1) Dynamical Pauli principle 1

Confine a pair of electrons in a double potential well in the triplet state, where both spins are polarized down. The two electrons are localized in the two wells, and they are very far away from each other so their interaction is negligible. Exchange them adiabatically by exchanging the two potential wells, such that their interaction is negligible all along the way.

What is the geometrical phase factor picked up by the two-electron wave function by the end of the exchange?

(A) +1
(B) -1 (Dude, electrons are fermions!)
(C) i or -i
(D) The question does not make sense.

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Single-particle states:

$$\psi_L(\boldsymbol{r})\chi_\uparrow(s), \tag{1}$$

with $\boldsymbol{r} \in \mathbb{R}^3$ and $s \in \{+, -\}$.

Notation: $(\boldsymbol{r}, s) \equiv x$.

Notation:

$$L_\uparrow(x) \equiv \psi_L(\boldsymbol{r})\chi_\uparrow(s). \tag{2}$$

Notation: $\Psi(\boldsymbol{r}_1, s_1, \boldsymbol{r}_2, s_2) \equiv \Psi(x_1, x_2) \equiv \Psi(1, 2)$.

Two-particle triplet state (modulo normalization)

$$\Psi(1, 2) = L_\uparrow(1)R_\uparrow(2) - R_\uparrow(1)L_\uparrow(2). \tag{3}$$

Adiabatic exchange results in $L_\uparrow \leftrightarrow R_\uparrow$:

$$\Psi'(1, 2) = R_\uparrow(1)L_\downarrow(2) - L_\downarrow(1)R_\uparrow(2) = -\Psi(1, 2). \tag{4}$$

Conclusion: minus sign picked up!

2) Dynamical Pauli principle 2

Confine a pair of electrons in a double potential well in the state where the electron on the left has spin up, and the other one has spin down. The two electrons are localized in the two wells, and they are very far away from each other so their interaction is negligible. Exchange them adiabatically by exchanging the two potential wells, such that their interaction is negligible all along the way.

What is the geometrical phase factor picked up by the two-electron wave function by the end of the exchange?

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Two-particle up-down state:

$$\Psi(1,2) = L_\uparrow(1)R_\downarrow(2) - R_\downarrow(1)L_\uparrow(2). \tag{5}$$

Adiabatic exchange results in $L_\uparrow \to R_\uparrow$, and $R_\downarrow \to L_\downarrow$:

$$\Psi'(1,2) = R_\uparrow(1)L_\downarrow(2) - L_\downarrow(1)R_\uparrow(2). \tag{6}$$

That has nothing to do with Ψ.

3) Dynamical Pauli principle 3

Confine a pair of electrons in a double potential well in the singlet state. The two electrons are localized in the two wells, and they are very far away from each other so their interaction is negligible. Exchange them adiabatically by exchanging the two potential wells, such that their interaction is negligible all along the way.

What is the geometrical phase factor picked up by the two-electron wave function by the end of the exchange?

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3) Dynamical Pauli principle 3

…singlet state…

What is the geometrical phase factor picked up by the two-electron wave function by the end of the exchange?

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(D) The question does not make sense.

Two-particle singlet state:

$$\begin{aligned}\Psi(1,2) &= [L_\uparrow(1)R_\downarrow(2) - R_\downarrow(1)L_\uparrow(2)] \\ &\quad - [L_\downarrow(1)R_\uparrow(2) - R_\uparrow(1)L_\downarrow(2)]\,. \end{aligned} \tag{7}$$

Adiabatic exchange results in $L_\uparrow \leftrightarrow R_\uparrow$, and $R_\downarrow \leftrightarrow L_\downarrow$:

$$\begin{aligned}\Psi'(1,2) &= [R_\uparrow(1)L_\downarrow(2) - L_\downarrow(1)R_\uparrow(2)] \\ &\quad - [R_\downarrow(1)L_\uparrow(2) - L_\uparrow(1)R_\downarrow(2)]\,. \end{aligned} \tag{8}$$

First term of Ψ matches fourth term of Ψ', etc.
Hence $\Psi' = \Psi$, despite dealing with 2 fermions.

4) Find the harmless mechanism

We perform a braiding-based quantum gate on a Majorana qubit defined with four Majorana zero modes. Which mechanism does *not* lead to gate error?

(A) Opening of a minigap, also known as Majorana hybridization.
(B) Unwanted tunnel coupling to an electronic reservoir.
(C) Changing the Hamiltonian too rapidly.
(D) Phonon-induced relaxation between the globally even and globally odd ground state.
(E) I can't make my final Hamiltonian exactly the same as the initial Hamiltonian.

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5) Make it longer (in just 60 seconds)

I want to improve my braiding-based quantum gate by making my wires longer. I don't want to slow down my topological quantum computer, so I keep the same braiding time in the longer device as I had in the shorter device. Thoughts?

(A) Makes sense.
(B) You could even shorten the braiding time in the longer device, it will do no harm, since the gate is topologically protected.
(C) No! That way you speed up your Majoranas, risking that you enhance quasiparticle creation!

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6) Braiding time

In the plots showing the results of our numerical simulations, the braiding time is shown as a dimensionless quantity. Assuming that v = Delta = 200 ueV (e.g., the superconducting gap of aluminium is around this value at subkelvin temperatures), what is the dimensionful braiding time corresponding to 10^4 dimensionless time units? (Hint: use wolframalpha for a quick calculation)

(A) cca. 3.2 ps
(B) cca. 3.2 ns
(C) cca. 32 ns
(D) cca. 320 ns

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$$\hbar = 0.66\,\mu\mathrm{eV\,ns}$$

$$\text{time unit: } \hbar/v \approx 3.3\,\mathrm{ps}$$

7) Qubits and Majoranas

You have the task to encode quantum information in 8 Majorana zero modes. Which is the *worst* strategy you could use?

(A) I encode 1 qubit per 2 Majoranas, so I can encode 4 qubits.
(B) I encode 1 qubit per 4 Majoranas, so I encode 2 qubits.
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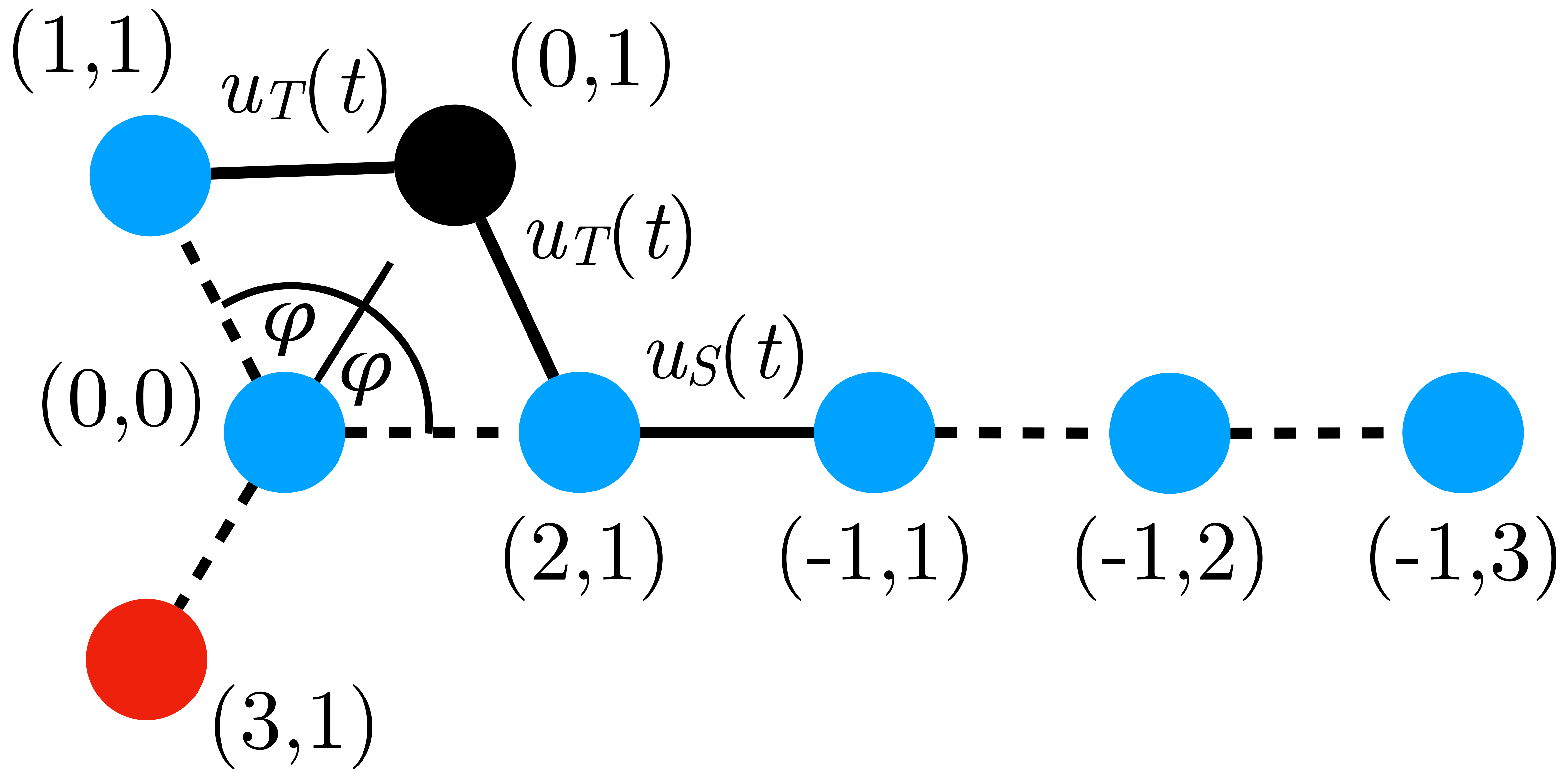
8) Fock-space Hamiltonian

What is the size of the Fock-space Hamiltonian matrix of the ball-and-stick model in our notes?

(A) 8 x 8
(B) 16 x 16
(C) 256 x 256
(D) 65536 x 65536



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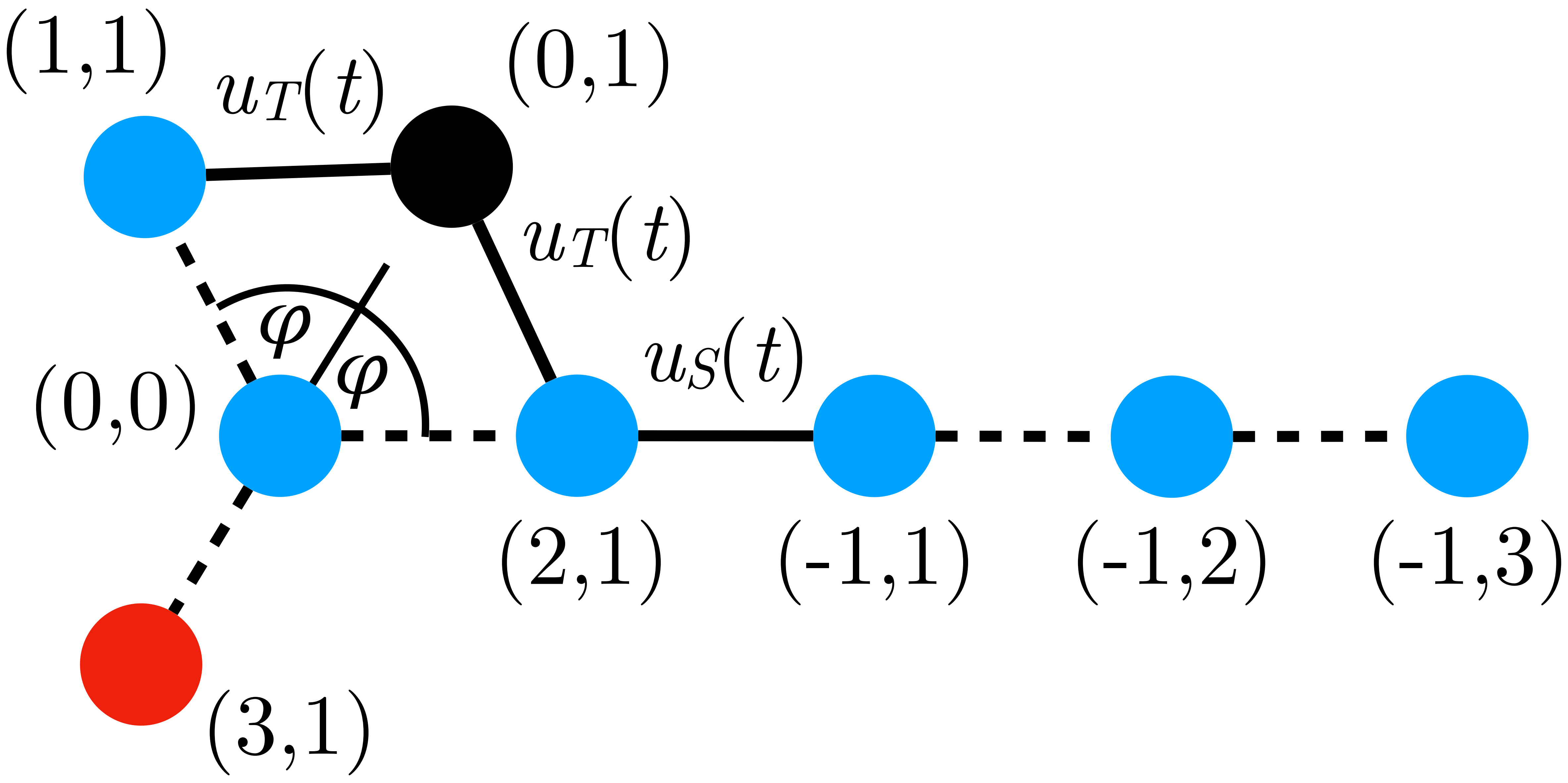

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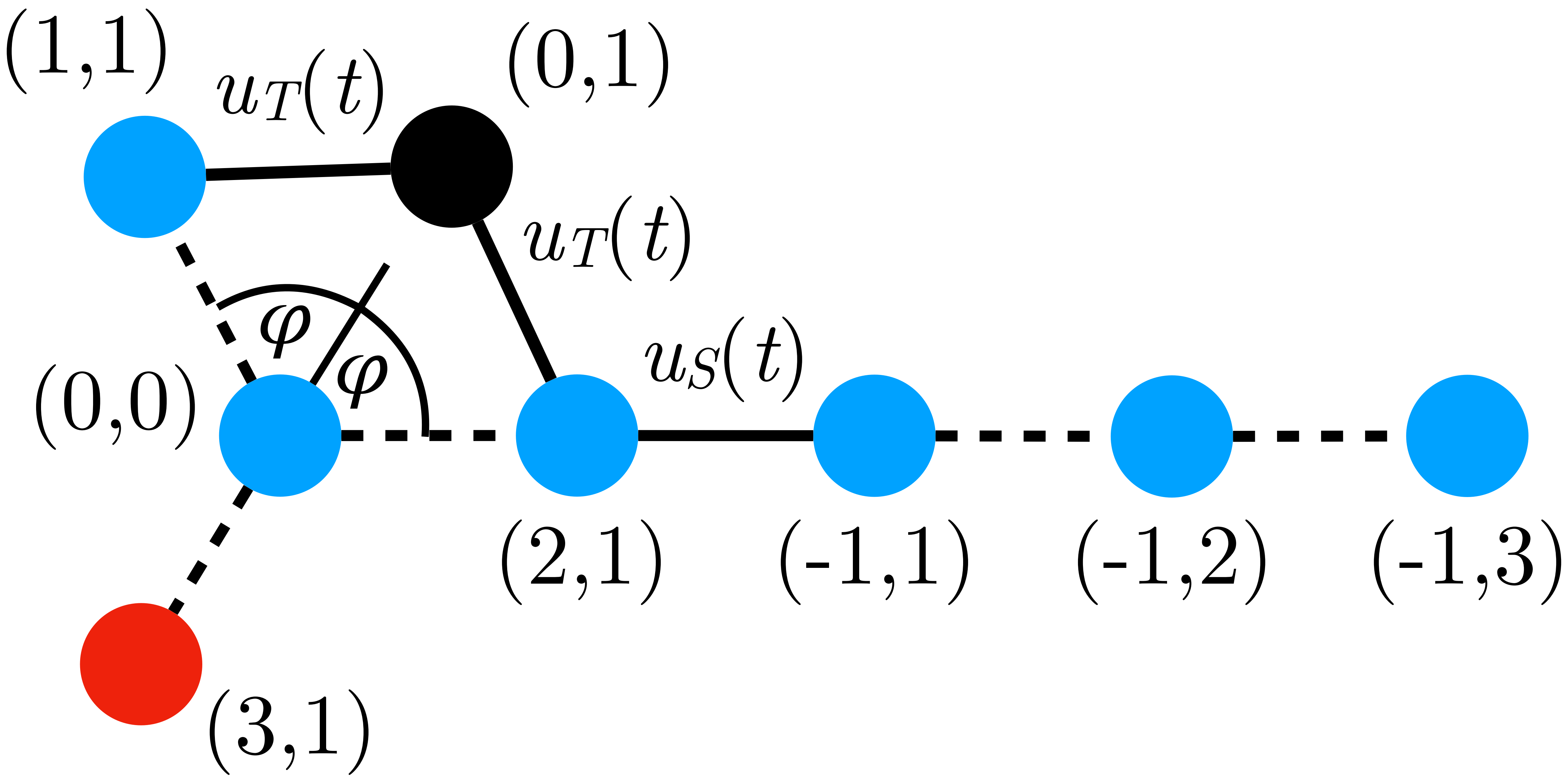

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