.

A3

Consider the famous “twin-paradox”. Let's call the twins Bobby and George. Bobby travels to one of the exo-planets of the Alpha Centauri system, whose distance is 4.5lightyears from the Earth, and it is almost in rest in the reference frame of the Earth. The maximal speed of the spacecraft is $0.75c$, and the accelaration and braking times are negligeble.

After reaching his destination, Bobby studies the exo-planet for 1year time and then he travels back to Earth.

a.) Draw the world lines of Bobby and George in the Minkowski plane.

b.) How many years does George age, who stayed on Earth, until Bobby gets back to the Earth?

c.) How many years does Bobby age at the same time?

A (wrong) explanation of the twin paradox states, that the different aging is caused by the acceleration of Bobby. Indeed, Bobby needs to have nonzero acceleration, if he wants to come home. However, using the following thought experiment we can exclude that explanation.

Let's suppose that Bobby and George both travel on the spacecraft, but at half distance George decides to stop – using the spacecrafts rescure cabin – and takes a long holiday at a space-motel that rests in the frame of Earth. When Bobby is traveling back, George accelerates his cabin to 0.75 c, joins Bobby in the spacecraft, and they arrive together back to Earth. We can see, that in this case George and Bobby can have exactly the same acceleration processes.

d.) Draw the modified world line of George in the figure!

e.) How much time does George spend in the motel?

f.) What is George's total aging during the travel?

B1

A pair of twins (Arnold and Bruce) have bought two interstellar spacecrafts.

After departing from Earth, Arnold accelerates to $0.5c$ and then with constant velocity he travels to the Alpha-Centauri system, where he lands on an exo-planet.

Bruce chooses a slightly different schedule. He accelerates to $0.99c$, but at half way he stops in the motel that was also mentioned in the problem above. After a (long) holiday he accelerates again to $0.99c$ and arrives to the Alpha Centauri at the same time as Arnold.

(a) Draw the world lines of Arnold and Bruce in the Minkowski plane.

(b) How much time does it take (measured in the Earth) for Arnold and Bruce to reach their destination?

(c) What is the aging of Arnold?

(d) How much time does it take for Bruce to reach the motel?

(e) How much time does Bruce spend in the motel?

(f) What is Bruce's total aging?

B2

The space destroyer of the tall blond aliens broke down, and now it travels with constant velocity in space. Their worst enemies, the small greys, in their space station realize the great opportunity, and target the blonds' destroyer. The scientists of the greys' space station have calculated that the destroyer will pass the space station with minimal distance of d, and its velocity is $0.5c$. They also calculated the time, when they have to shoot, if they want to hit the destroyer closest to the space station. In their calculations they use the reference frame of the space station, but they put the origin to the explosions position, and set $t = 0$ to the event of the explosion.

(a) Introduce a convenient coordinate system.

(b) Determine the time t0 ¡ 0, when the small greys have to shoot to hit the destroyer.

(c) Determine the bullets position r(t) as a function of time.

(d) The tall blond aliens have kidnapped Attila P., the famous Hungarian rockstar and amateur UFO-scientist. Mr. Attila P. developed the theory of "relativity of everithing", and withouth carrying any calculations he soothes the aliens by stating that in their frame of reference, the destroyer will not explode.

Determine the bullets position $r'(t')$ as a function of time in the destroyers frame of reference.

(e) Determine the exact time and position, when and where the bullet was shot.

(f) Should the blond aliens worry?

B3

Let Λ_1 be the Lorentz transformation that transforms to the frame of reference that moves in the direction $+x$ with velocity $0.6c$. Let Λ_2 be the trasformation that transforms to the frame of reference that moves in the direction $+y$ with velocity $0.6c$.

(a) Write down the matrices of Λ_1 and Λ_2.

(b) Apply the two transformation consecutively. Determine the matrices of the transformations $\Lambda = \Lambda_2\Lambda_1$ and $\Lambda' = \Lambda_2\Lambda_1$. Show that they are different.

(c) Consider a particle that rests in the system to where the Λ transformation leads. What is the velocity vector of this particle move in the original inertial system?

(d) Repeat the calculation of c.) for a particle that rests in the system to where the Λ' transformation leads. Are the velocity vectors of c.) and d.) the same?