OPTICS 1: GEOMETRICAL OPTICS

THEORY[1]

Geometrical optics, or ray optics, is a model of optics that describes light propagation in terms of rays. The ray in geometric optics is an abstraction useful for approximating the paths along which light propagates under certain circumstances.

The simplifying assumptions of geometrical optics include that **light rays**:

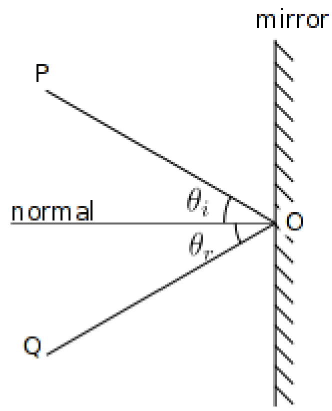
- propagate in straight-line paths as they travel in a homogeneous medium
- bend, and in particular circumstances may split in two, at the interface between two dissimilar media
- follow curved paths in a medium in which the refractive index changes
- may be absorbed or reflected.

Geometrical optics provides rules for propagating these rays through an optical system. The path taken by the rays indicates how the actual wave will propagate. This is a significant simplification of optics that fails to account for optical effects such as diffraction and polarization. It is a good approximation, however, when the wavelength is very small compared with the size of structures with which the light interacts. Geometrical optics can be used to describe the geometrical aspects of imaging, including optical aberrations.

Reflection

Glossy surfaces such as mirrors reflect light in a simple, predictable way. This allows for the production of reflected images that can be associated with an actual (real) or extrapolated (virtual) location in space.

With such surfaces, the direction of the reflected ray is determined by the angle the incident ray makes with the surface normal, a line perpendicular to the surface at the point where the ray hits. The incident and reflected rays lie in a single plane, and the angle between the reflected ray and the surface normal is the same as that between the incident ray and the normal. This is known as the ***Law of Reflection***.

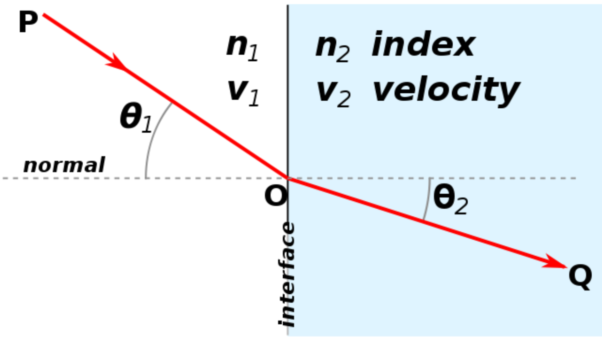


Mirrors with curved surfaces can be modeled by ray tracing and using the law of reflection at each point on the surface. For mirrors with parabolic surfaces, parallel rays incident on the mirror produce reflected rays that converge at a common focus. Other curved surfaces may also focus light, but with aberrations due to the diverging shape causing the focus to be smeared out in space. In particular, spherical mirrors exhibit spherical aberration. Curved mirrors can form images with a magnification greater than or less than one, and the image can be upright or inverted. An upright image formed by reflection in a mirror is always virtual, while an inverted image is real and can be projected onto a screen.

[1] The paragraphs and images were copied from Wikipedia: http://en.wikipedia.org/wiki/Optics, http://en.wikipedia.org/wiki/Geometrical_optics, https://en.wikipedia.org/wiki/Brewster's_angle

Refraction

Refraction occurs when light travels through an area of space that has a changing index of refraction. The simplest case of refraction occurs when there is an interface between a uniform medium with an index of refraction n_1 and another medium with an index of refraction n_2. In such situations, ***Snell's Law*** describes the resulting deflection of the light ray:



$$n_1 \sin\Theta_1 = n_2 \sin\Theta_2 \qquad [1]$$

where Θ_1 and Θ_2 are the angles between the normal (to the interface) and the incident and refracted waves, respectively. This phenomenon is also associated with a changing speed of light as seen from the definition of the index of refraction provided above which implies:

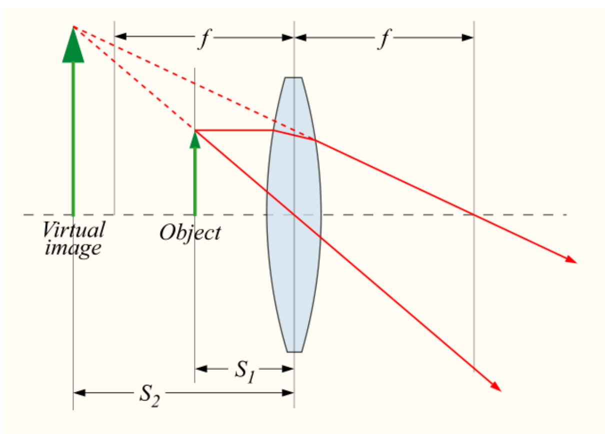
$$v_1 \sin\Theta_2 = v_2 \sin\Theta_1$$

where v_1 and v_2 re the wave velocities through the respective media.

Various consequences of Snell's Law include the fact that for light rays traveling from a material with a high index of refraction to a material with a low index of refraction, it is possible for the interaction with the interface to result in zero transmission. This phenomenon is called ***total internal reflection*** and allows for fiber optics technology. As light signals travel down a fiber optic cable, it undergoes total internal reflection allowing for essentially no light lost over the length of the cable. It is also possible to produce polarized light rays using a combination of reflection and refraction: When a refracted ray and the reflected ray form a right angle, the reflected ray has the property of "plane polarization". The angle of incidence required for such a scenario is known as *Brewster's angle*.

Snell's Law can be used to predict the deflection of light rays as they pass through "linear media" as long as the indexes of refraction and the geometry of the media are known. For example, the propagation of light through a ***prism*** results in the light ray being deflected depending on the shape and orientation of the prism. Additionally, since different frequencies of light have slightly different indexes of refraction in most materials, refraction can be used to produce ***dispersion spectra*** that appear as rainbows.

A device that produces converging or diverging light rays due to refraction is known as a ***lens***. Thin lenses produce focal points on either side that can be modeled using the lensmaker's equation. In general, two types of lenses exist: convex lenses, which cause parallel light rays to converge, and concave lenses, which cause parallel light rays to diverge. The detailed prediction of how images are produced by these lenses can be made using ray-tracing similar to curved mirrors. Similarly to curved mirrors, thin lenses follow a simple equation that determines the location of the images given a particular ***focal length (f)*** and **object distance d_{obj}** (S_1 on the figure):



$$\frac{1}{d_{obj}} + \frac{1}{d_{img}} = \frac{1}{f} \qquad [2]$$

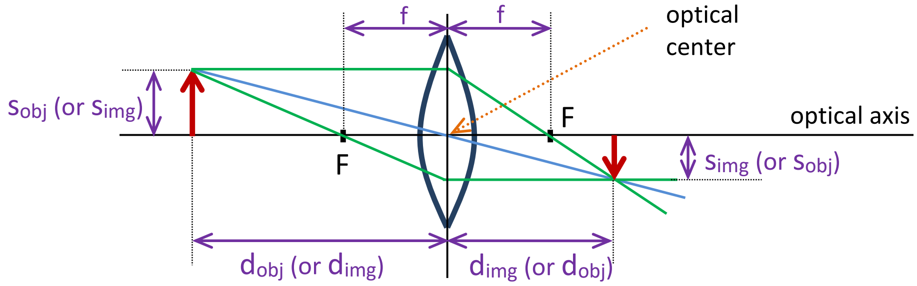
where d_{img} **is the distance associated with the image** (S_2 on the figure) and is considered by convention to be negative if on the same side of the lens as the object and positive if on the opposite side of the lens. The focal length f is considered negative for concave lenses.

Incoming parallel rays are focused by a convex lens into an inverted real image one focal length from the lens, on the far side of the lens. Rays from an object at a finite distance are focused further from the lens than the focal distance; the closer the object is to the lens, the further the image is from the lens. With convex lenses, incoming parallel rays diverge after going through the lens, in such a way that they seem to have originated at an upright virtual image one focal length from the lens, on the same side of the lens that the parallel rays are approaching on. Rays from an object at a finite distance are associated with a virtual image that is closer to the lens than the focal length, and on the same side of the lens as the object. The closer the object is to the lens, the closer the virtual image is to the lens.

Likewise, the **magnification** M of a lens is given by

$$M = -\frac{d_{img}}{d_{obj}} = -\frac{s_{img}}{s_{obj}} \qquad [3]$$

where s_{obj} and s_{img} are **the size of the object and the image**, respectively, and the negative sign is given, by convention, to indicate an upright object for positive values and an inverted object for negative values. Similar to mirrors, upright images produced by single lenses are virtual while inverted images are real.



Useful rays when finding the image of an object through a lens:
blue: the ray traveling from the object
passing directly through the optical center is not refracted;
green: the ray traveling from the object
parallel to the optical axis is refracted so that
it passes through the focal point if the lens is converging
(or in the case of a diverging lens it appears to come from the focal point).
Light travels along the same path between the "object" and the "image"
"forwards" and "backwards".

Lenses suffer from aberrations that distort images and focal points. These are both due to the geometrical imperfections and the changing index of refraction for different wavelengths of light (chromatic aberration).

Polarization

Polarization is a general property of waves that describes the orientation of their oscillations. For transverse waves such as many electromagnetic waves, it describes the orientation of the oscillations in the plane perpendicular to the wave's direction of travel. The oscillations may be oriented in a single direction (linear polarization), or the oscillation direction may rotate as the wave travels (circular or elliptical polarization). Elliptically polarized waves can rotate rightward or leftward in the direction of travel (chirality).

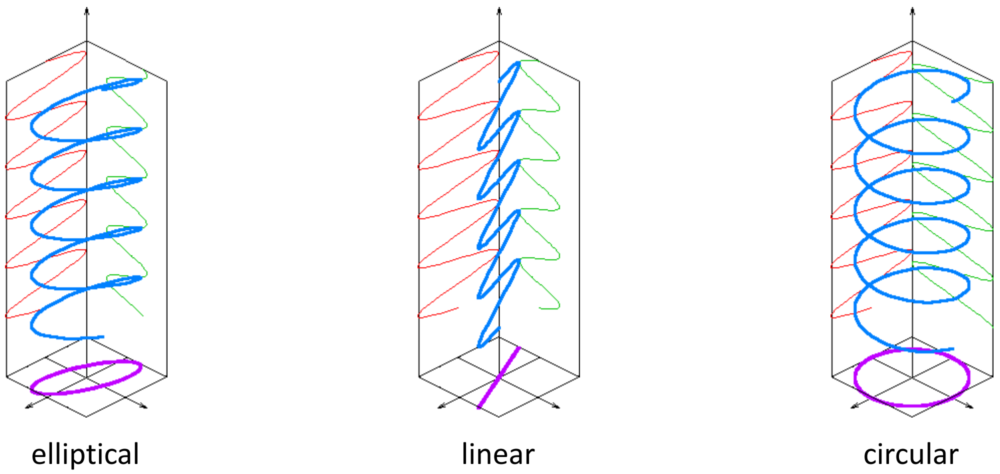
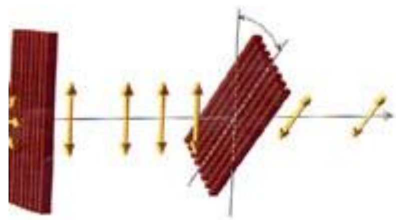


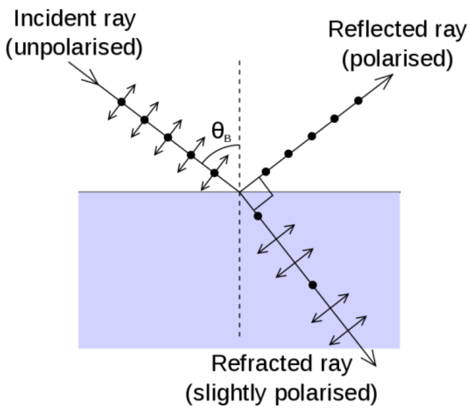
Figure: Evolution of the electric field vector (blue) with time (the vertical axes) at a particular point in space, along with its *x* (red/left) and *y* (green/right) components, and the path traced by the electric field vector in the *x* – *y* plane (purple).

There are media that have different indexes of refraction for different polarization modes; these are called *birefringent*. Media that reduce the amplitude of certain polarization modes are called *dichroic*. Devices that block nearly all of the radiation in one mode are known as *polarizing filters* or simply *polarizers*. By means of polarizers, the natural elliptically polarized light can be transformed to linearly polarized light.

A polarizing filter transforms natural elliptically polarized light to linearly polarized light.



A second polarizing filter rotates the plane of polarization by blocking the perpendicular radiation.



Brewster's angle

Brewster's angle (also known as the polarization angle) is an angle of incidence at which light with a particular polarization is perfectly transmitted through a transparent dielectric surface, with no reflection. When unpolarized light is incident at this angle Θ_B, the light that is reflected from the surface is therefore perfectly polarized.
At this angle the reflected and the refracted rays are perpendicular, so it is easy to show that

$$\mathrm{tg}\,\Theta_B = n \quad (n \text{ is the refractive index})$$

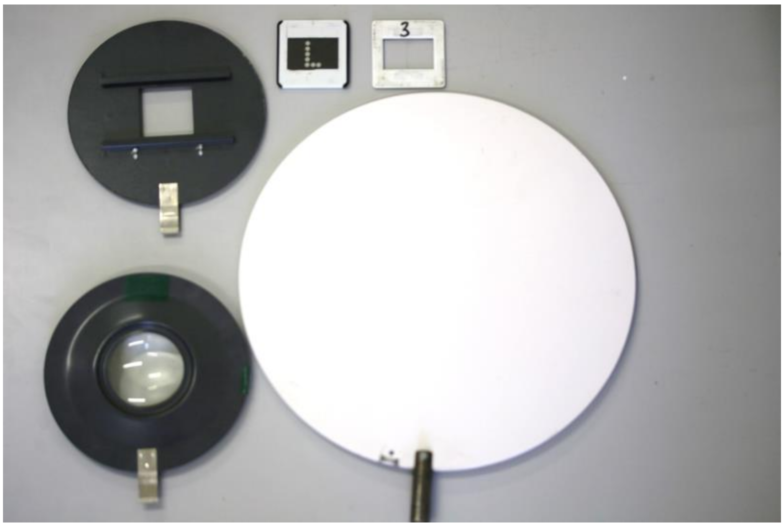
Optical rotation is the turning of the plane of linearly polarized light about the direction of motion as the light travels through certain materials. It occurs in solutions of chiral molecules such as sucrose (sugar), solids with rotated crystal planes such as quartz, and spin-polarized gases of atoms or molecules.

MEASUREMENTS

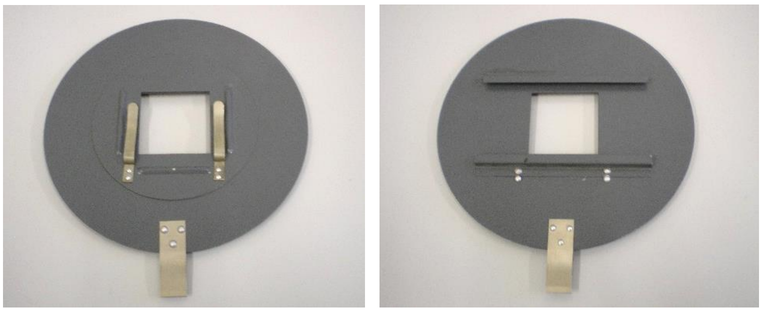
<u>Tools</u>

Optical bench with high and low holders.

Halogen lamp light source.



Screen, lens, slide holder, objects.



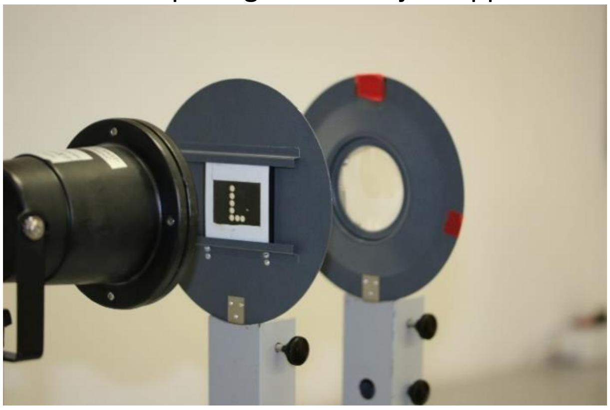
The two sides of the slide holder.

Gaps.

1. Determination of the focal length of a convex lens

Tools:
- optical bench
- halogen lamp light source
- slide holder
- a slide as object
- convex lens
- screen

Place the light source on one end of the optical bench and the screen on the other end. The teacher tells you the distance between the object and the screen. Place the object – i.e. the slide in the slide holder – accordingly between the lamp and the screen. Finally, place the lens between the object and the screen. Then by sliding the lens find that position or those positions, where a sharp image of the object appears on the screen.



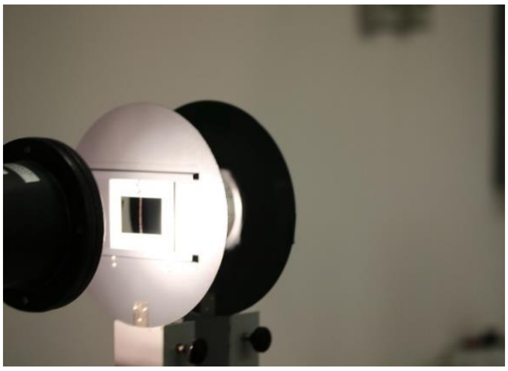

Measure the distance between the object and the lens d_{obj} and the distance between the image and the lens d_{img}, and measure (or estimate) the size of the image s_{img}.

Evaluation:
1/1. Calculate the focal length of the lens using the formula [2].
1/2. Calculate the error interval for the focal length using the error propagation formula.
Take 1 cm as the error of distance measurement for d_{obj} and d_{img}.
1/3. Calculate the magnification and the size of the object using the formula [3].
1/4. Draw a scaled diagram of the arrangement showing the most important rays.

2. Estimation of the thickness of a hair fiber using a lens

Tools:
- optical bench
- halogen lamp light source
- slide holder
- hair fiber in a slide
- convex lens, f = 50 mm
- screen

Place the light source on one end of the optical bench, the screen on the other end and in between the slide holder with the hair fiber and the lens. By sliding the lens find the largest possible sharp image.



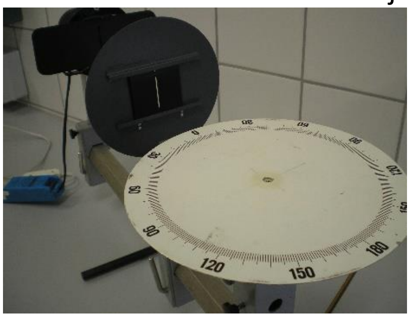
Measure the object distance d_{obj} and image distance d_{img}, and measure / estimate the thickness of the image of the hair fiber on the screen s_{img}.

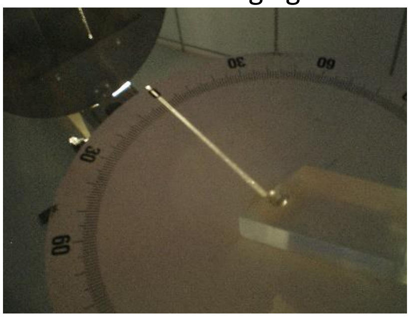
<u>Evaluation:</u>
2/1. Calculate s_{obj}, i.e. the thickness of a human hair.
2/2. Estimate the error of this calculation supposing that this error is mainly caused by the reading error of s_{img}.
Optional: 2/3. What else can contribute to the error of this calculation?

3. Determination of the refractive index of the prism

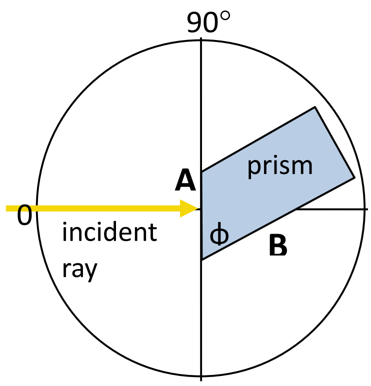
<u>Tools:</u>
- optical bench
- halogen lamp light source
- gaps
- slide holder
- rotating optical disc
- prism

Place the light source on one end of the optical bench and add the paper gap. Place a slide holder with the other gap 40-50 cm away and adjust the two gaps so that you get a ray parallel with the optical bench. Place the rotating disc in a second holder and adjust 0° to the incoming light.

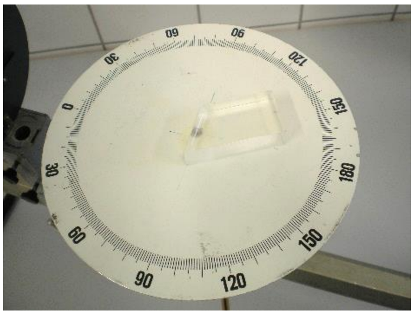


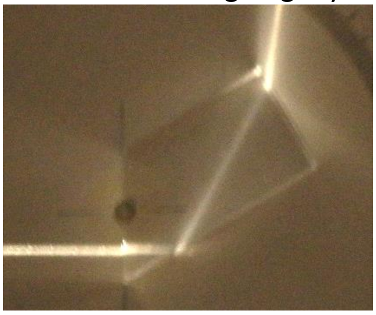


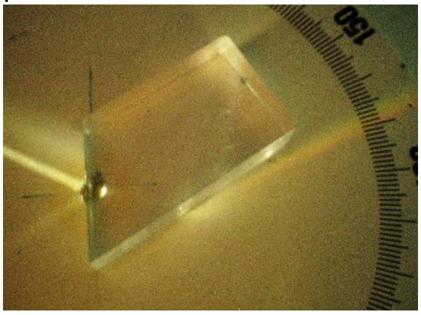
Place the prism on the rotating disc as shown in the figure:



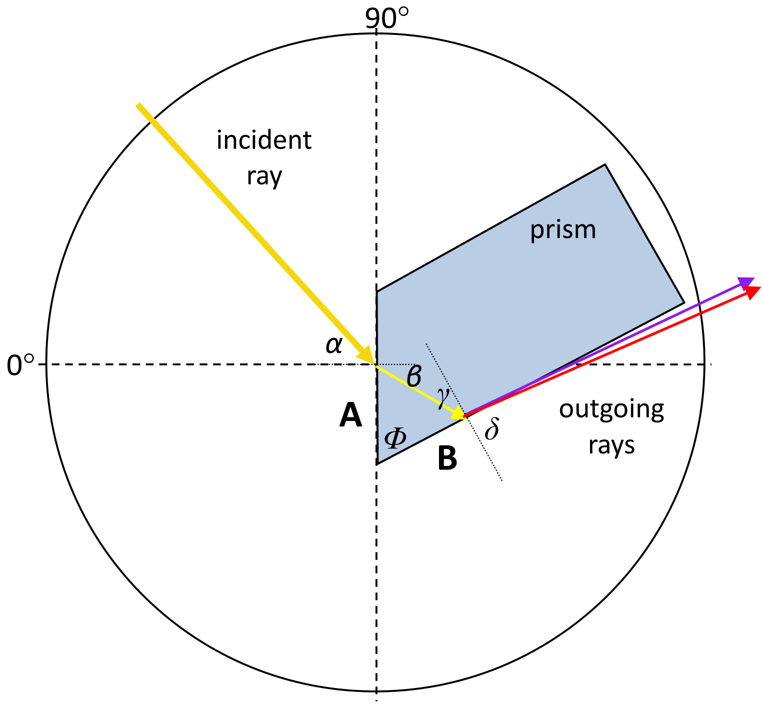
Now rotate the disc and find the position when outgoing rays appear.







Read the angle of incidence α at this position for the red and for the violet component.



Evaluation:

Derivation of the formula:

We know that

at the first interface $\sin\alpha = n\cdot\sin\beta$, at the second interface $n\cdot\sin\gamma = \sin\delta$, and from the geometry $\Phi = \beta + \gamma$.

For $\delta = 90°$ $\sin\delta = \sin 90° = 1$ $\rightarrow$ $n\cdot\sin\gamma = 1$;

and as $\gamma = \Phi - \beta$: $n\cdot\sin\gamma = n\cdot\sin(\Phi - \beta) = n\cdot(\sin\Phi\cdot\cos\beta - \cos\Phi\cdot\sin\beta) = n\cdot\sin\Phi\cdot\cos\beta - n\cdot\cos\Phi\cdot\sin\beta$.

Substituting $n\cdot\sin\beta = \sin\alpha$: $n\cdot\sin\Phi\cdot\cos\beta - \cos\Phi\cdot\sin\alpha = 1$ $\rightarrow$ $n\cdot\cos\beta = (1 + \cos\Phi\cdot\sin\alpha) / \sin\Phi$.

Adding the square of the last equation and the square of the equation $n\cdot\sin\beta = \sin\alpha$ we get

$n^2 = (1 + 2\cos\Phi\cdot\sin\alpha + \cos^2\Phi\cdot\sin^2\alpha)/ \sin^2\Phi + \sin^2\alpha = (1 + 2\cos\Phi\cdot\sin\alpha + \sin^2\alpha) / \sin^2\Phi$,

and expressing n we get the refractive index of the prism as a function of the angle of incidence α:

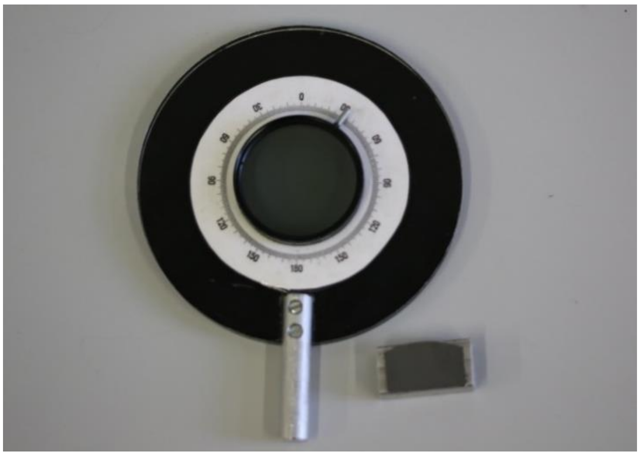
$$n = \sqrt{\frac{1+2cos\Phi sin\alpha+sin^2\alpha}{sin^2\Phi}}$$; in our case $\Phi = 60°$.

3/1. Calculate the refractive index of the prism for the red and violet component.

3/2. Take $\Delta\alpha = 1°$ as the error of the angle of incidence, and calculate the error of the refractive index using the error propagation formula. ($\Delta\alpha$ has to be converted to rad.)

4. Determining the refractive index of an unknown material by measuring Brewster's angle

Tools:
– optical bench
– halogen lamp
– gaps
– polarizer
– rotating disc
– prism
– unknown material
– paper



Polarizer and "unknown material".

The experimental setup is similar to the one we used to measure the refractive index of the prism but a polarizer is also placed in front of the prism.

Calculate the Brewster's angle of the prism, and rotate the disc in that position.
Set the polarizer so that the intensity of the reflected ray should be minimal.
Remove the prism and place the unknown material on the rotating disc.

By rotating the disc find the angle where the reflected ray's intensity is minimal. Read this angle.

Evaluation
Calculate the refractive index of the unknown material.
What can it be?

ENTRANCE TEST

Describe the measurements: tools, instructions, quantities measured and calculated, formulas.

Definitions, formulas:
- Law of Reflection
- Snell's Law
- refractive index
- equation [2], focal length, object distance, image distance
- magnification
- polarization
- Brewster's angle

Calculations:

1. The refractive index of diamond for red light is $n_{red} = 2.42$ and for blue light it is $n_{blue} = 2.45$. Calculate the angle of the refracted ray if the angle of the incident ray is 24.2° in case the light travels from the diamond to the air.

Solution:

$$n_1 \sin\Theta_1 = n_2 \sin\Theta_2$$

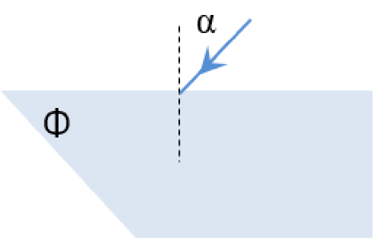
Medium 1 is diamond, medium 2 is air.
$n_1 = n_{red}$ or $n_1 = \text{blue}$; $\Theta_1 = 24.2°$;
$n_2 \approx 1$; Θ_2 is unknown.
For red light:

$$2.42 \sin 24.2° = 0.9920 = \sin\Theta_2 \rightarrow \Theta_2 = 82.75°.$$

For blue light:

$$2.45 \sin 24.2° = 1.004 > 1 \rightarrow \text{total internal reflection.}$$

2. Calculate Φ knowing that in case $\alpha < 36°$ there is no outgoing ray. The refractive index of the prism is $n = 1.33$.



Solution:
Let's apply Snell's Law:

$$\sin\alpha = n \cdot \sin\beta \rightarrow \beta = 26.23°.$$

If there is no outgoing ray then $\delta = 90°$:

$$\sin\delta = \sin 90° = 1$$

$$\sin\delta = n \cdot \sin\gamma = 1 \rightarrow \gamma = 48.75°.$$

Regarding the triangle containing Φ:

$$\Phi + (90°-\beta) + (90°-\gamma) = 180° \rightarrow \Phi = \beta + \gamma = 74.98°.$$