# 4. DC CIRCUIT REGULATION

Circuit regulation means that we change the current / potential difference / electric power on certain elements of the circuit.

*Basic knowledge (see summary at the end of the file):* Current, voltage, potential, resistance, Ohm's Law. Serial and parallel connection, resultant resistance. Power supplies, electromotive force, internal resistance, terminal voltage. Meters. Kirchhoff's Laws.

**NOTATIONS**

current: I
electric potential: V or φ; here φ
voltage: V or U; here U
  electromotive force: ε or E; here E
  terminal voltage: $U_t$
resistance: R
  internal resistance of the source: $R_i$
  internal resistance of the ammeter: $R_A$
  internal resistance of the voltmeter: $R_V$
  total resistance of the potentiometer: $R_p$
  total resistance of the helipot: $R_H$
  variable resistance: $R_v$ or $R_1$; here $R_1$

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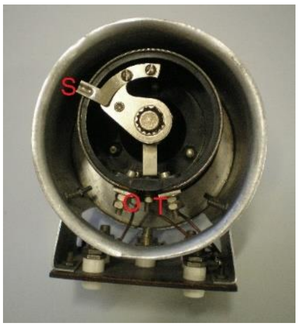
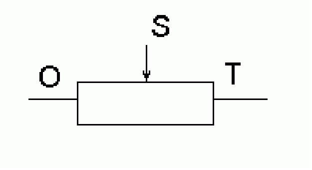
A potentiometer is usually used as a variable resistor.

<u>Potentiometer, helipot</u>

A *potentiometer* is a resistor equipped with a third, sliding contact.
It can serve as a *variable resistor* or as a *voltage divider.*



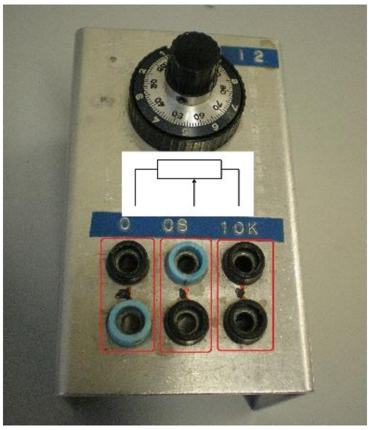

In the circuit diagrams, a potentiometer is drawn as:




By moving the sliding contact S from left to right, the resistance $R_{OS}$ between points O and S changes from zero to the maximum resistance of the potentiometer $R_{OT} = R_p$.
This is because $R = (\rho/A) \cdot \ell$, the resistance is proportional to the distance between the contacts, as in this case the resistivity and the cross-sectional area is constant.

We will use helical potentiometers – *helipots* – where the wire is wound in the form of a helix.






The position of the sliding contact can be read from a scale.

The resistance $R_{OS}$ is proportional to the scales read n:

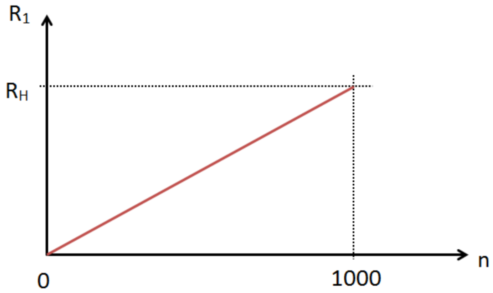
$$R_{OS} = \frac{n}{1000} R_H ,$$

where

$R_H$ is the resistance of the helipot (the resistance between points O and T),

and $0 \le n \le 1000$.

Below in the circuit diagrams $R_{OS}$ will be denoted by $R_1$.




$R_1 = \frac{n}{1000} R_H$

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**THEORY**

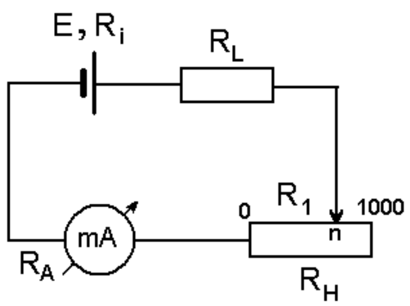
The current or voltage (or power) on a certain element of a circuit can be regulated using a variable resistor. Two typical arrangements are the serial regulation and the potentiometric regulation.

## 1. Serial regulation

The current (and potential difference and power) on the load with resistance $R_L$ is regulated using a variable resistor connected in series with the resistor.

The loop current

$$I = \frac{E}{R_1 + R_L + R_A + R_i}$$




depends on the value of $R_1$ (the variable resistance).

Remember that

$$R_1 = \frac{n}{1000} R_H .$$

For $n = 0$, i.e. $R_1 = 0$ we have maximum current:

$$I_{max} = \frac{E}{R_L + R_A + R_i} ,$$

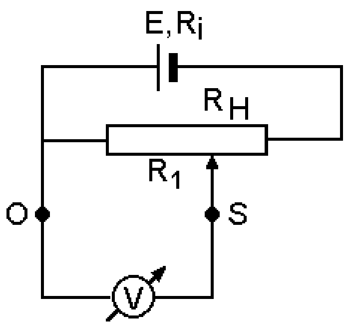
and for $n = 1000$, i.e. $R_1 = R_H$ the current is minimal:

$$I_{min} = \frac{E}{R_H + R_L + R_A + R_i} .$$

## 2. Potentiometric regulation, voltage divider

The *voltage divider* is often used to regulate the potential difference (and current and power) on an element.

### 2a. The potentiometer as a voltage divider




Consider the following circuit containing a helipot, a source, and an ideal voltmeter ($R_v = \infty$). Let us derive the formula for the voltage between the sliding contact S and the point O of the helipot in terms of $R_1 = R_{OS}$.
The total resistance in the circuit is $R_t = R_i + R_H$ , as no current is flowing across the ideal voltmeter and $R_1 + (R_H – R_1) = R_H$ . The current is

$$I = \frac{E}{R_t} = \frac{E}{R_i + R_H} = \text{const.},$$

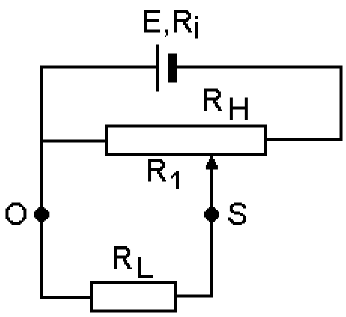
so the voltage across $R_{OS}$ is

$$U_{OS} = I\, R_{OS} = \frac{E}{R_i + R_H} R_{OS}\,.$$

The voltage changes linearly from zero to

$$U_{max} = \frac{E}{R_i + R_H} R_H = \frac{R_H}{R_i + R_H} E$$ as the sliding contact S moves from left to right.

### 2b. Potentiometric regulation of voltage when a loading resistor is applied




Above we have set up a variable-voltage source by means of a helipot, and now we apply this variable voltage to some device. This device “loads” the source by some resistance $R_L$.
In this case the total resistance of the circuit is

$$R_t = R_{OS} + (R_H – R_1) + R_i = \frac{R_1 R_L}{R_1 + R_L} + (R_H – R_1) + R_i\,,$$

the loop current is

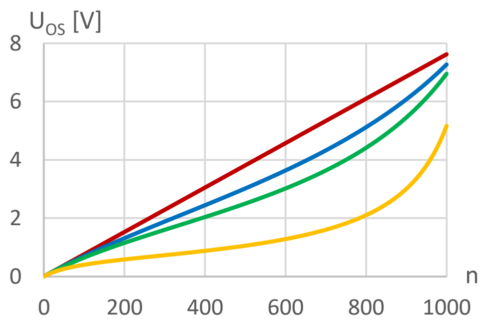
$$I_{(L)} = \frac{E}{R_t} = \frac{E}{\frac{R_1 R_L}{R_1 + R_L} + (R_H - R_1) + R_i}\,,$$

and the voltage across $R_{OS}$ is

$$U_{OS,(L)} = I\, R_{OS} = \frac{E}{\frac{R_1 R_L}{R_1 + R_L} + (R_H - R_1) + R_i} \cdot \frac{R_1 R_L}{R_1 + R_L} = \frac{\frac{R_1 R_L}{R_1 + R_L}}{\frac{R_1 R_L}{R_1 + R_L} + (R_H - R_1) + R_i} E\,.$$

This voltage depends on the position of the sliding contact so that the regulation is

- monotonous (the larger $R_1$ the larger $U_{OS,(L)}$);
- nearly linear in the vicinity of the two terminals of the potentiometer (n = 0 or 1000);
- nearly linear along the whole region if $R_L$ goes to infinity.




(Here $E = 8$ V, $R_i = 100\ \Omega$, $R_H = 2\ \text{k}\Omega$,
— without load; — $R_L = R_H$ ; — $R_L = 0.5\ R_H$ ; — $R_L = 0.1\ R_H$ )

## REPORT

**Introduction:**

<table>
<tr><td>date of the measurement</td><td colspan="2">name(s) of the student(s)</td><td>group no.</td></tr>
<tr><td colspan="2">4. DC circuit regulation</td><td colspan="2">name of the teacher</td></tr>
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**1. Serial regulation**

Draw the circuit diagram, and identify the elements of the circuit.

Describe the measurement tasks, what needs to be set and what needs to be read.

What is the purpose of the measurement? What quantities must be determined?

**2. Potentiometric regulation**

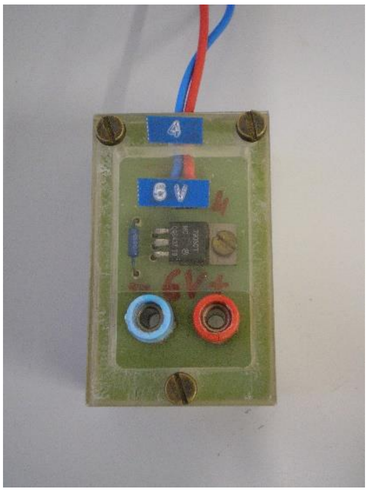
Draw the circuit diagram, and identify the elements of the circuit.

Describe the measurement tasks, what needs to be set and what needs to be read.

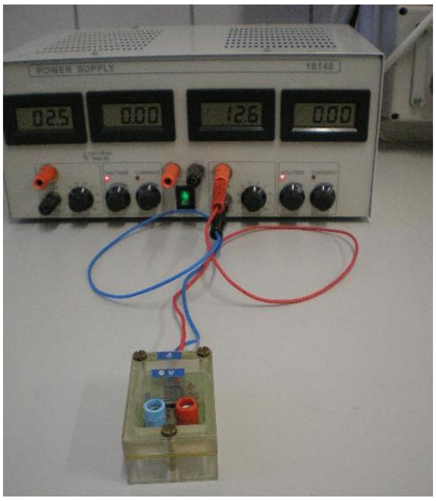
What is the purpose of the measurement? What quantities must be determined?

## MEASUREMENTS

<u>Tools</u>

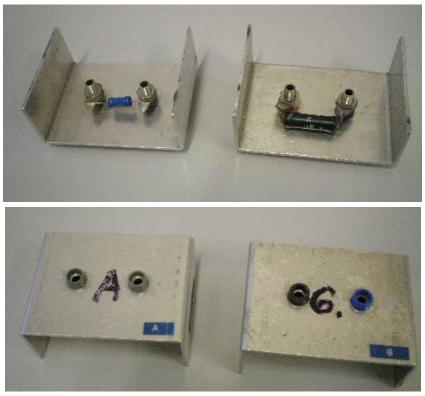


Source.
Nominal voltage: 6 V;
internal resistance: unknown.



The source has to be connected to the power supply before the measurement.

Universal digital multimeter



Resistors

Helipot

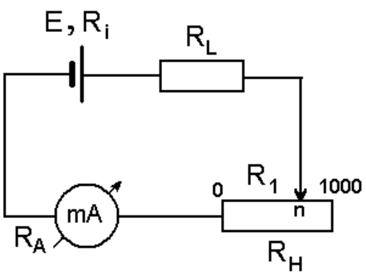
## 1. Serial regulation

*Tools:* source; helipot; resistor denoted by a number; multimeter; connecting wires.

Measure

the resistance of the "load" $R_L$ denoted by a number,
the total resistance $R_H$ of the helipot $R_H$, and
the terminal voltage $U_t$ of the source.

Set up the serial circuit:




The helipot is connected with one of the '0' sockets
and one of the 'CS' sockets.
The multimeter is connected with the 'COM' and
the 'VΩmA°C°F' sockets.
Set the multimeter to DC 'A' function
and choose the '20m' range.

Change the position of the sliding contact from 0 to 1000 scales in 100 scales steps. Measure the current at each scale setting. Collect the measurement data in the table.

**EVALUATION**

Calculate $R_1$ using the formula $R_1 = \frac{n}{1000} R_H$ .

Optional task: Plot the current I *vs.* $R_1$ in SciDAVis and determine the value of the electromotive force E and the sum of the internal resistances $R_A + R_i$ by fitting the function

$$I = \frac{E}{R_1 + R_L + R_A + R_i} .$$

The value of $R_L$ is known, it was measured. Pay attention to the units!

Mandatory task: Taking the reciprocal of the current, it is

$$\frac{1}{I} = \frac{1}{E} R_1 + \frac{R_L + R_A + R_i}{E} .$$

This is a linear relation between the variable resistance and the reciprocal current.
Plot 1/I *vs.* $R_1$ (in Excel), calculate the slope and the intercept, then calculate the value of the electromotive force E and the sum of the internal resistances $R_A + R_i$ from the values of the slope and the intercept.

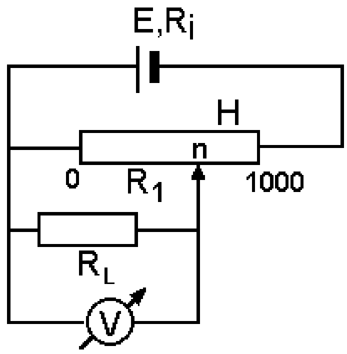
Optional task: Calculate the error interval for E and $R_A + R_i$ based on the standard deviation of the slope and the intercept using the error propagation method.

## 2. Potentiometric regulation

*Tools:* source; helipot; resistor denoted by a letter; multimeter; connecting wires.
The internal resistance of the voltmeter is about 50 MΩ, high compared to $R_H$ so it does not appreciably change the current flowing in the circuit

Measure the resistance of the "load" $R_L$ denoted by a letter.

Set up the following circuit:




The helipot is connected so that the wires coming from the source are connected to one of the '0' sockets and one of the '10k' (or '10000') sockets; the load and the multimeter are connected to the other '0' socket and one of the 'CS' sockets. The multimeter is connected with the 'COM' and the 'VΩmA°C°F' sockets.
Set the multimeter to DC 'V' function and choose the '2000m' or '20' range.

Change the position of the sliding contact and read the voltage at scale divisions as shown in the table. Then disconnect the load $R_L$ and measure the voltage again at n = 300, 600 and 1000. Collect the measurement data in the table.

**EVALUATION**

Plot U *vs*. $R_1$ for both cases, with and without the load.

Without the load it should be a straight line:

$$U = \frac{E}{R_i + R_H} R_1 .$$

Calculate the slope of the line using the least squares method. (Use the formula derived in the curve fitting practice for the slope of $y = ax$ .)

Calculate $R_i$, the internal resistance of the source, from the slope.

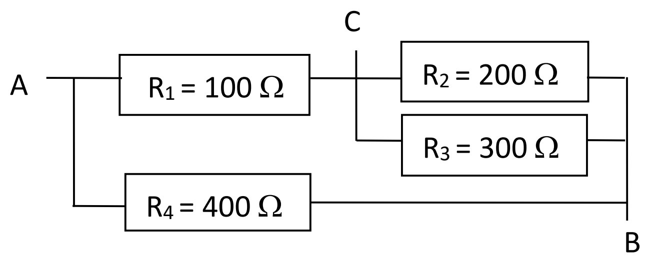
Above, in the serial regulation the sum of $R_i$+$R_A$ was determined.
Calculate $R_A$, the internal resistance of the ammeter.

**ENTRANCE TEST**

Describe the measurements: tools, instructions, quantities measured and calculated, formulas.

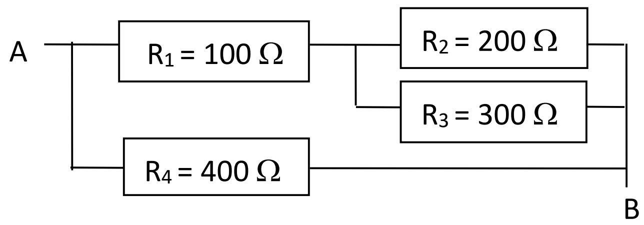
Calculations:

**1.** Calculate the resultant between points
**a)** A and B;
**b)** A and C;
**c)** B and C!



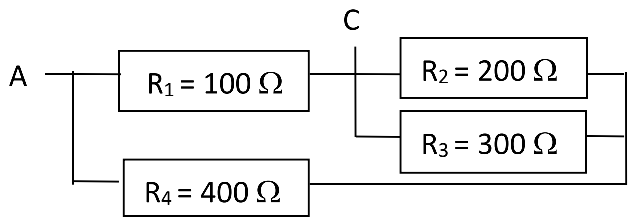

*Solution:*

**a)** $R_1$ is connected in series with the parallel resultant of $R_2$ and $R_3$; then $R_4$ is connected in parallel.



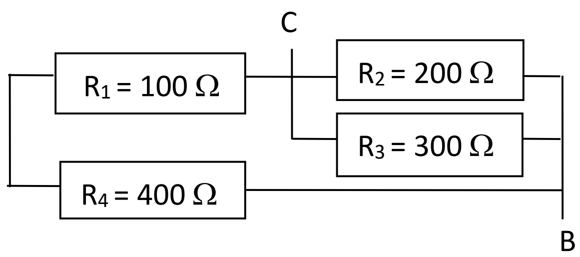

$\frac{200\cdot300}{200+300} = 120\ \Omega$;  $120+100 = 220\ \Omega$;  $R_{AB} = \frac{220\cdot400}{220+400} = 141.9\ \Omega$ .

**b)** $R_4$ is connected in series with the parallel resultant of $R_2$ and $R_3$; then $R_1$ is connected in parallel.

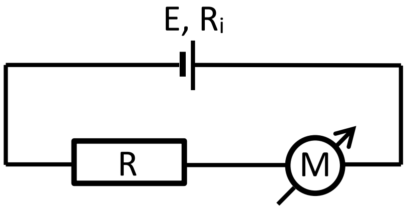



$\frac{200\cdot300}{200+300} = 120\ \Omega$;  $120+400 = 520\ \Omega$;  $R_{AC} = \frac{520\cdot100}{520+100} = 83.9\ \Omega$ .

**c)** $R_1$ and $R_4$ are connected in series; their resultant is connected in parallel with $R_2$ and $R_3$.




$\frac{1}{R_{BC}} = \frac{1}{200} + \frac{1}{300} + \frac{1}{400+100} \quad \rightarrow \quad R_{BC} = 96.8\ \Omega.$




**2.** The emf of the battery is E = 10 V, and its internal resistance is 2 Ω. R = 88 Ω. M is a digital multimeter.
**a)** M is used as a voltmeter. In this case its internal resistance is very high compared to R.
What voltage can we read on its display?
**b)** M is used as an ammeter. We set the 200 mA range where its internal resistance is 10 Ω. What current can we read on its display?
*Solution:*
**a)** As the resistance of the voltmeter is very high, no current is flowing in the circuit. The voltage across the voltmeter equals the emf E, so we can read 10 V.
**b)** All resistances are connected in series, the total resistance is $R_t$ = 2+88+10 = 100 Ω, and the current is $I = E / R_t$ = 10 V / 100 Ω = 0.1 A = 100 mA.

**3.** We set up a circuit connecting an ideal battery of E = 12 V with a constant resistor R = 100 Ω and a variable resistor $R_v$ in series. $0 \le R_v \le 500$ Ω. Determine the maximum and minimum current and voltage on the constant resistor.
*Solution:*
The total resistance of the circuit is $R_t = R + R_v = 100 + R_v$ [Ω].
The current is inversely proportional to $R_t$: $I = E / R_t$ , i.e.

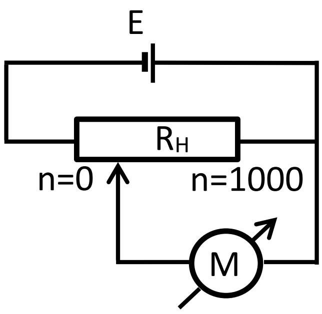
$I_{max} = E / R_{t,min} = E / R = 12 / 100 = 0.12$ A;

$I_{min} = E / R_{t,max} = E / (R + R_v) = 12 / (100+500) = 0.02$ A.

The voltage across the resistor is proportional to the current: $U_R = I R = E R / R_t$ ;

$U_{max} = I_{max} R = E = 12$ V;

$U_{min} = I_{min} R = E R / (R + R_v) = 12\cdot100/(100+500) = 2$ V.




**4.** The emf of the battery is E = 10 V, and its internal resistance is negligible. $R_H$ = 1 kΩ. The sliding contact is set to n = 400 (n = 0 is on the left and n = 1000 is on the right).
M is a digital multimeter that can be considered as an ideal meter.
What can we read when the multimeter is set as
**a)** a voltmeter?
**b)** an ammeter?

*Solution:*
**a)** An ideal voltmeter has infinite internal resistance, no current is flowing through an ideal voltmeter. This means that current only flows through the helipot, and

$I = E / R_H$ = 10 V / 1000 Ω = 0.01 A = 10 mA.

The voltmeter is connected to the right part of the helipot that has

$R_{H,r} = (1000–400)/1000 \cdot R_H = 600/1000 \cdot 1000$ Ω = 600 Ω resistance, so

$U = I R_{H,r} = 0.01\cdot600 = 6$ V, this voltage can be read on the meter.

**b)** An ideal ammeter has zero resistance (there is no potential drop on an ideal ammeter). This creates a short cut around the right side of the helipot, i.e. no current is flowing on the right side of the helipot, at the branching point all current goes to the branch of the ammeter. The total resistance of the circuit is the resistance of the left side of the helipot,

$R_{H,r} = 400/1000 \cdot R_H = 0.4 \cdot 1000\ \Omega = 400\ \Omega$ , so

$I = E / R_{OC} = 10\ V / 400\ \Omega = 0.025\ A = 25\ mA$.

**5.** We set up a circuit connecting a battery of E = 36 V and unknown internal resistance, a constant resistor R = 180 Ω, a helipot $R_H$ = 1 kΩ as a variable resistor, and an ideal ammeter in series. When the sliding contact of the helipot is set to its middle position the ammeter reads 45 mA.
**a)** Calculate the internal resistance of the battery.
**b)** Calculate the potential drop across the constant resistor for this helipot position.
**c)** Calculate the potential drop across the variable resistor for this helipot position.
**d)** Calculate the terminal voltage of the battery for this helipot position.
**e)** Draw the circuit, including a voltmeter that measures the terminal voltage of the battery.
**f)** Determine the maximum and minimum current in the circuit.
**g)** Calculate the voltage across the constant resistor and the variable resistor, and the terminal voltage of the battery when the current is at maximum.
**h)** Calculate the voltage across the constant resistor and the variable resistor, and the terminal voltage of the battery when the current is at minimum.

*Solution:*
**a)** The total resistance of the circuit can be calculated from Ohm's Law written for the whole circuit:

$R_t = E / I = 36\ V / 45 \cdot 10^{-3}\ A = 800\ \Omega$.

The resistance of the helipot is $R_H/2 = 0.5\ k\Omega = 500\ \Omega$; $R_A = 0$ (ideal ammeter); so

$R_t = R_i + R + R_H/2 \qquad \rightarrow \qquad R_i = R_t - R - R_H/2 = 800 - 180 - 500 = 120\ \Omega$.

**b)** Applying Ohm's Law for the constant resistor:

$U_R = I\,R = 45 \cdot 10^{-3}\ A \cdot 180\ \Omega = 8.1\ V$.

**c)** Applying Ohm's Law for the variable resistor:

$U_{Rv} = I\,R_v = 45 \cdot 10^{-3}\ A \cdot 500\ \Omega = 22.5\ V$.

**d)** The terminal voltage can be calculated in two ways:

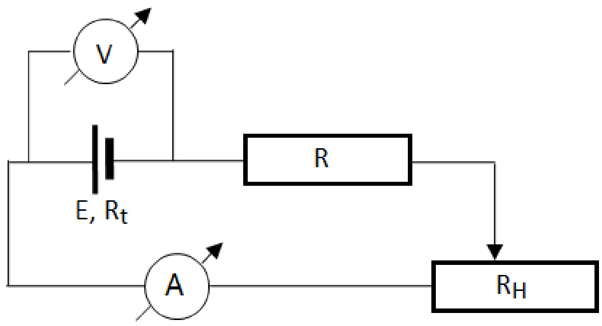
- We know that along a closed loop, the sum of all potential differences is zero (Kirchhoff's second law for loops), or, alternatively, the sum of potential drops equals the potential increase, i.e. the voltage across the terminals of the battery:

  $U_t = U_R + U_{Rv} = 8.1 + 22.5 = 30.6\ V$.

- The voltage across the terminals of the battery having an internal resistance $R_i$ is less than its electromotive force because of the potential drop on its internal resistance:

  $U_t = E - I\,R_i = 36\ V - 45 \cdot 10^{-3}\ A \cdot 120\ \Omega = 30.6\ V$

**e)**

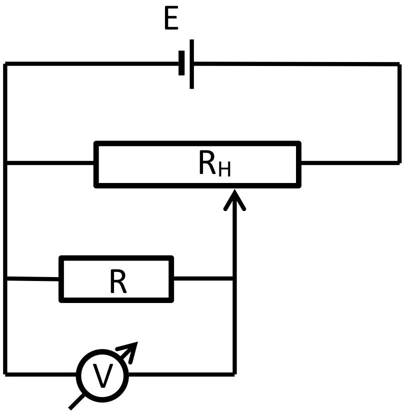



**f)** $I_{max} = E / R_{t,min} = E / (R_i + R + 0) = 36 / (120+180) = 0.12$ A = 120 mA;
$I_{min} = E / R_{t,max} = E / (R_i + R + R_H) = 36 / (120+180+1000) = 0.02769$ A = 27.69 mA.

**g)** $I_{max} = 0.12$ A:
$U_R = 0.12 \cdot 180 = 21.6$ V
$U_{Rv} = 0.12 \cdot 0 = 0$ V
$U_t = U_R + U_{Rv} = 21.6+0 = 21.6$ V; or $U_t = E - I\,R_i = 36 - 0.12 \cdot 120 = 21.6$ V.

**h)** $I_{min} = 0.02769$ A:
$U_R = 0.02769 \cdot 180 = 4.985$ V
$U_{Rv} = 0.02769 \cdot 1000 = 27.69$ V
$U_t = U_R + U_{Rv} = 4.985+27.69 = 32.68$ V; or $U_t = E - I\,R_i = 36 - 0.02769 \cdot 120 = 32.68$ V.

**6.** $R_H = 2000\ \Omega$; $R = 1200\ \Omega$; $E = 4.2$ V, the source is ideal ($R_i = 0$).
The sliding contact is set so that the left side of the helipot (parallel to R) has $R_1 = 800\ \Omega$.
What does the ideal voltmeter read?




*Solution:*
R and $R_1$ are connected in parallel

$$R_p = \frac{1}{\frac{1}{R}+\frac{1}{R_1}} = \frac{1200 \cdot 800}{1200+800} = 480\ \Omega,$$

then $R_H - R_1 = 2000 - 800 = 1200\ \Omega$ is connected in series,
the total resistance of the circuit is

$R_t = R_p + (R_H - R_1) = 480 + 1200 = 1680\ \Omega$.

The current flowing through the source is

$I = E / R_e = 4.2\ \text{V} / 1680\ \Omega = 0.0025$ A = 2.5 mA.

The voltmeter is connected in parallel to R but also to $R_1$. The resistance between these points is $R_p = 480\ \Omega$, so

$U = I\,R_p = 0.0025 \cdot 480 = 1.2$ V.

## BASIC ELECTRICITY

### Voltage

Assume a charge q moves in a static electric field **E** from point A to point B. The field exerts the force **F** = q**E** on the charge and the work done by the field is

$$W_{AB} = \int_A^B \mathbf{F} \cdot d\mathbf{r} = q \int_A^B \mathbf{E} \cdot d\mathbf{r} \ .$$

The static electric field is conservative, so the work does not depend on the path taken, and the work along a closed loop is zero. The work is determined by the initial and final points of motion A and B and on the charge itself. We define the ratio of work $W_{AB}$ to the charge q as the voltage $U_{AB}$ between points A and B:

$$U_{AB} = \frac{W_{AB}}{q} \ .$$

The unit of voltage is the Volt (V).

### Electric potential

We can assign a scalar property – the electric potential φ – to any point in the field as the work done on unit positive charge when it moves from the given point to the place where this potential is zero. The voltage between the points A and B equal to

$$U_{AB} = \varphi_A - \varphi_B \ .$$

### Electric current

The charged particles – charge carriers – can move freely in conductors, as metals, semiconductors, or electrolytes, but the average of their velocities is zero in the absence of external fields. If there is an electric field in the medium the charge carriers gain a velocity component parallel to the field and the average velocity no longer stays zero. But it does not increase infinitely by the effect of the electric force. The charge carriers will collide with the imperfections of the conductor or with each other and loose their excess velocity gained from the field. As a result, the charge carriers will have a constant average velocity – called *drift velocity* – parallel to the electric field. The stream of charge carriers constitutes a current in the conductor. The intensity of the current is the net charge that flows across the cross section of the conductor in unit time:

$$I = \frac{dq}{dt} \ .$$

The unit of electric current is the Amper (A)

We speak about stationary current when the current does not change with time. In order to maintain stationary current flowing in a conductor, a stationary electric field must be set up across it. Unlike the static case, there is nonzero electric field in a metal if current flows in it.

Both positive and negative charge carriers contribute to the electric current. The positive charges move in the direction of the field, the negative charges move in the opposite direction.

In a metal, the charge carriers are free electrons. Their drift velocity is proportional to the electric field intensity. In a metal wire, the electric field intensity is constant along the length of the wire, $E = U_{AB} / L$ , where L is the length of the piece of wire between points A and B.

**Ohm's Law** states that the current is proportional to the voltage across a conductor.
The ratio

$$R = \frac{U}{I} \quad \text{is called resistance.}$$

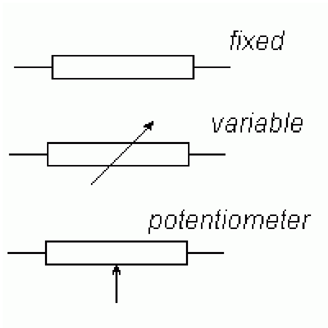
The unit of resistance is Ohm (Ω).

The resistance of a wire of length L and cross section A is

$$R = \rho \frac{L}{A},$$

where ρ is the resistivity of the material the wire is made of. The unit of resistivity is Ωm.

**Resistors** are parts of electric circuits with defined resistance ranging from about 1 Ω to several hundred MΩ. They are made from rods of a poor conductor (carbon grains mixed with insulating resin), or of resistance wire of or a thin strip of metal wound around some insulating support. The symbols for resistors in a circuit diagram are shown in the picture.




The **potentiometer** is a three-terminal device with a sliding contact as the third contact. The resistance between one end and the sliding contact can be varied.

**Power**

The work of the electric field when a charge dq moves from a place with potential $\varphi_A$ to a place with potential $\varphi_B$ is

$$W_{AB} = dq\,(\varphi_A - \varphi_B) = dq\,U_{AB}\text{ , where } U_{AB} \text{ is the voltage across A and B.}$$

The power of the electric force is

$$P = \frac{d\,W_{AB}}{d\,t} = U_{AB}\frac{d\,q}{d\,t} = U_{AB}\,I\,.$$

The work would increase the kinetic energy of the electrons in the metal, but they loose this excess KE during collisions. The energy gained from the electric field is transformed to the random motion of the atoms and other electrons of the metal, so it increases the internal energy, so the metal warms up. We call this process *dissipation.* The power dissipated in a resistor is $P = U\,I$, where I is the current and U is the voltage across the resistor.
The unit of power is watt (W). 1 W = 1 V · 1 A.
The dissipated power in a resistor can be calculated also as

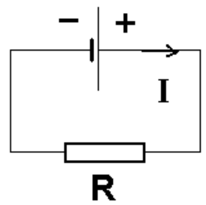
$$P = U\,I = I^2 R = U^2 / R\,.$$

The power used by an electric device – a cooker, heater, a loudspeaker, a motor, a lamp – is defined by the voltage it is connected to and the current that flows through it. This power is transformed to some other kind of energy – heat, sound, mechanical motion, light and so on.
The electric work is measured also in kWh. 1 kWh = 1000 W · 3600 s = $3.6\cdot 10^6$ J.

**Sources of electricity: galvanic cells, batteries, generators**

There are several means to produce electrical energy from other types of energy.
In a power plant, the generators convert mechanical energy into electric energy. The generators gain their mechanical energy from heat, or from the energy of wind or flowing waters. The commonly used galvanic cells and batteries convert chemical energy into electricity. The solar cells convert light energy – the energy of sunshine into electric energy.

A galvanic cell consists of two electrodes (metals or metal and carbon) immersed into some electrolyte. The ions of metals tend to dissolve in the electrolyte, leaving their outer electrons behind and making this electrode negative with respect to the electrolyte. Different metals dissolve more or less readily so there will be a potential difference between the electrodes.
The magnitude of the potential difference between the electrodes of a galvanic cell is called the *electromotive force (emf) E* of the cell. Connecting the electrodes by a metal wire, the electrons accumulated on the negative electrode start to flow toward the positive electrode: we get current in the wire the direction of which is just opposite to the drift velocity of the electrons.




To use the electrical energy accumulated in the cell, we connect some load to it: a light bulb, for example. The source and the load connected to it make the simplest electric circuit. The load is represented by a resistor, with resistance R.
The electric current flows always from + to – across a resistor.
According to Ohm's law, the voltage across the resistor is U = RI. This voltage usually is lower than the emf of the cell, as the cell itself has got some internal resistance, $R_i$. As the current flows around, it has to flow across the internal resistance, producing a voltage drop there. So we get Ohm's law for the whole circuit as

$$E = I\,(R + R_i)\ .$$

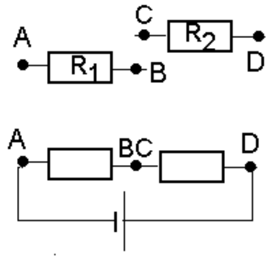
The voltage across the terminals of a loaded real source is called terminal voltage, $U_t$:

$$U_t = E - I\,R_i\ .$$

(The voltage across the terminals of an ideal voltage source is independent on the load, it is unchanged no matter how big resistance is connected across the terminals.)

**Connecting devices in series and in parallel; resultant resistance**

**Series connection**




Two resistors or other devices, elements of the circuit are connected *in series* when they are joined at one end but no other element is connected to their joining point. The free ends of the joined elements are connected into the circuit.
The same current flows through each element in the whole chain as there is no branching point:

$$I_{R1} = I_{R2} = I.$$

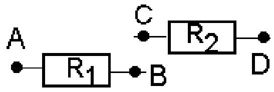
The voltages across the resistors add up:

$$U_{AD} = U_{R1} + U_{R2}\ .$$

The chain of resistors between A and D can be replaced by a single one with a resistance equal to the series resultant $R_s$ of the individual resistances. This series resultant can be calculated knowing that the voltages add up and the same current flows through both resistors, so

$$U_{AD} = U_{R1} + U_{R2} \quad \rightarrow \quad I\,R_s = I\,R_1 + I\,R_2 \quad \rightarrow \quad R_s = R_1 + R_2$$

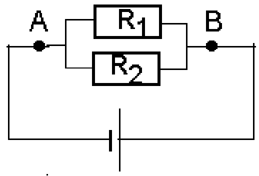
**Parallel connection**




Two resistors or other devices are connected in parallel if they are joined at both ends and there is no other element on the connecting wires.

The voltage is the same across both resistors as it is the difference between the potentials of that of the connecting wires:

$$U_{R1} = U_{R2} = U_{AB}\ .$$




The current flowing through the branches adds up:

$$I_{AB} = I_{R1} + I_{R2}\ .$$

The joined resistors can be replaced by a single one with a resistance equal to the parallel resultant $R_p$. The current through this resultant equals the current flowing out from and back into the source. This means that the sum of the current intensities flowing through both resistors is equal to the current flowing through the source.

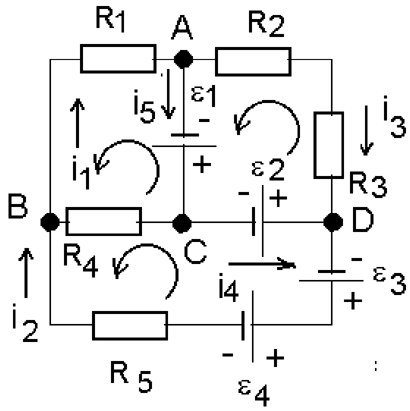
Combining the additivity of currents with Ohm's law, we get the resultant resistance:

$$I_{AB} = I_{R1} + I_{R2} \quad \rightarrow \quad \frac{U_{AB}}{R_p} = \frac{U_{AB}}{R_1} = \frac{U_{AB}}{R_2} \quad \rightarrow \quad \frac{1}{R_p} = \frac{1}{R_1} = \frac{1}{R_2}\ .$$

**Electrical circuits**

A more complicated *electrical circuit* can contain several sources and resistors and other devices, all connected with metal wires or strips. We will neglect the resistance of the connecting wires with respect to the other resistances in the circuit.

A circuit can contain several *loops:* these are non-crossing closed paths. There are *junctions (nodes)* like A, B, C, D, where several wires join. The part of the circuit between two neighbouring junction is called *branch*. In a branch, the same current flows through each element.




**Kirchhoff's Laws in electrical circuits**

<u>Kirchhoff's first law for junctions</u>

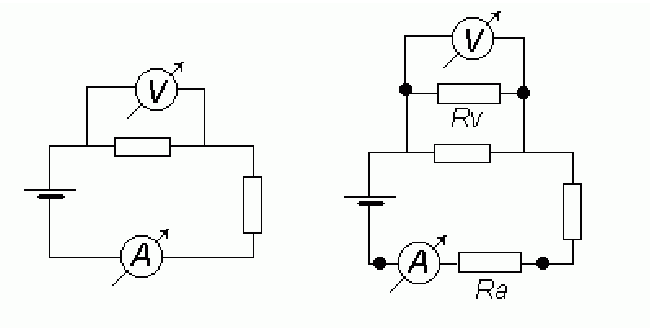
As the electric charge is conserved, and the charge cannot accumulate at a junction, the sum of the currents entering any junction must be equal to the sum of the currents leaving the junction. Using positive sign with the entering current and negative sine with the leaving currents, the net current at a junction must be zero.

<u>Kirchhoff's second law for loops</u>

The electric force is conservative in a static or stationary electric field, the work of the field along a closed path is zero. The same holds for a DC electric circuit. It is possible to assign a potential value ($\varphi$) to each point of a circuit and the work done by the electric field on a positive unit charge when it moves from point A to B is equal to the difference of the potentials $\varphi_A - \varphi_B$. Along a closed loop, the sum of all potential differences is zero.

**Connecting meters into the circuit**

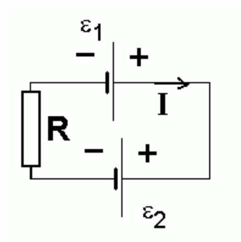
Voltmeters measure voltage <u>across two points</u> of a circuit, so they are connected <u>in parallel</u> to the points where we want to know the voltage.
Ammeters measure current <u>in a branch</u> of the circuit, so they are connected <u>in series</u> with the elements where we want to know the current (we have to break the circuit to insert the ammeter).
Ideal voltmeters have infinite resistance, ideal ammeters have zero resistance, but no meters are ideal so the current and voltage they measure will differ somewhat from the ones without the meter. To calculate the effect of the meter to currents and voltages in the circuit we replace the meter by its internal resistance in the circuit-diagram. The voltmeter reading then is equal to the voltage across its internal resistance. The reading of the ammeter is equal to the current flowing through its internal resistance.



**Measuring voltage with a potentiometer: the "ideal voltmeter"**

The emf of a source was defined as the voltage across the terminals when nothing is connected to them. But if we want to know the emf the voltmeter has to be connected, and the voltmeter has got some finite resistance. Therefore it will draw current from the source, that current flows through the internal resistance of the source and causes a potential drop across it. The voltmeter will measure less voltage than the emf. How can we measure real emf, without loading the source?

Consider a single-loop circuit containing two batteries connected with opposite polarities. The current in the circuit will be proportional to the difference of the emf's of both sources:



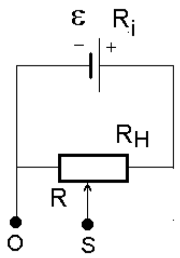

$I = (\varepsilon_1 - \varepsilon_2) / R$ ;

and no current will flow if the two emf's are equal.

If we had a source the emf of which could be controlled, and we connected it against an unknown battery, we could change the emf till the current in the loop becomes zero. In this case we would know that the unknown emf is equal to the controlled one.

How can we produce a variable-emf voltage source?

We have learnt about how to regulate voltage with a potentiometer, and found out that the voltage across the zero and the sliding contact was proportional to the resistance between zero and the slide if nothing was connected between them:

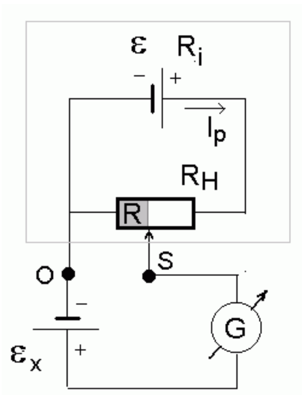



$U_{OS} = R / (R_i + R_H) \cdot \varepsilon$ .

This voltage can be controlled by moving the sliding contact.

Now we use this potentiometer circuit as a voltage source connected against a battery of unknown emf as shown in the following figure.

G is a very sensitive current-meter, called *galvanometer,* which is able to detect even microampers, but it is used here to indicate zero-current condition only.




If the galvanometer reads zero current, the variable voltage $U_{OS}$ and the unknown emf, $\varepsilon_x$ are equal.

$\varepsilon_x = I_p R = I_p (n_x/1000) R_H$ ,

where the position of the sliding contact is indicated by $n_x$ scales.

Usually we do not know $I_p$ because we do not know ε (the emf in the potentiometer circuit) precisely. However, $I_p$ does not depend on the emf measured when zero current is set.

We can replace the unknown battery $\varepsilon_x$ with a known emf source $\varepsilon_0$. Such are the so-called normal cells, like the Weston cell. (It is a cadmium-mercury cell, with very stable emf of $\varepsilon_0 = 1.01865$ V at room temperature.) Again, if the galvanometer reads zero current,

$\varepsilon_0 = I_p R = I_p (n_0/1000) R_H$ .

The ratio of both emf-s is

$\varepsilon_x / \varepsilon_0 = n_x / n_0$ .

So by performing two measurements, one with the unknown and one with the known emf source, setting zero current by moving the sliding contact of the potentiometer and reading its position, the unknown emf can be calculated as

$\varepsilon_x = \varepsilon_0 \cdot n_x / n_0$ .