

A ball is thrown up with a vertical initial velocity  $v_0$  and reaches a maximum height  $h_{\max}$ .

We collect data to see how the maximum height depends on the initial velocity.

x : $v_0$ (m/s)	4	6	8	9	10
y : $h_{\max}$ (m)	0.8	1.8	3.2	4.1	5.0

Determine the acceleration of gravity.

1. Set up the formula that expresses how the quantity in the second row of the table depends on the quantity in the first row of the table:  $y = F(x, par)$  2 points

Sol. 1:

$$v = v_0 - g t, \quad z = v_0 t - \frac{1}{2} g t^2;$$

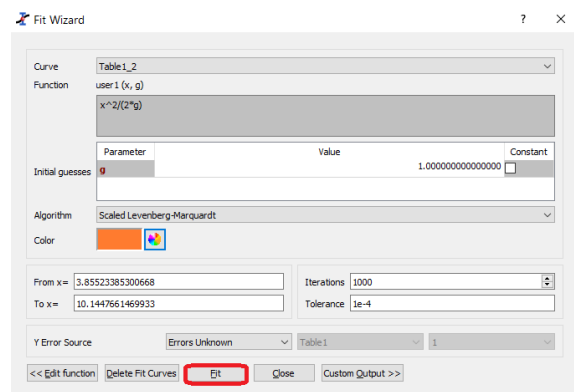
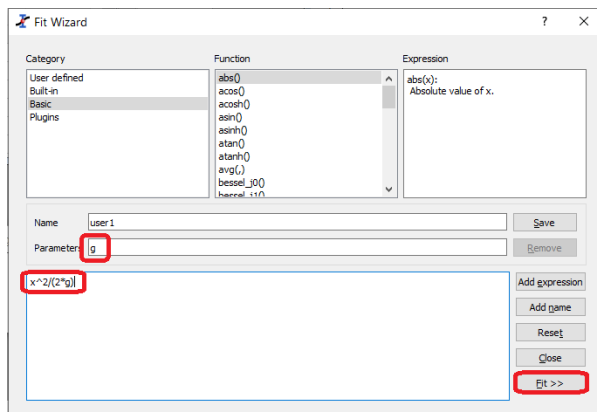
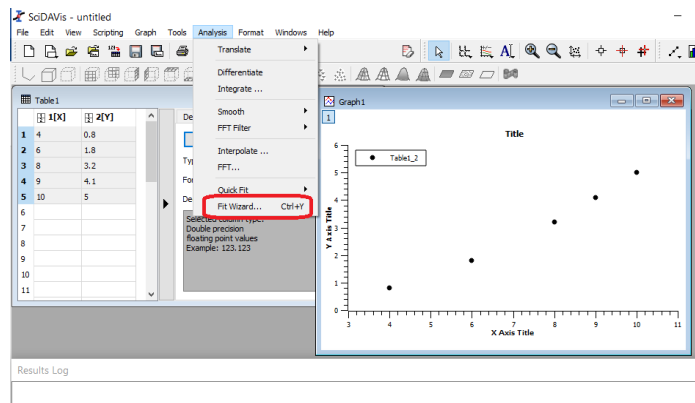
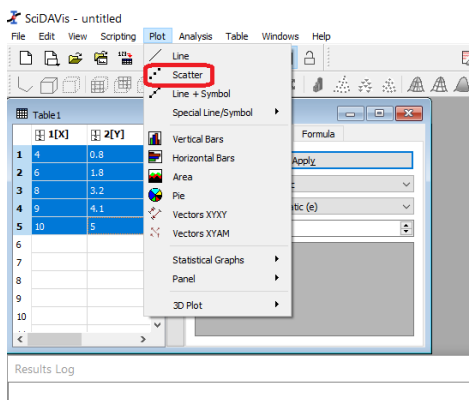
$$z = h_{\max} \quad \text{when } v = 0: v_0 - g t_h = 0 \rightarrow t_h = v_0/g \rightarrow z(t_h) = v_0 (v_0/g) - \frac{1}{2} g (v_0/g)^2 = \dots = v_0^2/(2g)$$

Sol. 2:

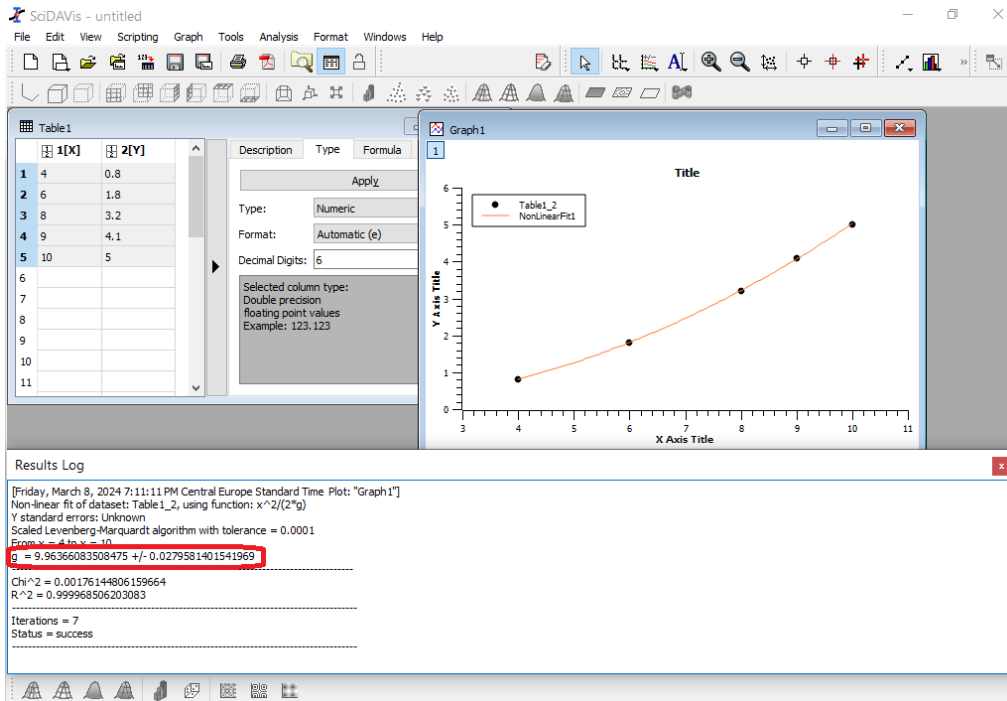
$$E_{\text{mech}} = \frac{1}{2} m v^2 + mg z = \text{const.}: \quad \frac{1}{2} m v_0^2 + 0 = mg h_{\max} + 0 \rightarrow$$

$$\rightarrow h_{\max} = \frac{v_0^2}{2g}$$

2. The expression is nonlinear. Use SciDAVis to plot your data, then type in the appropriate formula and fit the curve. Read the value of the unknown parameter. Upload a screenshot showing the graph and the results log window. 2 points



Close the Fit Wizard window.



Check the graph, and the Results Log window:

Non-linear fit of dataset: Table1\_2, using function:  $x^2/(2 \cdot g)$   
 Y standard errors: Unknown  
 Scaled Levenberg-Marquardt algorithm with tolerance = 0.0001  
 From x = 4 to x = 10  
 $g = 9.96366083508475 \pm 0.0279581401541969$  read this value

-----  
 Chi<sup>2</sup> = 0.00176144806159664  
 $R^2 = 0.999968506203083$  this should be close to 1  
 -----

Iterations = 7  
 Status = success

Fit Wizard:  $f(x) = x^2 / (2 \cdot g) \rightarrow g = (9.96366 \pm 0.02796) \text{ m/s}^2$

3. Transform your expression to a linear form: a function of the second variable in the table should be a linear function of the first variable in the table:  $G(y) = a \cdot x$  2 points

linearization:  $\sqrt{h_{\max}} = \frac{1}{\sqrt{2g}} v_0$

$y = a x \quad (b = 0)$

where x:  $v_0$ ; y:  $\sqrt{h_{\max}}$ ; a:  $\frac{1}{\sqrt{2g}}$

4. Derive the general formula for calculating the slope of the line  $y = ax$  (i.e. when the intercept is zero) using the least squares method. 2 points

$S(a) = \sum_{i=1}^N (ax_i - y_i)^2 \rightarrow \min. \rightarrow ??? \rightarrow ??? \rightarrow ??? \rightarrow a = \frac{\overline{x \cdot y}}{\overline{x^2}}.$

5. Using the above formula calculate the slope of the line  $G(y) = ax$ . Upload the Excel file.

x		y	x <sup>2</sup>	x·y
v <sub>0</sub> (m/s)	h <sub>max</sub> (m)	$\sqrt{h_{\max}}$ ( $\sqrt{m}$ )	v <sub>0</sub> <sup>2</sup> (m <sup>2</sup> /s <sup>2</sup> )	v <sub>0</sub> · $\sqrt{h_{\max}}$ (m· $\sqrt{m}$ /s)
4	0.8	0.8944272	16	3.577709
6	1.8	1.3416408	36	8.049845
8	3.2	1.7888544	64	14.31084
9	4.1	2.0248457	81	18.22361
10	5	2.236068	100	22.36068
7.4		1.6571672	59.4	13.30454

$$a = 0.223982$$

$$a = \frac{\bar{x} \cdot \bar{y}}{\bar{x}^2} = \frac{13.3045}{59.4} = 0.223982 \text{ (s}/\sqrt{\text{m}})$$

6. From the slope calculate again the value of the unknown parameter. Compare it with the one determined with SciDavis.

2 points

$$a = \frac{1}{\sqrt{2g}} = 0.223982 \text{ s}/\sqrt{\text{m}} \quad \rightarrow \quad g = \frac{1}{2a^2} = 9.9665 \text{ m/s}^2$$

Nonlinear fit using SciDAVis gave  $g = (9.96366 \pm 0.02796) \text{ m/s}^2$

With linearization the result is  $(9.9665 - 9.96366) / 9.96366 = 3.4\%$  greater.

### Optional task

Calculate the standard deviation of the quantity you had to determine using the linearized formula:

first calculate the standard deviation of the slope of the line  $G(y) = ax$  using the formula

$$S_a = \sqrt{\frac{\sum_{i=1}^N (a \cdot x_i - y_i)^2}{N \cdot (N-1) \cdot x^2}},$$

and then use the error propagation formula.

4 extra points

x		y	x <sup>2</sup>	x·y		(a·x - y) <sup>2</sup>
v <sub>0</sub> (m/s)	h <sub>max</sub> (m)	√h <sub>max</sub> (√m)	v <sub>0</sub> <sup>2</sup> (m <sup>2</sup> /s <sup>2</sup> )	v <sub>0</sub> (m/s)		(a·v <sub>0</sub> - √h <sub>max</sub> ) <sup>2</sup> (m)
4	0.8	0.8944272	16	3.577709		2.3E-06
6	1.8	1.3416408	36	8.049845		5.1E-06
8	3.2	1.7888544	64	14.31084		9E-06
9	4.1	2.0248457	81	18.22361		8.1E-05
10	5	2.236068	100	22.36068		1.4E-05
7.4	2.98	1.6571672	59.4	13.30454	sum:	0.0001115

$$S_a = \sqrt{\frac{0.0001115 \text{ m}}{5 \cdot 4 \cdot 59.4 \text{ m}^2/\text{s}^2}} = 0.0003064 \text{ s}/\sqrt{\text{m}}$$

$$g = 1/(2a^2) \rightarrow \Delta g = \sqrt{\left(\frac{\partial g}{\partial a} \cdot \Delta a\right)^2} = \sqrt{\left(-\frac{2}{2a^3} \cdot \Delta a\right)^2} = 0.02727 \text{ m/s}^2.$$

So linearization gives  $g = (9.966 \pm 0.027) \text{ m/s}^2$ .