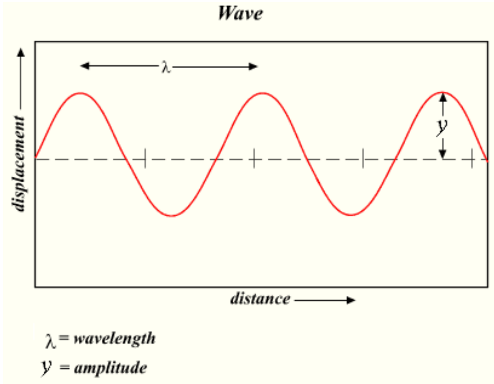
OPTICS 2: PHYSICAL OPTICS, WAVE OPTICS

THEORY

Light as a wave

Light is an electromagnetic wave composed of oscillating electric and magnetic fields. These fields continually generate each other, as the wave propagates through space and oscillates in time.

The **frequency** of a light wave is determined by the period of the oscillations. The frequency does not normally change as the wave travels through different materials ("media"), but *the speed of the wave depends on the medium*.



The speed, frequency, and wavelength of a wave are related by the formula

$v = \lambda f$,

where

v is the speed [m/s],
λ is the **wavelength** [m], and
f is the frequency [1/s = Hz].

Because the frequency is fixed, *a change in the wave's speed produces a change in its wavelength*.

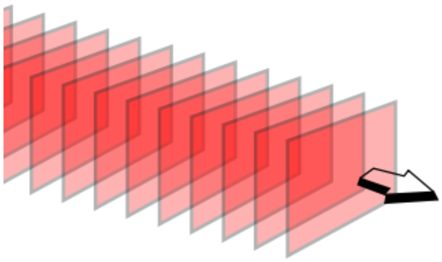
The **speed of light** in a medium is typically characterized by the **index of refraction, *n***, which is the ratio of the speed of light in vacuum, c, to the speed in the medium:

$n = c / v$.

The **speed of light in vacuum** is a constant, which is $c = 2.998 \cdot 10^8$ m/s.

Thus, a light ray with a wavelength of λ in a vacuum will have a wavelength of λ/n in a material with index of refraction n.

The **amplitude** of the light wave is related to the intensity of the light, which is related to the energy stored in the wave's electric and magnetic fields.

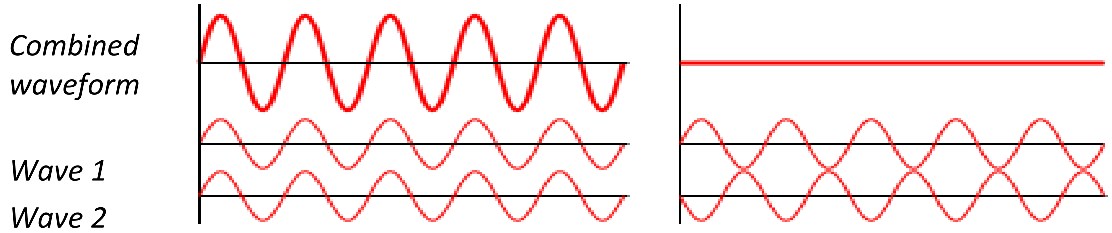


As a light wave travels through space, it oscillates in amplitude.
In this image, each maximum amplitude crest is marked with a plane to illustrate the wavefront.
The ray –used in geometrical optics– is the arrow perpendicular to these parallel surfaces.

Physical optics or wave optics builds on **Huygens's principle**, which states that every point on an advancing wavefront is the center of a new disturbance. When combined with the *superposition principle*, this explains how optical phenomena are manifested when there are multiple sources, or obstructions that are spaced at distances similar to the wavelength of the light.

Superposition and interference

The superposition principle can be used to predict the shape of interacting waveforms through the simple addition of the disturbances. This interaction of waves to produce a resulting pattern is generally termed "interference" and can result in a variety of outcomes.
If two waves of the *same wavelength and frequency* meet with a certain different phase shift the resulting wave will have the same wavelength and frequency but *the amplitude depends on the phase shift* between the two waves.
If the two waves are *in phase*, both the wave crests and wave troughs align. This results in *constructive interference* and an increase in the amplitude of the wave, which for light is associated with a brightening of the waveform in that location.
Alternatively, if the two waves of the same wavelength and frequency are *out of phase*, then the wave crests will align with wave troughs and vice-versa. This results in *destructive interference* and a decrease in the amplitude of the wave, which for light is associated with a dimming of the waveform at that location.



Two waves in phase *Two waves 180° out of phase*

Since Huygens's principle states that *every point of a wavefront is associated with the production of a new disturbance*, it is possible for a wavefront to interfere with itself constructively or destructively at different locations producing bright and dark fringes in regular and predictable patterns.

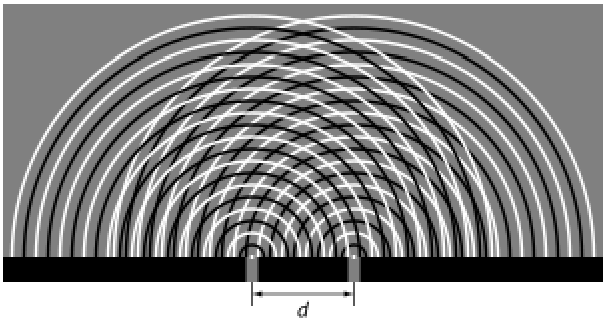
When oil or fuel is spilled,
colorful patterns are formed by thin-film interference.

Interferometry is the science of measuring these patterns, usually as a means of making precise determinations of distances or angular resolutions.

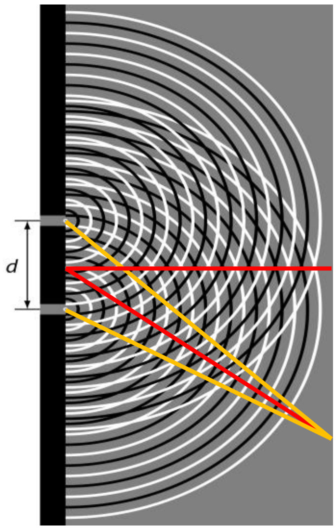
The *Michelson interferometer* was a famous instrument which used interference effects to accurately measure the speed of light.

Diffraction

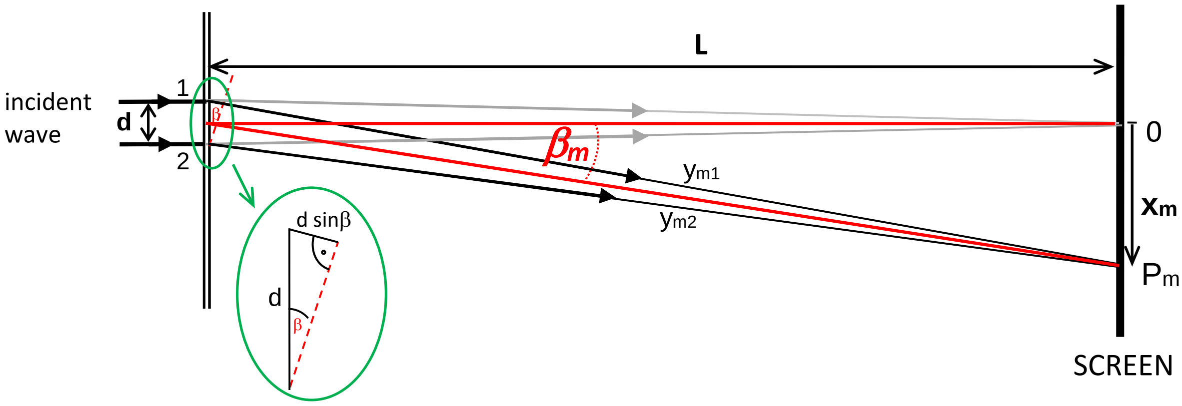
Diffraction is the process by which light interference is most commonly observed.



Diffraction on two slits separated by distance d.
The bright fringes occur along lines
where black lines intersect with black lines
and white lines intersect with white lines.
These fringes are separated by angle β
and are numbered as order n.



The simplest physical models of diffraction use equations that describe the angular separation of light and dark fringes due to light of a particular wavelength (λ).



Light fringes appear when

$m\,\lambda = d \sin\beta_m$

where d is the separation between two wavefront sources (e.g. two slits),
β_m is the angular separation between the central fringe and the mth order fringe (at the central maximum $m = 0$)

(The explanation for the above formula is that the phase of a wave at a given point P of the screen depends on the distance y between this point and the slit it travelled from. In case of a **coherent** light the incident wave at the two splits is in phase but at the screen there will be a phase difference $\Delta\varphi$ between the two waves arriving from the two splits owing to the path difference $\Delta y = y_1 - y_2$.
The relation between the path difference and the phase difference includes the wavelength λ:
$\Delta y = \lambda \cdot \Delta\varphi / 2\pi$. As it can be seen from the figure: $\Delta y = d \sin\beta_m$.
Light fringes appear at constructive interference where the phase difference is $\Delta\varphi = m \cdot 2\pi$, so these are those points of the screen where $m\,\lambda = d \sin\beta_m$ holds.)

Diffraction by reflection

Diffraction also occurs when a coherent beam hits a reflective grating.

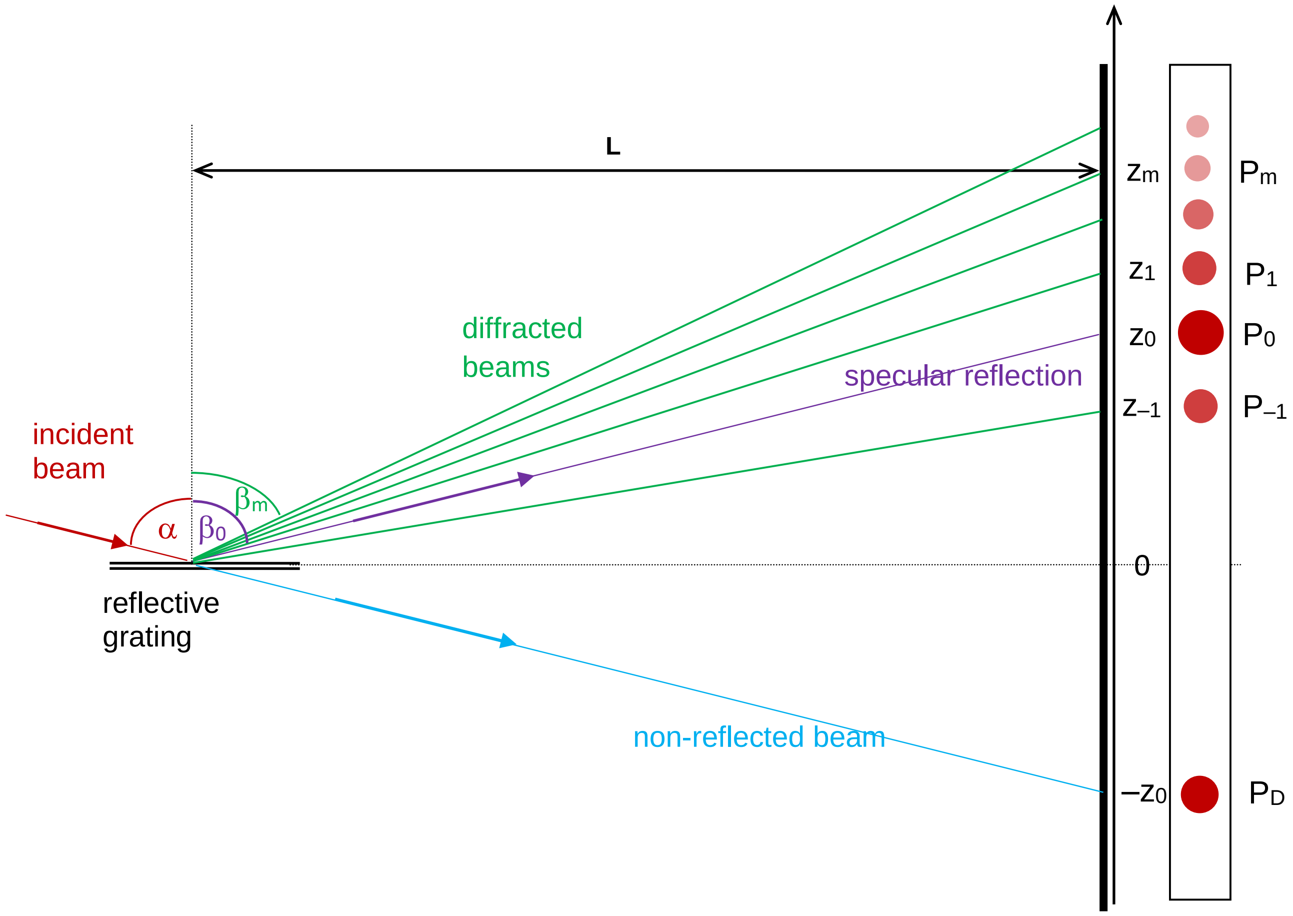


Figure 1.

If the path difference between the light from adjacent slits is equal to an integer multiple *m* of the wavelength, there is constructive interference. This condition for the grating is the so-called grating equation:

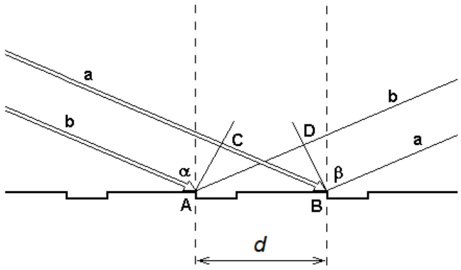
$$d\,(\sin\alpha - \sin\beta_m) = m\,\lambda \qquad [1]$$

These directions (β_m) depend on the spacing of the grating (d), the incident angle (α) of the beam and the wavelength (λ) of the laser. The light that corresponds to the specular reflection is called the zero order (m = 0), so $\beta_0 = \alpha$.

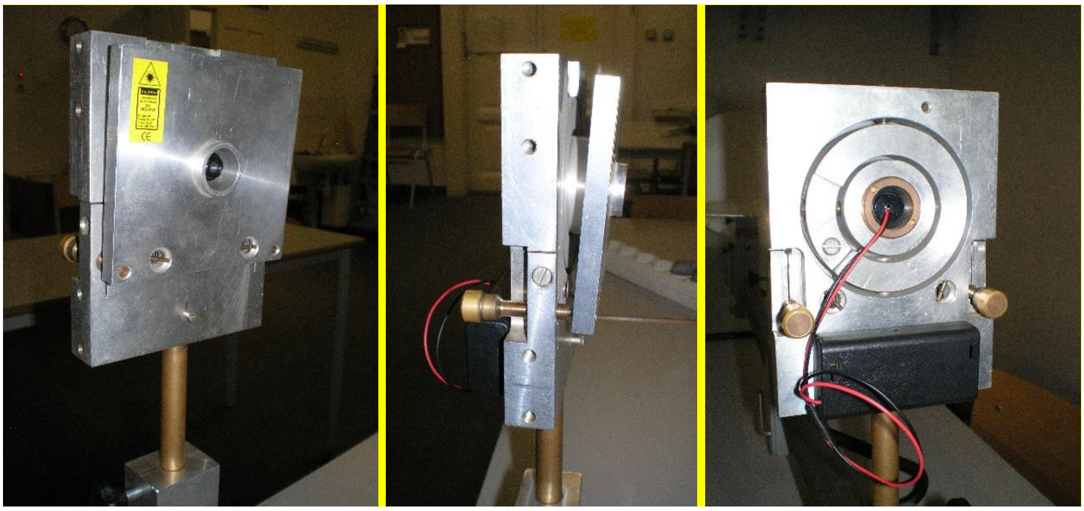
MEASUREMENTS

1. Determining the wavelength of a laser diode using a metal ruler as a reflective diffraction grating

We use a metal ruler as a reflection grating. The pattern of the metal ruler splits (diffracts) the single laser beam into several beams travelling in different directions.

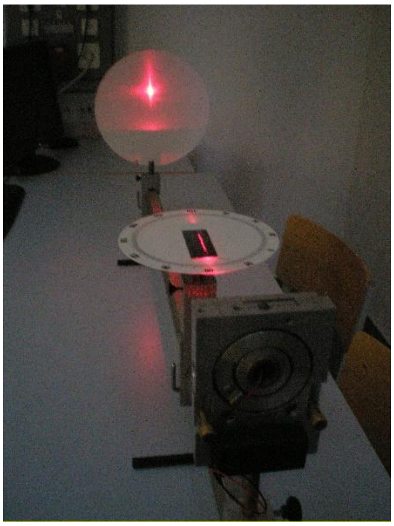


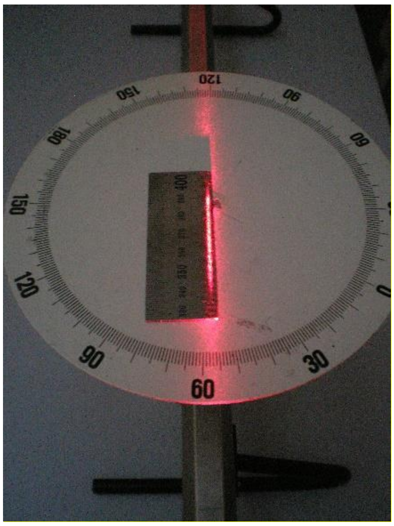
Tools:
- optical bench with high and low holders
- rotating disc
- laser
- metal ruler
- screen, mm-paper
- tape line

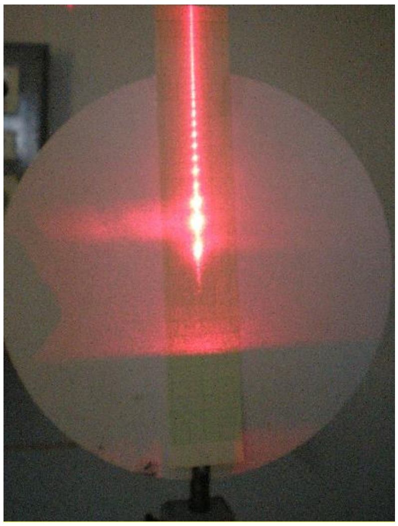


Laser. The position of the laser is adjusted by two screws.

Procedure
Place the laser on one end of the optical bench and the screen on the other end.
Place the rotating disc 20-30 cm away from the laser and put the metal ruler on it.
Adjust the metal ruler so that you see a clear diffraction image on the screen.







Measure L, the distance between the screen and the laser spot on the metal ruler.
Mark the spots of the diffracted laser beams on the mm paper on the screen (**P_0, P_1**, ..., **P_m**).
Remove the ruler and mark the non-reflected beam also (**P_D**).

<u>Evaluation</u>

From equation [1] we get that $\sin\beta_m$ is a linear function of m :

$$\sin\beta_m = \sin\alpha - \frac{\lambda}{d}\, m \qquad [2]$$

Plotting $\sin\beta_m$ vs. m gives a straight line with a slope of $-\lambda/d$.

As can be seen on Figure 1. $\sin\beta_m$ can be calculated from z_m and L as

$$\sin\beta_m = \frac{L}{\sqrt{L^2 + z_m^2}} \qquad [3]$$

Find $z = 0$ as the midpoint between the non-reflected beam and the specular reflected beam which is seen as the brightest diffracted beam (points **P_0** and **P_D**).
Read the values of the z_m distances according to Figure 1.
Calculate the values $\sin\beta_m$ from z_m and L for at least 6 decimals using formula [3].
Plot $\sin\beta_m$ as a function of m.
Calculate the slope using the least squares method:

$$a = \frac{\overline{x \cdot y} - \overline{x} \cdot \overline{y}}{\overline{x^2} - \overline{x}^2} \quad \text{where} \quad x = m \quad \text{and} \quad y = \sin\beta_m.$$

Calculate the wavelength λ from the slope knowing that

$a = -\lambda/d$ and $d = 0.5$ mm in our case.

Calculate the error of the slope using the formula

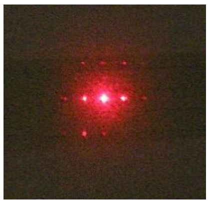
$$\Delta a = \sqrt{\frac{s_r^2}{n\cdot\left(\overline{x^2} - \overline{x}^2\right)}} \quad \text{where} \quad s_r^2 = \frac{\sum\left(a\cdot x_i + b - y_i\right)^2}{n-2}, \qquad b = \overline{y} - a\cdot\overline{x}$$

Calculate the error of the wavelength by error propagation.

2. Determining the grating constant of a transmission grating

<u>Tools:</u>
- optical bench with high and low holders
- laser
- slide holder and transmission grating
- screen, mm-paper
- tape line

<u>Procedure</u>



Place the laser on one end of the optical bench and the screen on the other end.
Place slide holder with the transmission grating so that you get a diffraction pattern on the screen.
Measure L, the distance between the screen and transmission grating.
Mark the bright spots of the diffracted laser beams on the mm paper.

<u>Evaluation:</u>
Determine Δz, the distance between two adjacent bright spots,
and calculate D, the grating constant:

$$D = \frac{\lambda L}{\Delta z}.$$

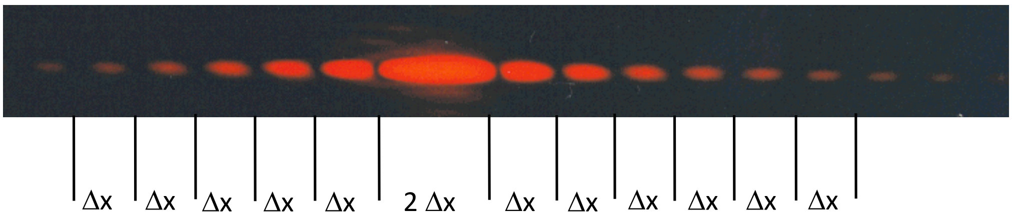
The value of λ was calculated in the previous measurement.

3. Determining the width of a hair by diffraction

As the width of a hair is commensurable with the wavelength of light the rays starting from the two "sides" of the hair interfere and a diffraction picture is obtained. Dark spots appear on the screen where the distance between two dark spots Δx depends on the width of the hair D:

$$\Delta x = \lambda \cdot L / D$$

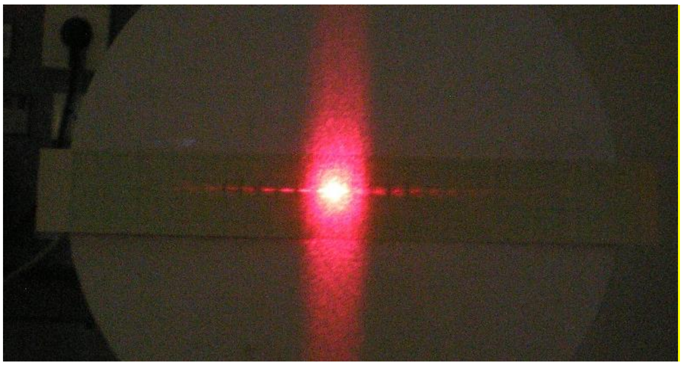
where λ is the wavelength of the laser and
L is the distance between the hair and the screen.



Observe that the center of the pattern is the brightest area, and not a dark area as we would see the shadow of an object without diffraction.

Tools:
- optical bench with high and low holders
- laser
- slide holder
- hair fiber in a slide
- screen, mm-paper
- tape line

Procedure
Place the laser on one end of the optical bench and the screen on the other end.
Place slide holder with the hair fibre so that you get a diffraction pattern on the screen.

Measure L, the distance between the screen and transmission grating.
Mark the dark spots of the diffracted laser beams on the mm paper.

Evaluation
Read the distance between the dark spots Δx,
calculate D, the width of the hair,
and compare it with the value determined with the lens (Optics 1).

4. Michelson interferometer
a) Demonstration.
b) Determining the temperature coefficient of a heated ceramic tube.
Temperature is determined by measuring the resistance of a Pt resistance thermometer:

$R(T) = R_0\,(1 + \alpha_{Pt}\,(T - T_0))$, where $R_0 = 1000\ \Omega$ at $T_0 = 0$ °C; $\alpha_{Pt} = 3.92\cdot10^{-3}\ \mathrm{K}^{-1}$

$\rightarrow\ \Delta T = \Delta R / (R_0 \cdot \alpha_{Pt}) = \Delta R / 3.92$ [K].

The elongation $\Delta\ell$ of the ceramic tube is determined with the interferometer:

$\Delta\ell = (N/2)\cdot\lambda$, where N is the number of the cycles while heating; $\lambda = 650$ nm.

The temperature coefficient is calculated using the formula

$\ell(T) = \ell_0\,(1 + \alpha\,\Delta T)\ \rightarrow\ \Delta\ell = \ell_0\,\alpha\,\Delta T$.

ENTRANCE TEST

Describe the measurements: tools, instructions, quantities measured and calculated.

Definitions, formulas

- Speed of light in vacuum.
- How are the speed, frequency, and wavelength of a wave are related?
- How do the speed, frequency, and wavelength change in different media?
- Conditions for constructive and destructive interference.
- Huygens principle.
- What is interference?

Calculations

A light ray is travelling from air to glass. Its wavelength in the air is λ_0 = 760 nm. The angle of incidence is 60° and the angle of reflection is 30°. Determine
a) the wavelength of the light in glass;
b) the speed of the light in glass;
c) the frequency of the light in glass; and
d) the frequency of the light in air.

Solution:
The refractive index is calculated using Snell's Law:

$n = \sin 60^\circ / \sin 30^\circ = 1.732$.

a) The wavelength of the light in glass is

$\lambda = \lambda_0 / n = 760 / 1.732 = 438.8$ nm $= 438.8 \cdot 10^{-9}$ m.

b) The speed of the light in glass is

$v = c / n = 2.998 \cdot 10^8 / 1.732 = 1.731 \cdot 10^8$ m/s.

c) The frequency of the light in glass is

$f = v / \lambda = 1.731 \cdot 10^8 / 438.8 \cdot 10^{-9} = 3.945 \cdot 10^{14}$ Hz.

d) The frequency of the light is independent of the medium

$f = c / \lambda_0 = v / \lambda = 3.945 \cdot 10^{14}$ Hz.