Optical spectroscopy in materials science 9. Symmetry and selection rules in vibrational spectroscopy

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Vibrational degrees of freedom

For a molecule with N atoms: 3N – 6 (3N- 3 translations– 3 rotations) For linear molecules: 3N – 5 (rotation around axis does not cause atomic displacement)

Selection rules:



Basic concepts - symmetry

Symmetry operations:

- identity
- rotations
- reflections

Symmetry point group: all symmetry operations which leave at least one point in the object fixed

Symmetry elements and operations: (Schoenflies' notation)

- C_n^m axis of rotation: rotation by $2\pi/n$ m times
- σ:

- mirror plane
- σ_v : vertical
- σ_h : horizontal
- σ_d : diagonal (vertical, bisects two C₂ axes perpendicular to principal axis)
- S_n^m : improper rotation (C_n^m , then σ_h) rotation-reflection
- •*i*: inversion center

Basic concepts - groups

Symmetry operations form a group Group multiplication: subsequent application of operations

Group:

$$AB \in G$$

$$A(BC) = (AB)C$$

$$E \in G$$

$$A^{-1}A = E, A \in G, A^{-1} \in G$$

closed associative contains identity element contains inverse element

Crystallographic point groups

	notation	n	number
Rotation	C _n	1,2,3,4,6	5
$C_n + \sigma_h$	C _{nh}	1,2,3,4,6	5
$C_n + \sigma_v$	C _{nv}	2,3,4,6	4
$C_n + S_n$	S _n	2,4,6	3
$C_n + C_2$	D _n	2,3,4,6	4
$C_{nh}+C_2$	D _{nh}	2,3,4,6	4
$S_n + C_2$	D _{nd}	2,3	2
Cubic	no principal axis	4C ₃	5

For molecules: any n (5,7,10...)

 $C_{_{\infty\nu}}$ (HCI), $D_{_{\infty h}}$ (Cl₂), l_h (C₆₀)

Point group determination



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Examples









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Ammonia

 C_{3v}

Source: Wikipedia



 $\begin{array}{c} \text{Fullerene} \ C_{60} \\ I_h \end{array}$



Multiplication tables

C _{2v}				
C_{2v}	E	C_2	$\sigma_{_{v}}$	σ_v
E	E	C_2	σ_v	σ_v
C_2	C_2	E	σ_v	σ_{v}
$\sigma_{_{\!v}}$	σ_{v}	σ_v	E	C_2
σ_v	σ_v	σ_v	C_2	Ε



C_{3v}



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Representations

- Symmetry operations can be represented by matrices
- The trace of the matrices (*the character*) is invariant with respect to similarity transformations
- Reducible representation: matrices attain block format on a suitably chosen basis
- Irreducible representation: cannot be simplified (reduced) further
- Character table: characters of irreducible representations (irreps) by symmetry operation for individual point groups



Representations - example: NH₃

Representation on Cartesian basis:

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} C_3 = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} C_3^2 = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$\sigma_v'' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \sigma_v'' = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \sigma_v''' = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \sigma_v''' = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Representation on r_1 , r_2 , r_3 basis:

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} C_3 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} C_3^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$
$$\sigma_v'' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \sigma_v''' = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Character table:

C _{3v}	E	2C ₃	3σ _v
Γ ₁	1	1	1
Γ ₂	1	1	-1
Γ ₃	2	-1	0

Great Orthogonality Theorem (GOT) (Fundamentális ortogonalitási tétel (FOT))

$$\sum_{R} \Gamma_{i}(R)_{mn}^{*} \Gamma_{j}(R)_{op} = \frac{h}{l_{i}} \delta_{ij} \delta_{mo} \delta_{np}$$

where:

 Γ_i , Γ_i are two non-equivalent irreducible representations

- h is the order of the group (number of elements)
- I_i is the dimension of the matrices

These relations for the characters follow from the GOT:

$$(\chi_R = \sum_k \Gamma_i(R)_{kk}):$$

$$\sum_{R} \chi_{i}(R)^{*} \chi_{j}(R) = h \delta_{ij}$$
$$\sum_{i} l_{i}^{2} = h$$

Reduction

Reduction of reducible representations:

$$\chi(R) = \sum_{i} n_i \chi_i(R)$$

 $\chi(R)$ being the character of the reducible representation, n_i the coefficient of the *i*th irrep



Vibrational modes

Vibrational Hamiltonian:

$$H_{vib} = \sum_{i} H_{i} = \frac{1}{2} \left(-\hbar^{2} \frac{\partial}{\partial q_{i}^{2}}\right) + \omega_{i} q_{i}^{2}$$

 q_i normal coordinates $\longrightarrow \omega_i$ eigenfrequencies

 $V = \frac{1}{2} \sum_{i} \omega_i^2 q_i^2$

Neumann principle: V and ω_i show the symmetry of the point group

• non-degenerate case:
$$V \sim q_i^2$$
, $Rq_i = \pm q_i$

•degenerate case: $V \sim (q_j^2 + q_k^2 + q_l^2) \quad R |q_j, q_k, q_l| \Rightarrow |\pm q_j, \pm q_k, \pm q_l|$

The normal coordinates form bases of the irreps of the point group; the dimension of the representation equals the degeneracy

Selection rules

IR:
$$\frac{\partial \mu}{\partial q} \neq 0$$

 $\mu_x \sim x$, $\mu_v \sim y$, $\mu_z \sim z$ are always basis functions for an irrep of the point group

Since basis functions of inequivalent representations are orthogonal, this selection rule means: those vibrations are IR active, whose normal coordinate belongs to the same irrep as one of the x,y,z coordinates

Raman:
$$\frac{\partial \alpha}{\partial q} \neq 0$$

 $\alpha_{xx} \sim x^2$, $\alpha_{xy} \sim xy$,...and the products of the coordinates are also always basis functions for an irrep of the point group, this selection rule means: those vibrations are Raman active whose normal coordinate belongs to the same irrep as one of the binary products of the x,y,z coordinates

Notation of irreps

- A, B 1-dimensional
- E 2-dimensional
- T(F) 3-dimensional
- G 4-dimensional
- H 5-dimensional

In point groups containing inversion: Subscript g (gerade = even) or u (ungerade = odd) refers to the character of *i* in the given irrep: $\chi(i) > 0$ g, $\chi(i) < 0$ u

number in subscript is just a "serial number"

"Counting" of vibrational modes



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C _{2v}	E	C ₂	σ _v (xz)	σ _v (yz)	
A ₁	1	1	1	1	z, x ² , y ² , z ²
A ₂	1	1	-1	-1	R _x , xy
B ₁	1	-1	1	-1	x, R _x , xz
B ₂	1	-1	-1	1	y, R _z , yz

Representation on a 3N-dimensional basis



Character contributions of atoms situated on symmetry elements

R	χ(R)	R	χ(R)
C _n ^k	$1+2\cos(2\pi k/n)$	\mathbf{S}_{n}^{k}	$-1+2\cos(2\pi k/n)$
$E=C_1^k$	3	$\sigma = S_1^1$	1
C_{2}^{1}	-1	$i=S_{2}^{1}$	-3
C_{3}^{1}, C_{3}^{2}	0	S_{3}^{1}, S_{3}^{5}	-2
C_4^{1}, C_4^{3}	1	S_4^{1}, S_4^{3}	-1
C_6^{1}, C_6^{5}	2	S_6^{1}, S_6^{5}	0

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Reduction

$$n_j = \frac{1}{h} \sum_R \chi_j(R)^* \chi(R)$$

For water, $\chi_r(E) = 9$, $\chi_r(C_2) = -1$, $\chi_r(\sigma_{xz}) = 1$, $\chi_r(\sigma_{yz}) = 3$. Applying the reduction formula and using the character table:

 $n(A_1) = 3$, $n(A_2) = 1$, $n(B_1) = 2$, $n(B_2) = 3$

C _{2v}	E	C ₂	σ _v (xz)	σ _v (yz)	
A ₁	1	1	1	1	z, x ² , y ² , z ²
A ₂	1	1	-1	-1	R _z , xy
B ₁	1	-1	1	-1	x, R _y , xz
B ₂	1	-1	-1	1	y, R _x , yz

	Е	C ₂	$\sigma_v(xz)$	σ _v (yz)
n	3	1	1	3
Γ	9	-1	1	3

Vibrational modes: $\Gamma = 2A_1 \oplus B_2$

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Normal vibrations of the water molecule



C₆₀, the truncated icosahedron



http://www.seed.slb.com/en/scictr/watch/fullerenes2/saved.htm





Vibrational modes of C₆₀

C60

Character table:

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I_h	E	$12C_5$	$12 C_5^2$	$20C_3$	$15C_2$	i	$12\mathrm{S}_{10}$	$12\mathrm{S}^3_{10}$	$20S_6$	$15\sigma_d$	
A_g	1	1	1	1	1	1	1	1	1	1	Raman
A_u	1	1	1	1	1	-1	-1	-1	-1	-1	
F_{1g}	3	$\frac{(1+\sqrt{5})}{2}$	$\frac{(1-\sqrt{5})}{2}$	0	-1	3	$\frac{(1-\sqrt{5})}{2}$	$\frac{(1+\sqrt{5})}{2}$	0	-1	rot
F_{1u}	3	$\frac{(1+\sqrt{5})}{2}$	$\frac{(1-\sqrt{5})}{2}$	0	-1	-3	$-\frac{(1-\sqrt{5})}{2}$	$-\frac{(1+\sqrt{5})}{2}$	0	1	IR
\mathbf{F}_{2g}	3	$\frac{(1-\sqrt{5})}{2}$	$\frac{(1+\sqrt{5})}{2}$	0	-1	3	$\frac{(1+\sqrt{5})}{2}$	$\frac{(1-\sqrt{5})}{2}$	0	-1	
F_{2u}	3	$\frac{(1-\sqrt{5})}{2}$	$\frac{(1+\sqrt{5})}{2}$	0	-1	-3	$-\frac{(1+\sqrt{5})}{2}$	$-\frac{(1-\sqrt{5})}{2}$	0	1	
G_{1g}	4	-1	-1	1	0	4	-1	-1	1	0	
G_{1u}	4	-1	-1	1	0	-4	1	1	-1	0	
H_{g}	5	0	0	-1	1	5	0	0	-1	1	Raman
$\tilde{\mathrm{H}_{u}}$	5	0	0	-1	1	-5	0	0	1	-1	

Characters and reduction of 3N dimensional representation

Counting of C₆₀ vibrational modes:

Characters of representation defined by Cartesian unit vectors on atoms:

l _h	E	12C ₅	12C ₅ ²	20C ₃	15C ₂	i	12S ₁₀	12S ₁₀ ³	20S ₆	15σ _d
χ _Γ (R)	180	0	0	0	0	0	0	0	0	4

From here, using the reduction formula, we get for the A_q mode:

 $n(A_g) = 1/120(1*180) - 12(1*0) + 12(1*0) + 20(1*0) + 15(1*0) + 1*0 + 12(1*0) + 20(1*0) + 15(1*4) = 2$

The final result of the reduction is:

$$\Gamma = 2A_g \oplus 4T_{1g} \oplus 4T_{2g} \oplus 6G_g \oplus 8H_g \oplus 1A_u \oplus 5T_{1u} \oplus 5T_{2u} \oplus 6G_u \oplus 7H_u$$

Estimation of the number of vibrations

IR
$$\frac{\partial \mu}{\partial Q} \neq 0$$
 Raman $\frac{\partial \alpha}{\partial Q} \neq 0$



degeneration

Principle of mutual exclusion:

if the molecule possesses an inversion center, IR-active modes are not Raman active and vice versa





odd (ungerade) and even (gerade) modes are orthogonal

Symmetry analysis gives the maximum number of spectral lines (it can decrease by incidental degeneration, intensity below detection threshold)

Infrared spectrum of C₆₀



Take-home message

- Point group: set of symmetry operations leaving at least one point in the object fixed
- Symmetry operations form a group in the mathematical sense
- Matrix representations reducible, irreducible
- Starting from the molecular structure, using the character tables and the reduction formula, the number of vibrational modes can be predicted
- IR and Raman activity: symmetry analysis – selection rules – principle of mutual exclusion

Összefoglalás

- Pontcsoport: azon szimmetriaműveletek halmaza, amik egy objektumban legalább egy pontot mozdulatlanul hagynak
- A szimmetriaműveletek matematikai értelemben csoportot alkotnak
- A szimmetriaműveletek mátrixokkal ábrázolhatók reducibilis, irreducibilis
- A szerkezetből kiindulva, a karaktertáblák és a redukciós formula alkalmazásával a rezgési módusok száma meghatározható
- IR és Raman-aktív módusok: szimmetria-analízis – kiválasztási szabályok – kölcsönös kizárás elve

Emission spectroscopy

> Atomic emission spectroscopy (AES)

- o Flame test
- Flame emission photometry
- Atomic absorption spectrophotometry
- Inductively coupled plasma
- Molecular spectroscopy
 - IR emission spectroscopy
 - Luminescence

Emission spectroscopy



Emission by excited atom Higher the transition energy, shorter the wavelength



Atomic emission spectroscopy - Flame test

Walt Wolland, Bellevue Community College

http://www.800mainstreet.com/s/s.html



Ba Ca K Li Na Rb

Quantitative methods: flame photometry, atomic absorption spectroscopy (AAS)

Atomic emission spectroscopy



- Excitation: thermal energy of the flame
- Flame: most frequent: acetylene and air
 - vaporization \rightarrow homogeneous atomic cloud
 - excitation, but not ionization, of atoms
- Vaporizer \rightarrow sample solution
- **Temperature:** 2000 3000 °C
- **<u>Requirement</u>**: constant composition, temperature and structure of the flame

<u>Atomic Absorption Spectroscopy (AAS)</u>





http://www.cee.vt.edu/ewr/environmental/teach/smprimer/aa/aa.html http://www.resonancepub.com/atomicspec.htm

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Detectable elements



Inductively coupled plasma (ICP)







ICP Kitchen area



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ICP plasma



ICP-AES – Detection limits



Emission spectra: effect of temperature





Infrared emission spectroscopy

Temperature: 100 – 200 ° C

INTERFEROMETER

Keresztury Gábor, Mink János, Kristóf János MTA Kémiai Kutatóközpont, Veszprémi Egyetem





Strong emission "BLACKBODY"

TITANIUM plate



The model of layer structure

Self-absorption

The spectral shape depends on:

- effective thickness of the layers (d₁, d₂)
- thickness of emitting, absorbing layers (d_e, d_a)



G. Keresztury, J. Mink, J. Kristóf: *Anal. Chem.* **67**, 3782 (1995)



Figure 2. Simulation of the effect of self-absorption using eq 12 for different effective thicknesses of emitting bulk (x_b) and absorbing surface layer (x_b). (Lorentzian band shapes with $\tilde{\nu}_0 = 600$, $l_0 = 2000$, and $w = 30 \text{ cm}^{-1}$.)

 t_{b} – transmittance of the bulk

- t_s transmittance of the surface
- E emittance

Molecular spectroscopy – Luminescence

Light emission by excited molecules



Fluorescence of different sized CdSe quantum dots

Joseph R. Lakowicz – Principles of fluorescence spectroscopy, 3rd edition



Types of luminescence

(a) Excitation Mode	Luminescence Type
absorption of radiation (UV/VIS)	photoluminescence
chemical reaction	chemiluminescence, bioluminescence
thermally activated ion recombination	thermoluminescence
injection of charge	electroluminescence
high energy particles or radiation	radioluminescence
friction	triboluminescence
sound waves	sonoluminescence
(b) Excited State (Assuming Singlet State)	Luminescence Type
first excited singlet state	fluorescence, delayed fluorescence
lowest triplet state	phosphorescence

Chemiluminescence/bioluminescence

Chemiluminescence – is the emission of light as the result of a chemical reaction

Bioluminescence – one type of chemiluminescence; the light is produced and emitted by a living organism



e.g. firefly, deep-sea fish, jellyfish, squids, bacteria, planktons, mushrooms

Luminescence

Fluorescence

Phosphorescence

- > Emission: From excited singlet state
- > Transition: Allowed
- ▷ Emission rate: Fast: 10⁸ s⁻¹
- > Average lifetime: 1 10 ns

From excited triplet state "Forbidden" Slow: $10^3 - 10^0 \text{ s}^{-1}$ ms – s

> Example:





1 sec



640 sec



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0 sec

Fluorescence – Typical fluorophores



Fluorescence – the beginning



Sir John Fredrich William Herschel 1792 – 1871 On a case of superficial colour presented by a homogeneous liquid internally colourless. By Sir John Frederick William Herschel, Philosophical Translation of the Royal Society of London (1845) 135:143–145. Received January 28, 1845 — Read February 13, 1845.

"The sulphate of quinine is well known to be of extremely sparing solubility in water. It is however easily and copiously soluble in tartaric acid. Equal weights of the sulphate and of crystallised tartaric acid, rubbed up together with addition of a very little water, dissolve entirely and immediately. It is this solution, largely diluted, which exhibits the optical phenomenon in question. Though perfectly transparent and colourless when held between the eye and the light, or a white object, it yet exhibits in certain aspects, and under certain incidences of the light, an extremely vivid and beautiful celestial blue colour, which, from the circumstances of its occurrence, would seem to originate in those strata which the light first penetrates in entering the liquid, and which, if not strictly superficial, at least exert their peculiar power of analysing the incident rays and dispersing those which compose the tint in question, only through a very small depth within the medium.

Fluorescence of quinine is the most widely used example up to now

Luminescence – Jablonski diagram



Professor Alexander Jablonski 1898 – 1980



One form of a Jablonski diagram.



Franck-Condon principle R: configuration coordinate absorption (vertical) relaxation emission (vertical) relaxation

http://en.wikipedia.org/wiki/Franck-Condon_principle

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Luminescence in molecules

http://www.shsu.edu/~chemistry/chemiluminescence/JABLONSKI.html



Jablonski diagram

Intersystem crossing: singlet– triplet Internal conversion: into vibrationally excited state of higher singlet Fluorescence: singlet - singlet Phosphorescence: singlet - triplet (delayed)

Fluorescence basics

Mirror image





During an electronic transition a change from one vibrational energy level to another will be more likely to happen if the two vibrational wavefunctions overlap more significantly

Electronic excitation does not greatly alter the nuclear geometry

Fluorescence – Stokes shift



Sir George Gabriel Stokes 1819 – 1903



Experimental schematic for detection of the Stokes shift.

Visual observation of Stokes shift

Fluorescence – Lifetime (τ) and quantum yield (Q)

- These are the two most important characteristics

quantum yield = Q = $\frac{number of emitted photons}{number of absorbed photons} = \frac{\Gamma}{\Gamma + k_{nr}}$

where Γ – emission rate of fluorophores

k_{nr} – number of non-radiative transitions to ground state S₀

if there is Stokes shift, Q < 1

Lifetime: average time between excitation and emission

if k_{nr} = 0, *intrinsic* lifetime



Fluorescence – Quenching

Quenching – intensity of fluorescence decreases



Jablonski diagram with collisional quenching and fluorescence resonance energy transfer (FRET). The term Σk_i is used to represent non-radiative paths to the ground state aside from quenching and FRET.

Reasons:

collision with other molecules

(quenchers)

- Formation of non-fluorescent complexes
- resonance energy transfer (RET)

Fluorescence – Resonance energy transfer

- > emission spectrum of donor overlaps with absorption spectrum of acceptor
- > no intermediate photon
- > dipole-dipole interaction between donor and acceptor

By Alex M Mooney - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=23197114







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Fluorescence – Eu-based fluorophores



White llight illumination

UV light (365 nm)





Spectrofluorimeter – Fluorolog 3





Take-home message

- Atomic emission spectroscopy: flame test, flame emission photometry, atomic absorption spectroscopy, inductive coupled plasma
- Infrared emission spectroscopy: vibrational levels, self-absorption
- Molecular spectroscopy: types of luminescence
- Jablonski diagram: absorption, fluorescence, phosphorescence, internal conversion, intersystem crossing
- Quantum yield and lifetime
- Resonance energy transfer

Összefoglalás

- Atomi emissziós spektroszkópia: lángfestés, lángfotometria, atomabszorpciós spektroszkópia
- Infravörös emissziós spektroszkópia: rezgési szintek, önabszorpció
- Molekulaspektroszkópia: lumineszcencia típusok
- Jablonski-diagram: abszorpció, fluoreszcencia, foszforeszcencia, belső konverzió, intersystem crossing
- Kvantumhatásfok és élettartam
- Rezonáns energiatranszfer