Andris's Q1)

Take a bulk conventional superconductor in the z<0 half space, and place a semiconducting nanowire on top of it along the x axis. This sample is inside a solenoid, which would produce a 1 Tesla homogeneous magnetic field in the absence of the sample.

What is the B-field profile in the sample and in its vicinity?

1. It is homogeneous.
2. It is zero in the superconductor and homogeneous outside the superconductor, including the nanowire.
3. It is zero in the superconductor and inhomogeneous outside the superconductor.
4. It is more complicated than any of those.

Andris's Q2)

Perhaps you remember from your relativistic quantum mechanics studies that a rather general form of spin-orbit interaction is

$$H_{so} = \lambda\vec{\sigma} \cdot (\vec{E} \times \vec{p}),$$

where $\vec{E}$ is a (potentially inhomogeneous) static electric field, $\vec{p}$ is the momentum of the electron, $\vec{\sigma}$ is the spin of the electron, and λ is a prefactor. Assume that this simple picture holds also for the electrons in our semiconducting nanowire aligned with the x axis. How to orient a homogeneous E-field so that we have the Rashba Hamiltonian as specified in the topocondmat chapter, that is in the y direction?

1. Along x.
2. Along y.
3. Along z.
4. None of those will work.

Q1) Bogulyubov-de Gennes matrix in momentum space

Recall that in real space the BdG matrix takes the form

$$\mathcal{H} = \begin{pmatrix} \boldsymbol{h} & \boldsymbol{\Delta} \\ -\boldsymbol{\Delta}^* & -\boldsymbol{h}^* \end{pmatrix}.$$

Consider a system where the "particle part" (upper left) h of the hamiltonian in momentum space takes the form

$$h = \cos(k)\sigma_0 + \sigma_y \sin(k).$$

What is the "hole part" (lower right) of the momentum space BdG matrix?

1. $-\cos(k)\sigma_0 - \sigma_y \sin(k)$
2. $\cos(k)\sigma_0 - \sigma_y \sin(k)$
3. $-\cos(k)\sigma_0 + \sigma_y \sin(k)$
4. Only the pairing terms can be propotional to $\sin(k)$ and thus the question is meaningless.

Q2) EFA Hamiltonian

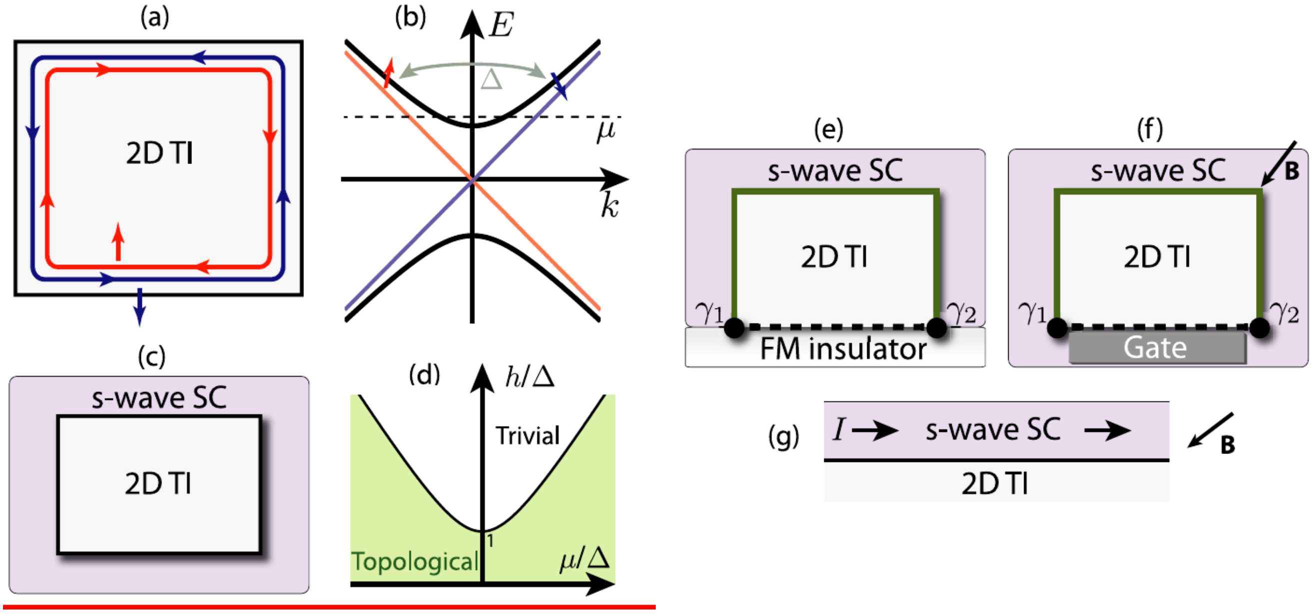
Consider the EFA Hamiltonian

$$\mathcal{H}^{\mathrm{EFA}} = \begin{pmatrix} \hat{p}-\mu & 0 & 0 & -\Delta \\ 0 & -\hat{p}-\mu & \Delta & 0 \\ 0 & \Delta & \hat{p}+\mu & 0 \\ -\Delta & 0 & 0 & -\hat{p}+\mu \end{pmatrix}.$$

Which statement below is true?

1. $\mathcal{H}^{\mathrm{EFA}}$ can not be realized in a realistic systems since this BdG does not have particle-hole symmetry.
2. $\mathcal{H}^{\mathrm{EFA}}$ is realized by the surface states of two dimensional Chern insulators, thus making them an ideal candidate for realizing Majorana fermions.
3. $\mathcal{H}^{\mathrm{EFA}}$ is realized in CNTs where the spectrum is also linear.
4. Neither of the above statements are true.

L. Fu and C. L. Kane, Phys. Rev. B, **79**, 161408(R) (2009) & Phys. Rev. Lett. **100**, 096407 (2008)



Q3) Lutchyn wire - I

Consider an infinite Lutchyn wire. At $x<0$ we set the parameters of the system such that they are in the trivial regime, while for $x>0$ the parameters of the system are tuned to be in the topological regime.

Which statement is true?

1. This device will show a quantized zero bias conductance peak.
2. This device can not show a zero bias conductance peak.
3. This device can show a zero bias conductance peak.
4. This device can not show a quantized zero bias conductance peak because we need p-wave type superconductivity for that.

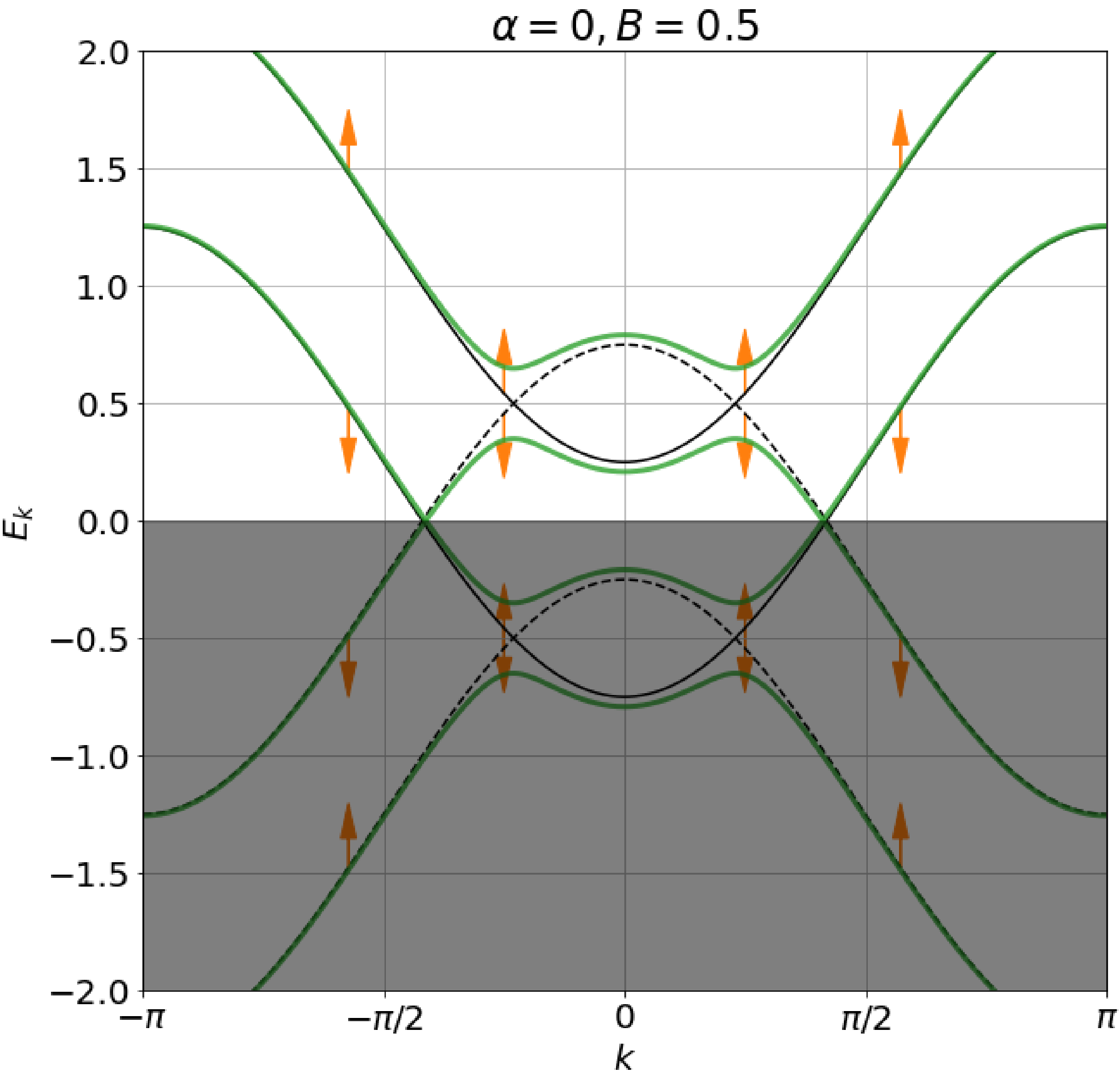
Q4) Lutchyn wire - II

The value of the topological invariant for the Lutchyn wire was calculated as

$$Q = \mathrm{sign}\left[\left(B_z^2 - \Delta^2 - (t+\mu)^2\right)\left(B_z^2 - \Delta^2 - (t-\mu)^2\right)\right].$$

Notice that the strength α of the spin-orbit coupling is missing from this expression. Which of the following statements is true:

1. The topological invariant calulated above only characterizes the system for finite values of α.
2. The topological invariant calulated above characterizes the system for arbitrary values of α.
3. The expression above does not contain α because spin orbit coupling in this model vanishes at time reversal invariant momenta.
4. More than one of the above statements is true.



$\alpha = 0, B = 0.5$
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
E_k
$-\pi$
$-\pi/2$
0
$\pi/2$
π
k

Q5) Lutchyn wire III

If we were to characterize the phases of the Lutchyn wire which have a gap at zero enregy in the BgG spectrum based on the characteristic behaviour of the conductivity of NINS junctions frabricated from it how many phases can we discern ?

1.
2.
3.
4.

Q6) Lutchyn wire - extra

The EFA of the particle (upper left) block of the BdG matrix of the Lutchyn wire is given by

$$h_{Lutchyn} = \left(\frac{p^2}{2m} - \mu\right)\sigma_0 + \alpha p \sigma_x + B_z \sigma_z$$

Consider an NINS junction with fabricated from a Lutchyn wire and let the S region be in the topological phase. Assume that the impurity potential in the I region is propotional to a $B_n \vec{n} \cdot \vec{\sigma}$.

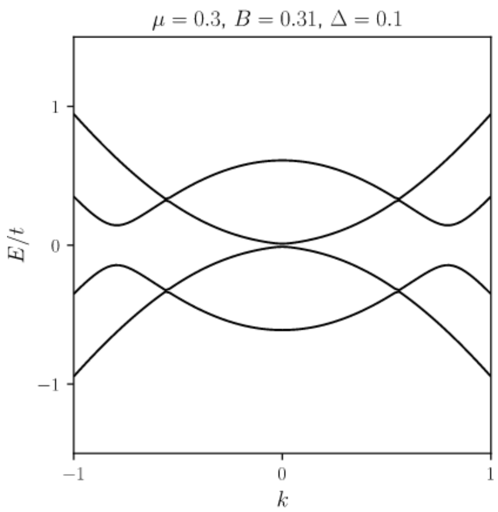
1. The ZBCP can be supressed if we carefully align $\vec{n}$.
2. The ZBCP is undisturbed by the presence of any disorder, thus it will be also independent of $\vec{n}$.
3. Such term would break the particle hole symmetry of the BdG matrix and thus we can not describe its consequences in the BdG approach.
4. This term breaks time reversal symmetry, and since the topological invariant of the Kitaev and Lutchyn wires is $\mathbb{Z}_2$ it will force the topological phase to turn in to trivial.

Exercises for lecture 8

Gap closing at k=0

When playing with the first slider in the topocondmat chapter, notice how the gap seems to close at the time-reversal invariant momentum k=0. Why? Because...

$$H = (k^2/2m - \mu - B\sigma_z)\tau_z + 2\Delta\tau_y k.$$



A) Kramers degeneracy → this is the only point where the gap can close.

B) For a gap closing we need the first term of the first bracket to vanish.

C) For k≠0, the superconducting gap cannot be closed.

D) The Hamiltonian has to be self-adjoint.

Useful alternative definition of Nambu spinor

In the topocondmat chapter, an alternative definition of the of the BdG Hamiltonian:

$$H_{\mathrm{BdG}} = \begin{pmatrix} H & \Delta \\ -\Delta^* & -H^* \end{pmatrix} \qquad \begin{matrix} \mathcal{T} = U\mathcal{K} \\ \Delta' = \Delta U^\dagger \end{matrix} \qquad H_{\mathrm{BdG}} = \begin{pmatrix} H & \Delta' \\ \Delta'^\dagger & -\mathcal{T}H\mathcal{T}^{-1} \end{pmatrix}$$

According to our definition, the so-called Nambu spinor can be written as $\hat{\mathbf{c}}^\dagger = \left(\hat{c}^\dagger_{m\uparrow}, \hat{c}^\dagger_{m\downarrow}, \hat{c}_{m\uparrow}, \hat{c}_{m\downarrow}\right)$

In the alternative definition above, what is the corresponding Nambu spinor?
Use that time reversal is $\mathcal{T} = i\sigma^y\mathcal{K}$

$$\hat{\mathbf{c}}^\dagger = \left(\hat{c}^\dagger_{\uparrow}, \hat{c}^\dagger_{\downarrow}, \hat{c}_{\uparrow}, \hat{c}_{\downarrow}\right)$$

A) $\hat{\mathbf{c}}^\dagger = \left(\hat{c}^\dagger_{m\uparrow}, \hat{c}^\dagger_{m\downarrow}, \hat{c}_{m\uparrow}, \hat{c}_{m\downarrow}\right)$

B) $\hat{\mathbf{c}}^\dagger = \left(\hat{c}^\dagger_{m\uparrow}, -\hat{c}^\dagger_{m\downarrow}, \hat{c}_{m\uparrow}, \hat{c}_{m\downarrow}\right)$

C) $\hat{\mathbf{c}}^\dagger = \left(\hat{c}^\dagger_{m\uparrow}, \hat{c}^\dagger_{m\downarrow}, \hat{c}_{m\uparrow}, -\hat{c}_{m\downarrow}\right)$

D) $\hat{\mathbf{c}}^\dagger = \left(\hat{c}^\dagger_{m\uparrow}, -\hat{c}^\dagger_{m\downarrow}, \hat{c}_{m\uparrow}, -\hat{c}_{m\downarrow}\right)$

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Take a quantum wire, with ↑ described by Kitaev wire, ↓ by a constant. I tried to write the BdG Hamiltonian in this alternative basis, with time reversal $\mathcal{T} = i\sigma^y\mathcal{K}$

$$H(k) = \begin{pmatrix} 2v\cos k - \mu & 0 \\ 0 & E_0 \end{pmatrix}$$

$$\Delta(k) = \begin{pmatrix} 2i\Delta\sin k & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \quad H_{\mathrm{BdG}}(k) = \begin{pmatrix} 2v\cos k - \mu & 0 & 0 & -2i\Delta\sin k \\ 0 & E_0 & 0 & 0 \\ 0 & 0 & -E_0 & 0 \\ 2i\Delta\sin k & 0 & 0 & -2v\cos k - \mu \end{pmatrix}$$

How many mistakes did I make? Think: signs, shape of H and Δ part of HBdG.

A) 0 B) 1 C) 2 D) more than 2