Superconducting nanostructures

Literature:

- Y.V. Nazarov, Y.M. Blanter: Quantum transport (Cambridge University Press, 2009)
- S. Datta, P.F. Bagwell, M.P. Anantram: Scattering Theory of Transport for Mesoscopic Superconductors (http://docs.lib.purdue.edu/ecetr/107/)
- R. Cron Ph.D. thesis (Atomic contacts: a Test-Bed for Mesoscopic Physics, CEA Saclay)
- J. Cserti: Superconducting mesoscopic systems (lecture notes)

"Survival kit for superconductivity"

•Below certain temperature (T_C, the SC transition temperature) the electrical resistance of certain materials becomes zero.

•Some conventional superconductors: Nb (9.26K), Pb (7.19K), V (5.3K), Ta (4.48K), Hg (4.15K), Sn (3.72K), In (3.41K), Al (1.2K), Zn (0.85K), Ti (0.39 K), W (0.015K) (See http://hyperphysics.phy-astr.gsu.edu/hbase/solids/scond.html).

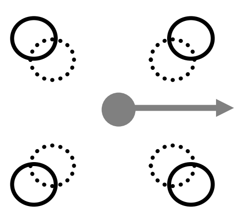
•Superconductors are ideal diamagnets, weak magnetic fileds do not penetrate the bulk of the superconductor (Meissner effect). High magnetic field destroys superconductivity. The critical field, H_C varies between ~10^{-4} T (W) to ~0.2 T (Nb).

•There is a narrow layer at the boundary, where the external magnetic field decreses exponentially to zero. Characteristic length: λ (penetration depth).

•In the SC state the specific heat depends exponentially on the temperature.

•Microscopic BCS theory (Bardeen, Cooper, Schrieffer):

•the electron-phonon coupling can introduce an attractive interaction between the electrons which may overcome Coulomb repulsion. The phonon mediated attraction is a local interaction, $V_{e-ph}=-(2\lambda/\nu)\delta(r_1-r_2)$.

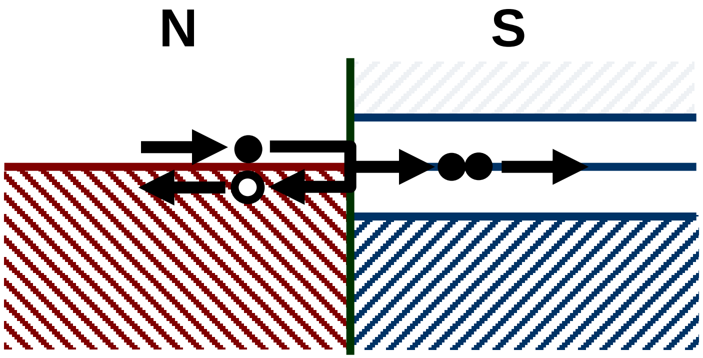


Naive picture: an electron moving in the lattice attracts the ions, which will than attract the next electron passing by.

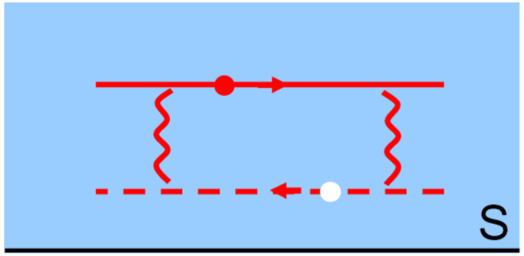
•The ground state of two electrons with attraction is a bound state with $E=-2\Delta$, where $\Delta=\hbar\omega_D\exp(-1/\lambda)$ is the superconducting energy gap. ($\Delta(T=0)\approx 1.76k_BT_C$, approaching T_C it vanishes by $(T_C-T)^{1/2}$.) In the SC state bound states of electron pairs with **k**↓ and **-k** are formed (Cooper pairs)

•The superconducting order parameter is a complex number with the absolute value equal to the gap, and the phase ϕ.

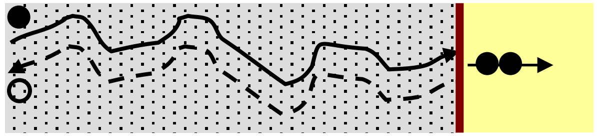
Andreev reflection



At a **normal metal – superconductor (NS) interface** the Andreev reflection is the basic process for charge conversion. An incident up-spin electron with energy $E=\varepsilon_F+\varepsilon$ *drags* with it a down-spin electron of energy $E=\varepsilon_F-\varepsilon$ to form a Cooper pair in the superconductor leaving behind a hole in the spin-down band.



In a bulk superconductor repeated Andreev reflections lead to the modification of the energy eigenstates, and the formation of the superconducting condensate with Cooper pairs.



In a normal metal coupled to a superconductor Andreev reflections introduce superconducting correlations, these are called **proximity effects**.

The process of Andreev reflection can be described by the Bogoliubov – de Gennes (BdG) equations. The BdG equations could be formally derived from the BCS theory of superconductivity.

"Electron - hole description" of superconductivity

Superconductivity is described by the BCS mean field Hamiltonian:

$$\hat{H}_S = \sum_k \xi_k \left(c^+_{k\uparrow} c_{k\uparrow} + c^+_{k\downarrow} c_{k\downarrow} \right) + \underbrace{\Delta c^+_{k\uparrow} c^+_{-k\downarrow} + \Delta^* c_{-k\downarrow} c_{k\uparrow}}_{\text{pairing}}, \qquad \Delta = V \left\langle c_{-k\downarrow} c_{k\uparrow} \right\rangle$$

V describes the attractive interaction, ξ is measured from the Fermi energy.

This hamiltonian is strange in the sense, that the pairing terms do not conserve the particle number. Usually the Hamiltonians contain c^+c -type terms. This structure can be recovered with the transformation:

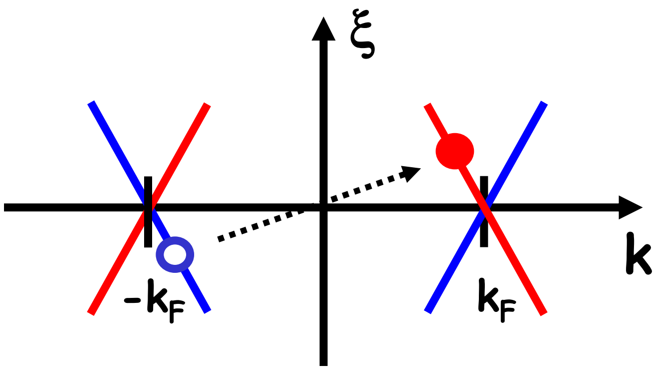
$$e_k = c_{k\uparrow}, \quad h_k = c^+_{-k\downarrow}$$

The "h^+" operator annihilates an electron, i.e. it creates a hole. The spin label is not necessary any more as the operators "e" stand for spin up electrons and the operators "h" stand for spin down "holes".

Let us first look at the normal state Hamiltonian (Δ=0) with the new set of operators:

$$\hat{H}_N = \sum_k \xi_k \left(e^+_k e_k - h^+_k h_k \right) + \sum_k \xi_k \qquad (h_k h^+_k + h^+_k h_k = 1)$$

A -k, down spin electron state is converted to k down spin hole state. The quasiparticle energy of the hole state is opposite to that of the electron state.



Electron state:
- -e charge
- $|k|>k_F \Rightarrow \xi_k>0$; $|k|<k_F \Rightarrow \xi_k<0$
- sign of k and v~dξ/dk are the same

Hole state:
- +e charge
- $|k|>k_F \Rightarrow \xi_k<0$; $|k|<k_F \Rightarrow \xi_k>0$
- sign of k and v~dξ/dk are opposite! (positive k hole state propagates in negative direction!)

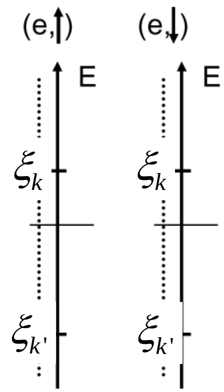
Vacuum state, ground state in the normal metal ($\Delta=0$)

Electron description:

($|k|>k_F$, $|k'|<k_F$)

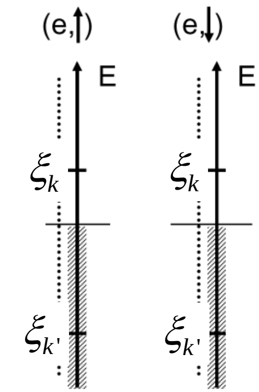
Vacuum state:
no electron states are filled

$$c_k|0\rangle_e = 0$$



The ground state of a normal metal (Fermi sphere) is obtained by filling all the electron states below the Fermi energy

$$|G\rangle_N = \prod_{\xi_k<0} c^+_{k\uparrow} c^+_{k\downarrow} |0\rangle_e$$

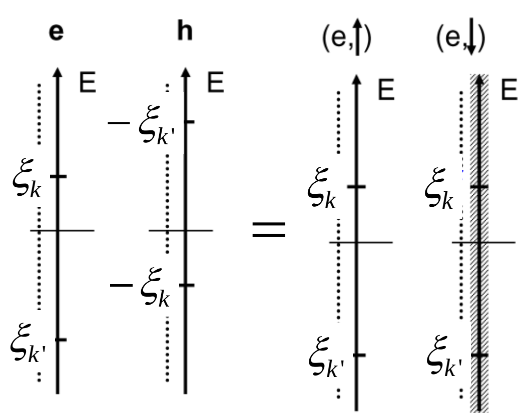


Electron-hole description:

New vacuum state in the electron-hole picture:

$$h_k|0\rangle_{eh} = 0,$$

$$e_k|0\rangle_{eh} = 0$$



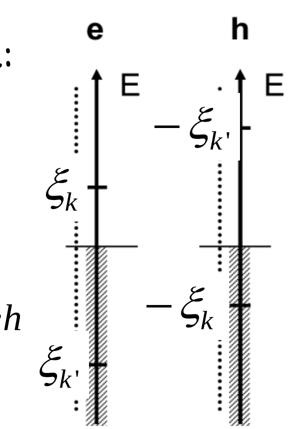
Transformation from the e to the e-h vacuum state:

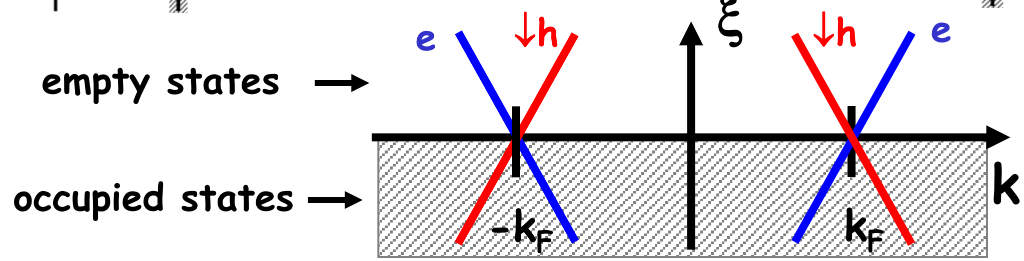
$$|0\rangle_{eh} = \prod_k c^+_{k\downarrow}|0\rangle_e$$

By filling all the states below the Fermi energy both in the electron and the hole band we get the same ground state:

$$|G\rangle_N = \prod_{\xi_k<0} e^+_k h^+_k |0\rangle_{eh}$$

$$|G\rangle_N = \prod_{\xi_k>0} c_{-k\downarrow} \prod_{\xi_k<0} c^+_{k\uparrow} |0\rangle_{eh}$$





Quasiparticles in the superconducting state

The Hamiltonian in the electron-hole representation (the constant $\Sigma\xi_k$ term is omitted) :

$$\hat{H}_S = \sum_k \xi_k \left(e_k^+ e_k - h_k^+ h_k\right) + \underbrace{\Delta e_k^+ h_k + \Delta^* h_k^+ e_k}_{\text{Andreev reflection}}$$

In this new representation the particle number is conserved (but the charge is not!). The interaction term corresponds to Andreev reflection, where an electron is converted to a hole travelling on the time-reversed path or vica versa.

In order to diagonalize the Hamiltonian new operators are introduced:

$$\gamma_{k1} = u_k e_k + v_k h_k$$

$$\gamma_{k0} = -v_k^* e_k + u_k^* h_k$$

$$\hat{H}_S = \sum_k E_k \left(\gamma_{k1}^+ \gamma_{k1} - \gamma_{k0}^+ \gamma_{k0}\right) + \eta_k \gamma_{k1}^+ \gamma_{k0} + \eta_k^* \gamma_{k0}^+ \gamma_{k1},$$

$$\begin{cases} E_k = \xi_k \left(|u_k|^2 - |v_k|^2\right) + \Delta u_k v_k^* + \Delta^* u_k^* v_k \\ \eta_k = -2\xi_k u_k v_k + \Delta u_k^2 - \Delta^* v_k^2 \end{cases}$$

The diagonalization condition is η_k=0. Taking u_k as a real number, and imposing E>0:

$$\boxed{E_k = +\sqrt{\xi_k^2 + |\Delta|^2}}$$

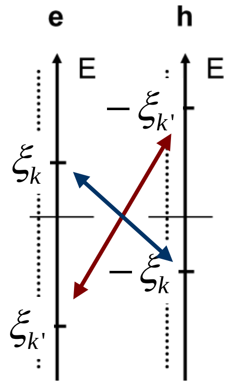
$$u_k = \frac{1}{\sqrt{2}}\left(1 + \frac{\xi_k}{|E_k|}\right)^{1/2}, \quad |v_k| = \frac{1}{\sqrt{2}}\left(1 - \frac{\xi_k}{|E_k|}\right)^{1/2}, \quad \frac{v_k}{|v_k|} = +\frac{\Delta}{|\Delta|}$$

The quasiparticles of the superconductor (γ_{k0} and γ_{k1} are coherent superpositions of electron and hole normal quasiparticles. For $\xi_k \gg |\Delta|$ the normal state quasiparticles are recovered. For ξ_k=0 the quasiparticles are an equally weighted superposition of electron and hole normal quasiparticles.

A gap of 2Δ opens in the quasiparticle spectrum!

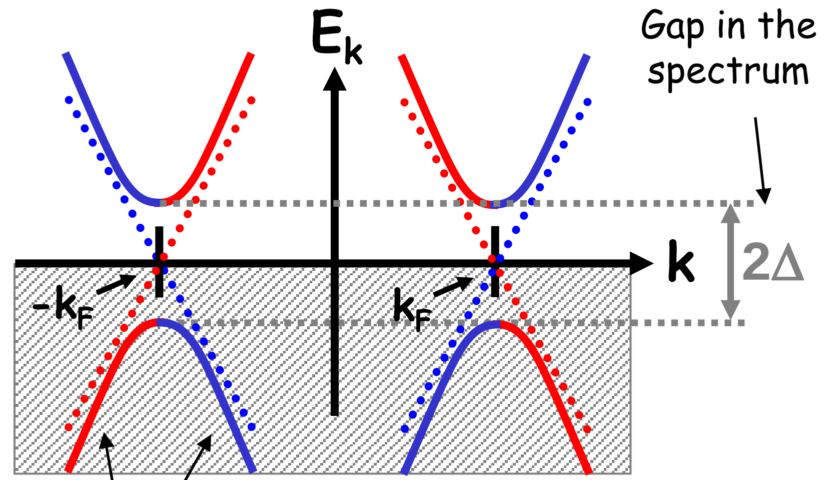
Superconducting ground state

Due to the Andreev reflection term the el. states are coupled to the phase conjugated hole states with opposite spin

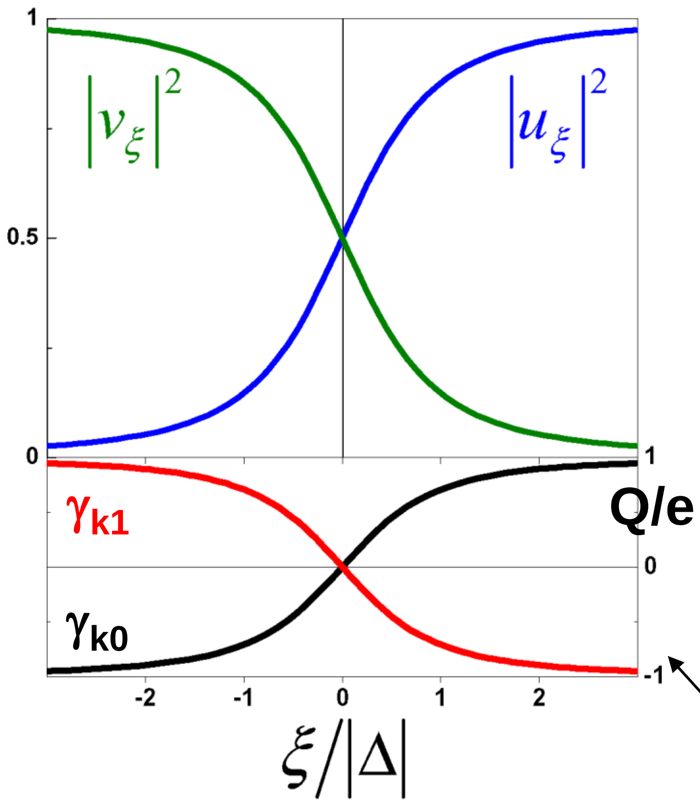


The positive energy states are created by the γ^+_{k1} operator

The negative energy states are created by the γ^+_{k0} operator



The red parts are hole-like states in the sense that the sign of k is opposite to the sign of $v \sim dE_k/dk$. However, in all states the electron (e_k) and the hole (h_k) quasiparticles are superimposed. Similarly the blue parts are electron-like states.



The superconducting ground state:

$$\left|G\right\rangle_{SC} = \Pi\,\gamma^+_{k0}\left|0\right\rangle_{eh} = \Pi(-v_k^* e_k^+ + u_k^* h_k^+)\left|0\right\rangle_{eh} =$$

$$= \ldots = \Pi(-v_k^* c^+_{k\uparrow} c^+_{-k\downarrow} + u_k^*)\left|0\right\rangle_e$$

BCS ground state

The evolution of the quasiparticle charge as a function of ξ

Bogoliubov - de Gennes equation

The Schrödinger equation: $i\hbar\frac{\partial\psi}{\partial t} = H\psi, \quad H = \frac{(\mathbf{p}-e\mathbf{A})^2}{2m} + eV(\mathbf{r})$

The wave equation for holes is the complex conjugate of the corresponding equation for electrons: $i\hbar\frac{\partial\psi_h}{\partial t} = -H^*\psi_h$

An unoccupied electron state with energy E correspond to an occupied hole state with energy -E

Now we describe the system with up-spin electrons (u') and down-spin holes (v'): $i\hbar\frac{\partial u'}{\partial t} = Hu', \quad i\hbar\frac{\partial v'}{\partial t} = -H^*v'$

In the Andreev reflection the incoming electron has an energy $E=\varepsilon_F+\varepsilon$, whereas the reflected hole has an energy $E=-(\varepsilon_F-\varepsilon)$. The coupling between these two can be accomplished by the following equation:

$$\begin{pmatrix} H & \Delta e^{-i2\varepsilon_F t/\hbar} \\ \Delta^* e^{+i2\varepsilon_F t/\hbar} & -H^* \end{pmatrix}\begin{pmatrix} u' \\ v' \end{pmatrix} = i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u' \\ v' \end{pmatrix}$$

where Δ is the pairing potential and the time dependence is needed to cause a change in the energy from $\varepsilon_F+\varepsilon$ to $-(\varepsilon_F-\varepsilon)$. This time dependence can be suppressed by the gauge transformation:

$$u = u'e^{+i\varepsilon_F t/\hbar}, \quad v = v'e^{-i\varepsilon_F t/\hbar}$$

So finally the BdG equation reads:

$$\boxed{\begin{pmatrix} H-\varepsilon_F & \Delta \\ \Delta^* & -(H^*-\varepsilon_F) \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u \\ v \end{pmatrix}}$$

Note: if more SC terminals are present with different chemical potentials then the time dependence of the pairing potential cannot be transformed away -> AC Josephson effect

The same equation can be directly derived from the superconducting Hamiltonian!

Derivation of the BdG equation from the SC Hamiltonian (for a k-state)

$$\hat{H}_S=\sum_k \xi_k\left(e_k^+e_k-h_k^+h_k\right)+\Delta e_k^+h_k+\Delta^* h_k^+e_k \qquad \psi_k=\left(u\ e_k^+ + v\ h_k^+\right)|0\rangle_{eh} \qquad \hat{H}_S\psi_k=E\psi_k$$

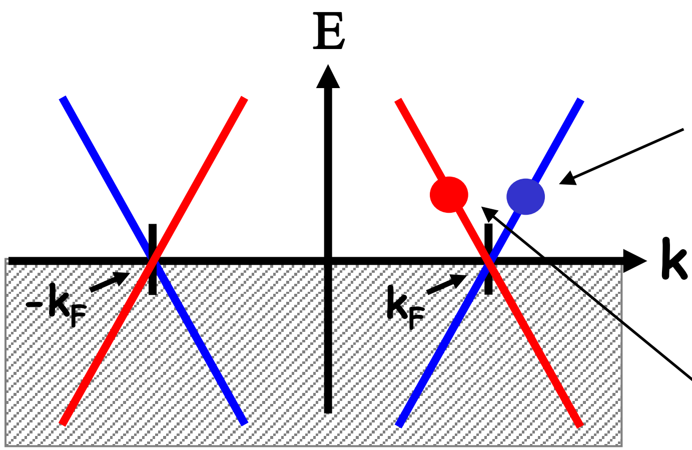
Help: $e_k|0\rangle=h_k|0\rangle=0,\quad e_ke_k^+|0\rangle=\left(1-e_k^+e_k\right)|0\rangle=1\cdot|0\rangle,\quad \{e_k,h_k\}=0$

After some algebra: $\xi_k u e_k^+|0\rangle-\xi_k v h_k^+|0\rangle+\Delta v e_k^+|0\rangle+\Delta^* u h_k^+|0\rangle=Eue_k^+|0\rangle+Evh_k^+|0\rangle$

In a matrix form: $\psi_k=\left(ue_k^+ + vh_k^+\right)|0\rangle_{eh} \Rightarrow \begin{pmatrix} u \\ v\end{pmatrix}\begin{matrix}\leftarrow \text{elektror} \\ \leftarrow \text{hole}\end{matrix} \Rightarrow \begin{pmatrix}\xi_k & \Delta \\ \Delta^* & -\xi_k\end{pmatrix}\begin{pmatrix} u \\ v\end{pmatrix}=E\begin{pmatrix} u \\ v\end{pmatrix}$

Solution in a normal metal

We consider positive energy solutions, the negative energy states are occupied (Fermi sphere).



Electron eigenfunction:

$$\begin{pmatrix} p^2/2m-\varepsilon_F & 0 \\ 0 & -(p^2/2m-\varepsilon_F)\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}e^{ik_ex}=E\begin{pmatrix}1\\0\end{pmatrix}e^{ik_ex}$$

$$\frac{\hbar^2k_e^{\ 2}}{2m}-\frac{\hbar^2k_F^2}{2m}=E \Rightarrow |k_e|>k_F \text{ at } E>0 \qquad k_e=k_F\sqrt{1+E/\varepsilon_F}$$

Hole eigenfunction:

$$\begin{pmatrix} p^2/2m-\varepsilon_F & 0 \\ 0 & -(p^2/2m-\varepsilon_F)\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}e^{ik_hx}=E\begin{pmatrix}0\\1\end{pmatrix}e^{ik_hx}$$

$$\frac{\hbar^2k_F^2}{2m}-\frac{\hbar^2k_h^{\ 2}}{2m}=E \Rightarrow |k_h|<k_F \text{ at } E>0 \qquad k_h=k_F\sqrt{1-E/\varepsilon_F}$$

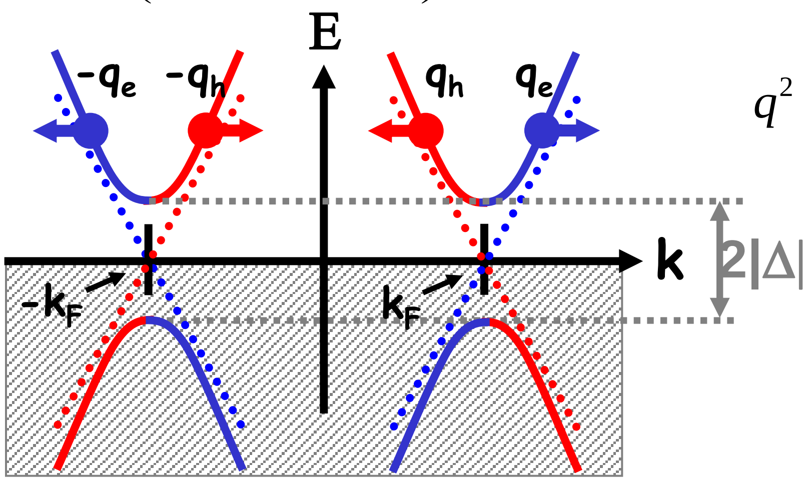
Solution in a superconductor

$$\begin{pmatrix} p^2/2m-\varepsilon_F & |\Delta|e^{i\phi} \\ |\Delta|e^{-i\phi} & -(p^2/2m-\varepsilon_F) \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} e^{iqx} = E\begin{pmatrix} u \\ v \end{pmatrix} e^{iqx}$$

$$\left(\frac{\hbar^2 q^2}{2m}-\varepsilon_F - E\right)u + |\Delta|e^{i\phi}v = 0 \quad \rightarrow \quad v = -\frac{\left(\frac{\hbar^2 q^2}{2m}-\varepsilon_F - E\right)}{|\Delta|e^{i\phi}}u$$

$$|\Delta|e^{i\phi}u - \left(\frac{\hbar^2 q^2}{2m}-\varepsilon_F + E\right)v = 0$$

$$\boxed{E^2(q) = \left(\frac{\hbar^2 q^2}{2m}-\varepsilon_F\right)^2 + |\Delta|^2}$$



$$q^2 = k_F^2\left(1 \pm \frac{\sqrt{E^2-|\Delta|^2}}{\varepsilon_F}\right)$$

"+" $\Rightarrow$ electron–like, $|q_e| > k_F$

"–" $\Rightarrow$ hole–like, $|q_h| < k_F$

For $|\varepsilon|<|\Delta|$:

$$q^2 = k_F^2\left(1 \pm i\frac{\sqrt{|\Delta|^2-E^2}}{\varepsilon_F}\right)$$

Eigenvectors: (u,v)=? (actually it is enough to calculate v/u)

For $|E|>|\Delta|$:

electron-like:

$$\frac{v_e}{u_e} = \left(\frac{E-\sqrt{E^2-|\Delta|^2}}{|\Delta|e^{i\phi}}\right)$$

hole-like:

$$\frac{v_h}{u_h} = \left(\frac{E+\sqrt{E^2-|\Delta|^2}}{|\Delta|e^{i\phi}}\right)$$

For $|E|<|\Delta|$:

electron-like:

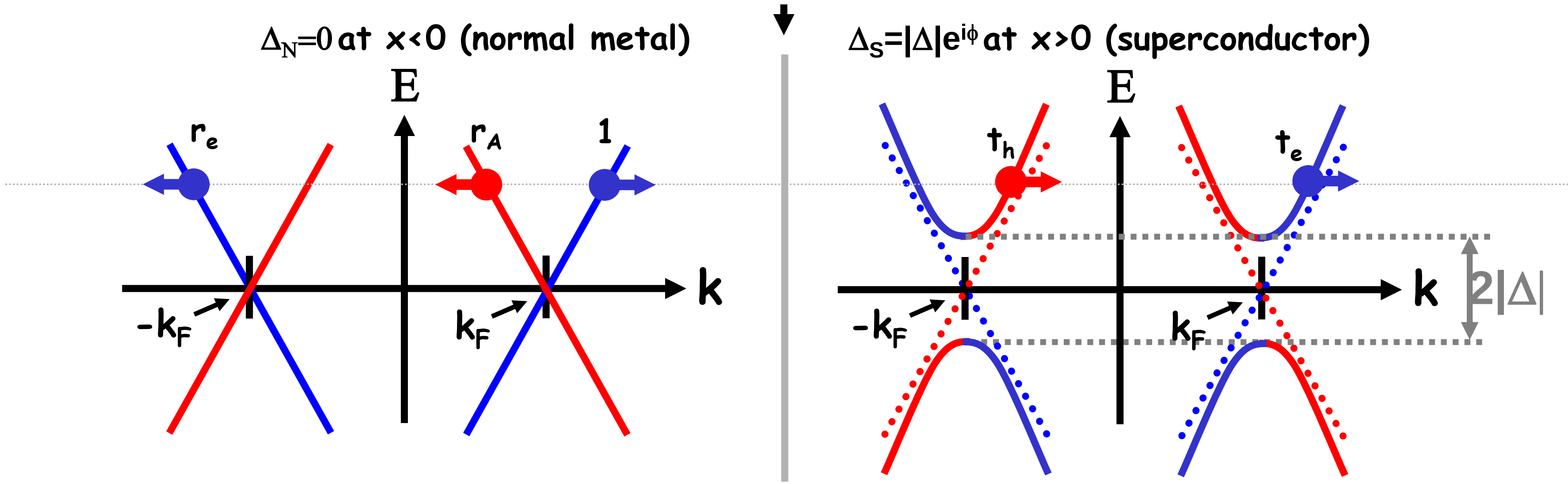
$$\frac{v_e}{u_e} = \left(\frac{E-i\sqrt{|\Delta|^2-E^2}}{|\Delta|e^{i\phi}}\right)$$

hole-like:

$$\frac{v_h}{u_h} = \left(\frac{E+i\sqrt{|\Delta|^2-E^2}}{|\Delta|e^{i\phi}}\right)$$

Solution for an NS-interface

NS interface at x=0, which has perfect transparency (T=1) if the superconductivity is suppressed



$$\Psi_N(x) = 1\begin{pmatrix}1\\0\end{pmatrix}e^{ik_e x} + r_e\begin{pmatrix}1\\0\end{pmatrix}e^{-ik_e x} + r_A\begin{pmatrix}0\\1\end{pmatrix}e^{ik_h x}$$

Incoming electron; Normal reflected el.; Andreev reflected hole

$$\Psi_S(x) = t_e\begin{pmatrix}u_e\\v_e\end{pmatrix}e^{iq_e x} + t_h\begin{pmatrix}u_h\\v_h\end{pmatrix}e^{-iq_h x}$$

Transmitted electron and hole-like quasiparticles

Matching the wave functions:

$$\Psi_N(0) = \Psi_S(0) \qquad \Psi'_N(0) - \Psi'_S(0) = 0$$

$$1 + r_e = t_e u_e + t_h u_h \quad (1) \qquad k_e - r_e k_e = t_e u_e q_e - t_h u_h q_h$$

$$r_A = t_e v_e + t_h v_h \quad (2) \qquad r_A k_h = t_e v_e q_e - t_h v_h q_h$$

Andreev approximation:
$\Delta \ll E_F \Rightarrow k_e \approx k_h \approx q_e \approx q_h$ $\rightarrow$

$$1 - r_e = t_e u_e - t_h u_h \quad (3)$$

$$r_A = t_e v_e - t_h v_h \quad (4)$$

$$(2)-(4) \Rightarrow 2t_h v_h = 0 \Rightarrow t_h = 0$$

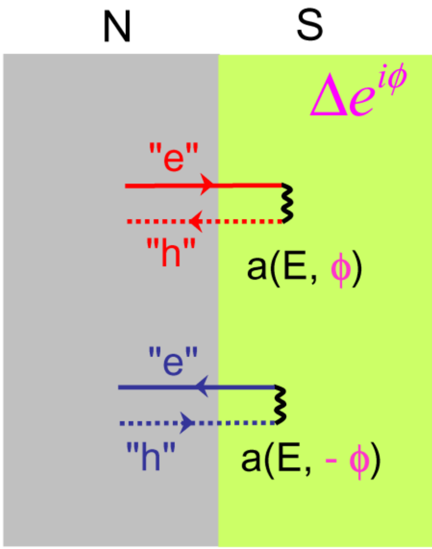
$$(1)-(3) \Rightarrow 2r_e = 2t_h u_h = 0$$

$$(1)+(3) \Rightarrow 2 = 2t_e u_e \Rightarrow t_e = 1/u_e$$

$$(2)+(4) \Rightarrow 2r_A = 2t_e v_e = 2v_e/u_e$$

$$\boxed{r_e = 0, \; r_A = v_e/u_e}$$

Andreev reflection amplitude for perfectly transmitting barrier



$$r_A(E,\phi)=\frac{v_e}{u_e}\begin{cases}\dfrac{1}{|\Delta|e^{i\phi}}\left(E-\operatorname{sgn}(E)\sqrt{E^2-|\Delta|^2}\right), & |E|>|\Delta|\\[2ex] \dfrac{1}{|\Delta|e^{i\phi}}\left(E-i\sqrt{|\Delta|^2-E^2}\right), & |E|<|\Delta|\end{cases}$$

It can be shown that for |E|<|Δ|:

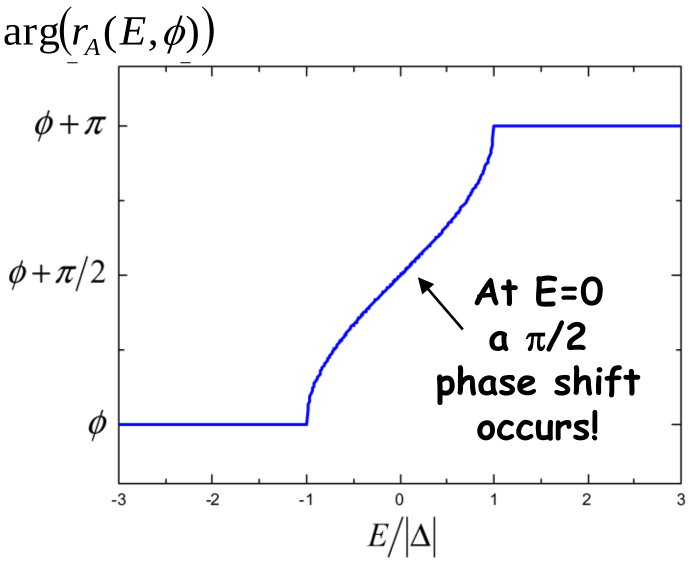
$$\begin{cases}\arg\left(r_A(E,\phi)\right)=\phi+\arccos\left(\dfrac{E}{|\Delta|}\right)\\ |r_A(E,\phi)|=1\end{cases}$$

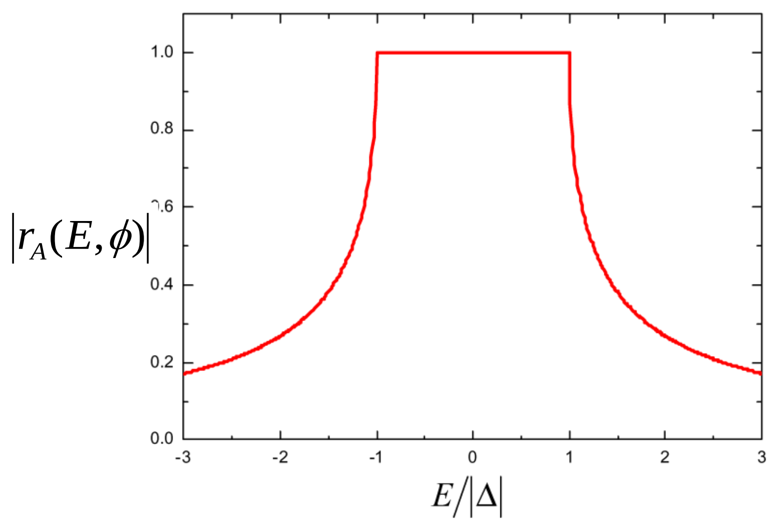
Whereas the Andreev reflection amplitude is 1, the normal reflection vanishes for a T=1 channel: $r_e(E,\phi)=0$

In an NS junction with perfect transmission the normal reflection is prohibited, and all the incoming electrons are Andreev reflected with a probability of one.

For each incoming electron a charge of 2e is transmitted

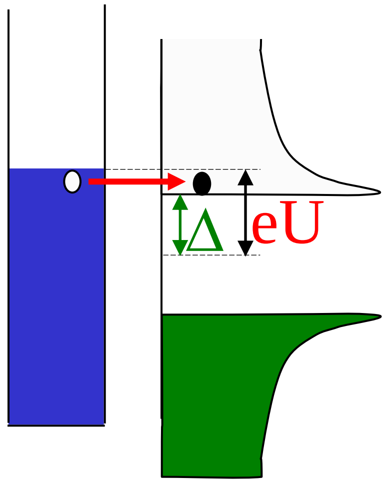
$$G_{NS}=2G_N$$



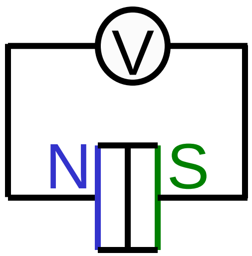


Opposite limit: a weakly transmitting tunnel barrier between the N and S electrode

For a tunnel barrier with low transmission (T<<1) the Andreev reflection is suppressed (the probability for the transmission of 2e charges ~T^2), thus the transport is dominated by the direct quasiparticle tunneling at $|eV|>|\Delta|$.



The dI/dV curve of the NS tunnel junction directly measures the superconducting density of states!

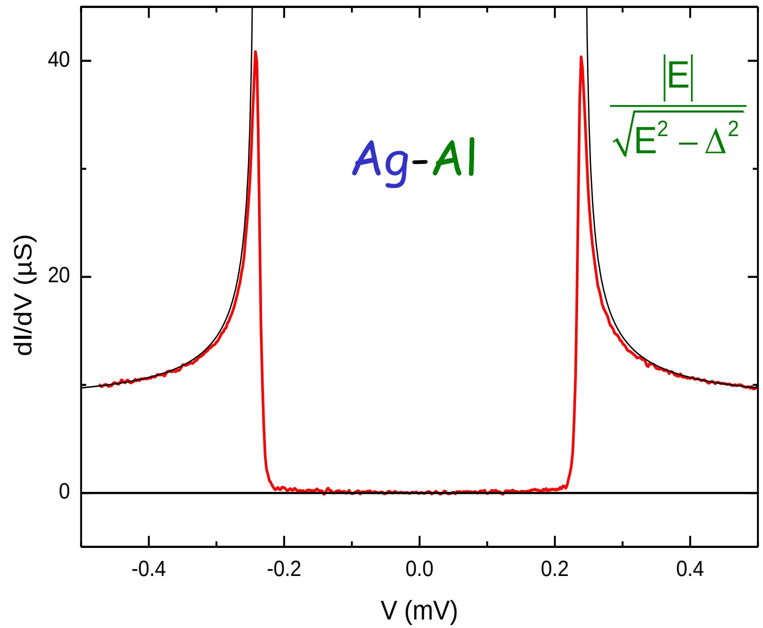


$$I^+ \sim T \cdot \int \mathrm{d}\varepsilon\, g_N(\varepsilon - eU)\, f_N(\varepsilon - eU) \cdot g_S(\varepsilon)\big(1 - f_S(\varepsilon)\big)$$

$$I^- \sim T \cdot \int \mathrm{d}\varepsilon\, g_S(\varepsilon)\, f_S(\varepsilon) \cdot g_N(\varepsilon - eU)\big(1 - f_N(\varepsilon - eU)\big)$$

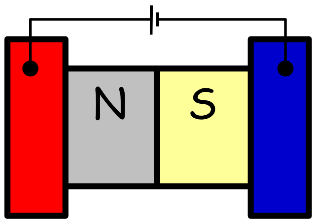
$$I = I^+ - I^- \sim T \cdot g_N(\varepsilon_F) \int \mathrm{d}\varepsilon\, g_S(\varepsilon)\big(f_N(\varepsilon - eU) - f_S(\varepsilon)\big)$$

$$\frac{\mathrm{d}I}{\mathrm{d}U} \sim T \cdot g_N(\varepsilon_F) \int \mathrm{d}\varepsilon\, g_S(\varepsilon)\, f_N'(\varepsilon - eU)$$



BTK theory

(conductance of a ballistic NS junction)



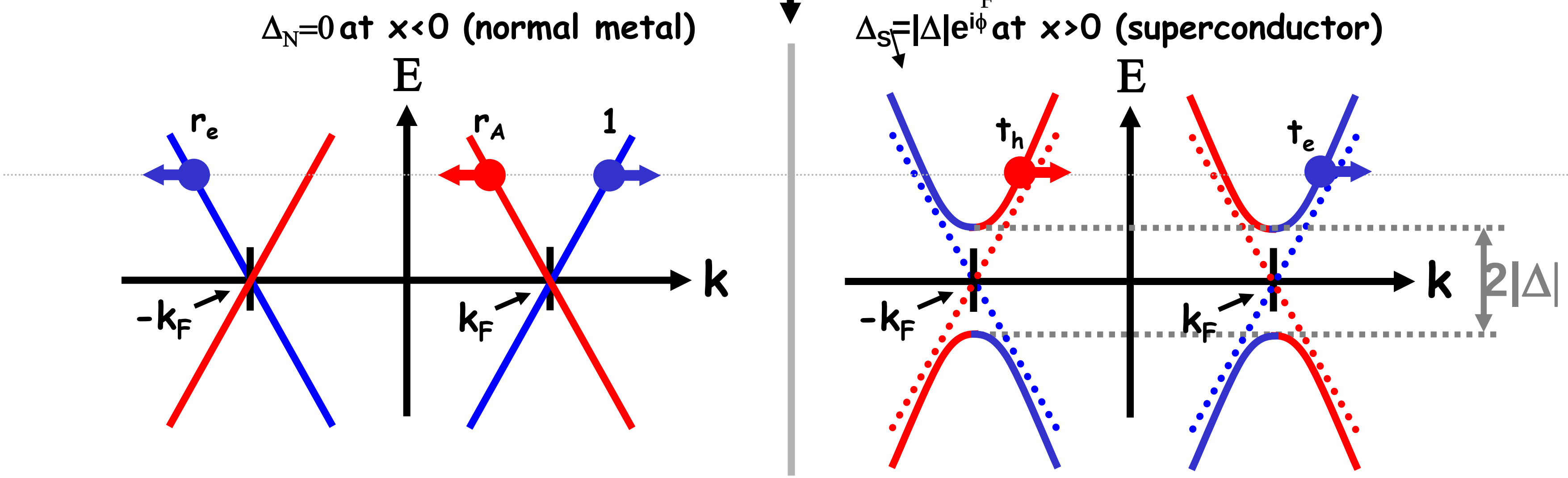
G.E. Blonder, M.Tinkham, T.M.Klapwijk, PRB **25**, 4515 (1982)

In a realistic junction between a normal metal and a superconductor a finite interface scattering has to be considered.

The BTK theory calculates the I-V curve of an NS junction by modeling the interface scattering with a Dirac-delta potential described by a dimensionless scattering strength, Z.

$$V(x) = Z\,\frac{2E_F}{k_F}\,\delta(x)$$

dimensionless „barrier strength"



$$\Psi_N(x) = 1\begin{pmatrix}1\\0\end{pmatrix}e^{ik_e x} + r_e\begin{pmatrix}1\\0\end{pmatrix}e^{-ik_e x} + r_A\begin{pmatrix}0\\1\end{pmatrix}e^{ik_h x}$$

$$\Psi_S(x) = t_e\begin{pmatrix}u_e\\v_e\end{pmatrix}e^{iq_e x} + t_h\begin{pmatrix}u_h\\v_h\end{pmatrix}e^{-iq_h x}$$

Andreev reflection

normal reflection

quasiparticle transmission

Matching the wave functions: $\Psi_N(0) = \Psi_S(0) \equiv \Psi(0), \quad \Psi'_N(0) - \Psi'_S(0) = Z\,\frac{2E_F}{k_F}\,\frac{2m}{\hbar^2}\,\Psi(0)$

Reflection probabilities:

The probability for Andreev reflection: $A = |r_A|^2$

The probability for normal reflection: $B = |r_e|^2$

$E < \Delta$	$E > \Delta$
$A = \dfrac{\Delta^2}{E^2 + (\Delta^2 - E^2)(1+2Z^2)^2}$	$A = \dfrac{\varepsilon^2 - 1}{\left[\varepsilon + (1+2Z^2)\right]^2}$
$B = 1 - A$	$B = \dfrac{4Z^2(1+Z^2)}{\left[\varepsilon + (1+2Z^2)\right]^2}$

$$\varepsilon = \frac{E}{\sqrt{E^2 - \Delta^2}}$$

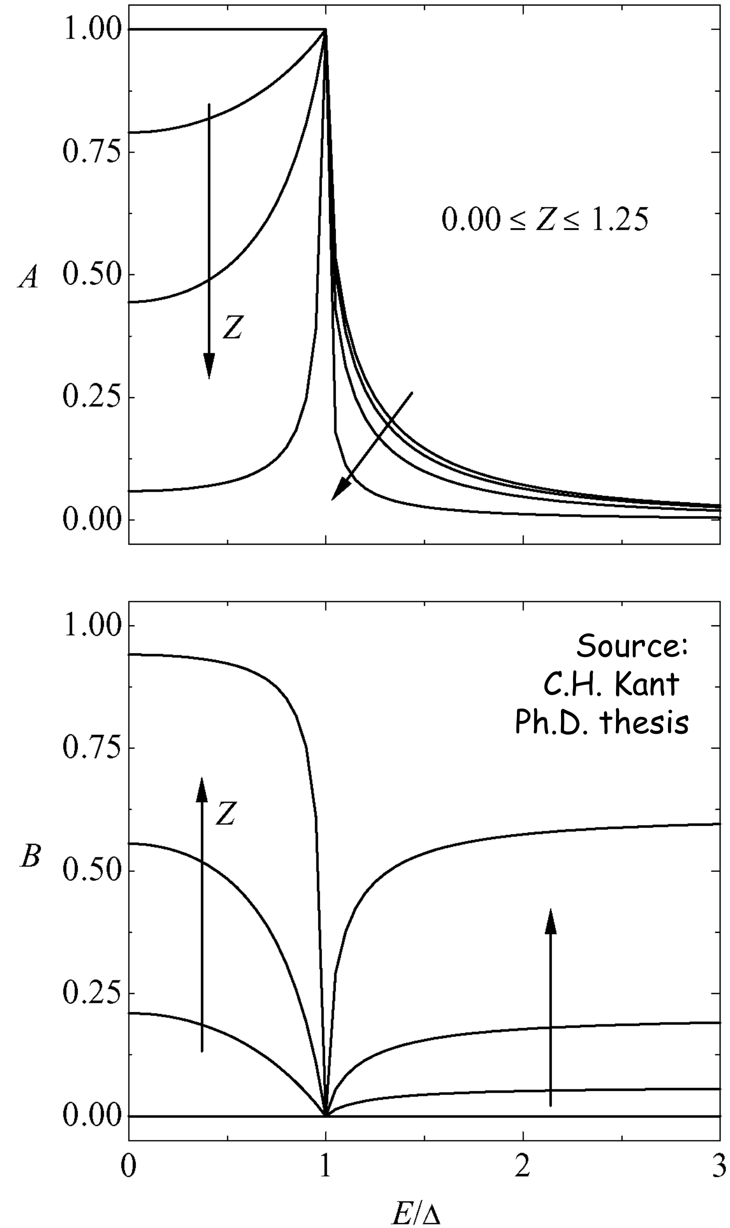
-For Z=0 and E<Δ all the incoming electrons are Andreev reflected

-At E<Δ the probability for quasiparticle transmission is zero, i.e. A+B=1.

-At E>>Δ A=0 and the probability for normal reflection is:

$$B(E \gg \Delta) = R_N = 1 - T_N = \frac{Z^2}{1+Z^2}$$

$$\Rightarrow T_N = \frac{1}{1+Z^2}; \quad Z^2 = \frac{1-T_N}{T_N}$$



-The Andreev reflection probability at zero energy:

$$A(E=0) = R_A = \frac{1}{(1+2Z^2)^2} = \left(\frac{T_N}{2-T_N}\right)^2$$

Calculation of the current

Let us calculate the current at the normal side:

$$I = e\underset{\uparrow}{S}\int v(E)\rho(E)\left[1 + A(E) - B(E)\right]\left[f(E-eV) - f(E)\right]\mathrm{d}E$$

The area of the contact

The conductance, G_{NS}=dI/dV:

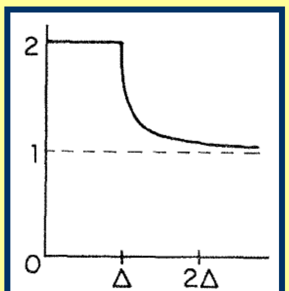
$$G_{\mathrm{NS}} = -e^2 S v_{\mathrm{F}} \rho_{\mathrm{F}} \int \left[1 + A(E) - B(E)\right] f'(E-eV)\mathrm{d}E$$

The normal state conductance (Δ=0):

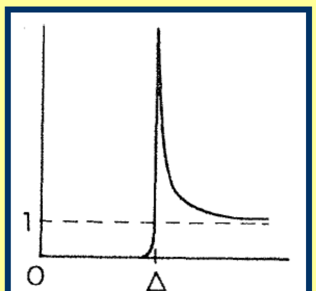
$$G_{\mathrm{NN}} = \frac{e^2 S v_{\mathrm{F}} \rho_{\mathrm{F}}}{1 + \mathrm{Z}^2}$$

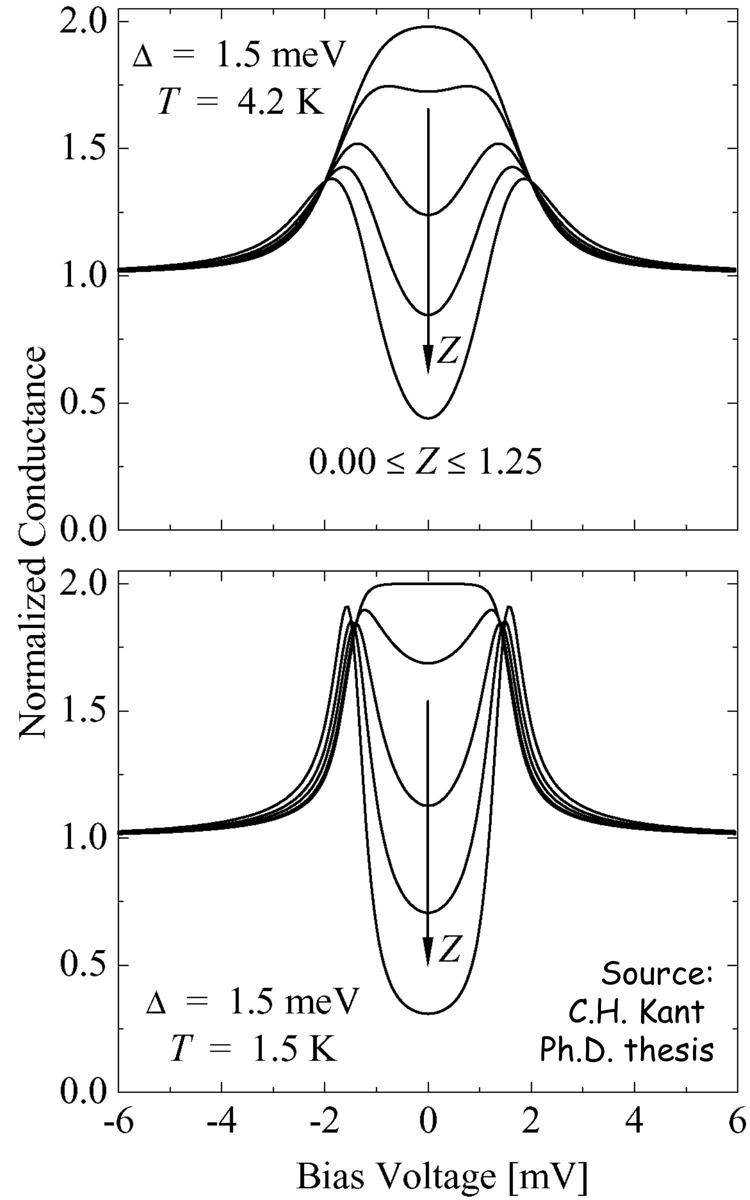
$$\frac{G_{\mathrm{NS}}}{G_{\mathrm{NN}}} = -(1 - Z^2)\int \left[1 + A(E) - B(E)\right] f'(E-eV)\mathrm{d}E$$

Z=0 limit: $G(eV \ll \Delta) = 2G_N$, for each incoming electron a hole is reflected, and a charge of 2e is transmitted

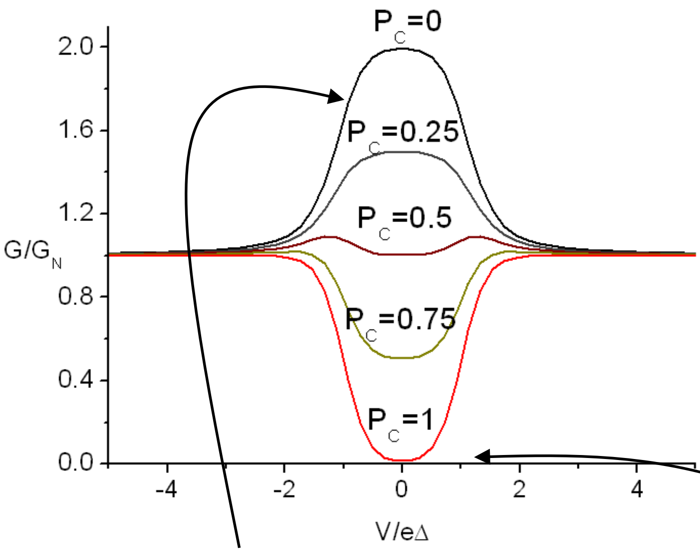


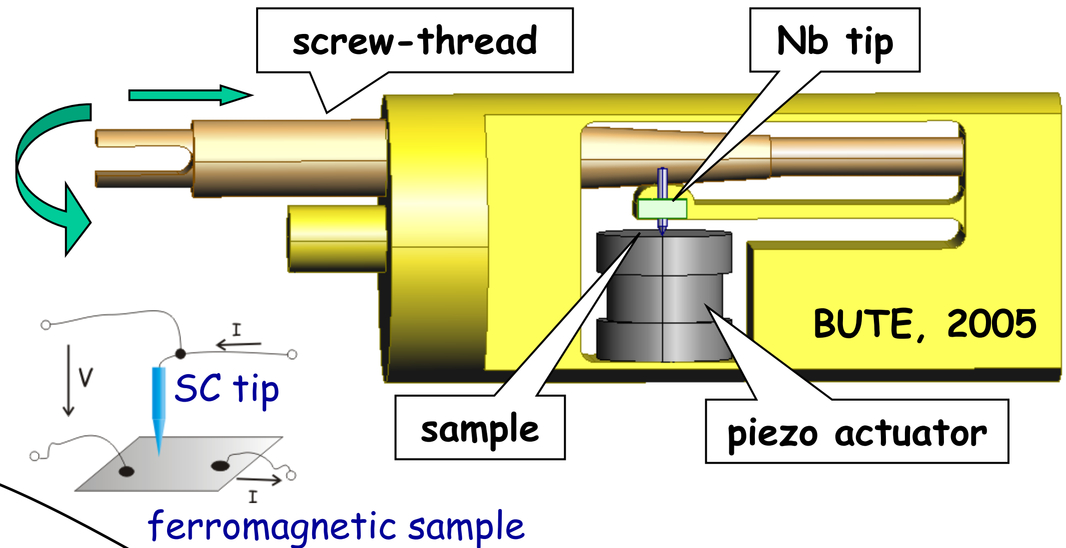
Z>>1 limit: conventional NIS tunneling curve, $G(eV < \Delta) = 0$, sharp peak at Δ





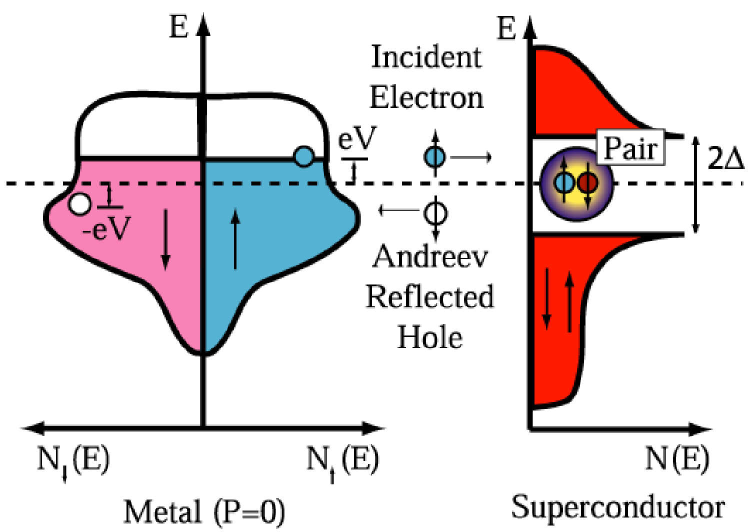
Application: measurement of spin-polarization with SF contact





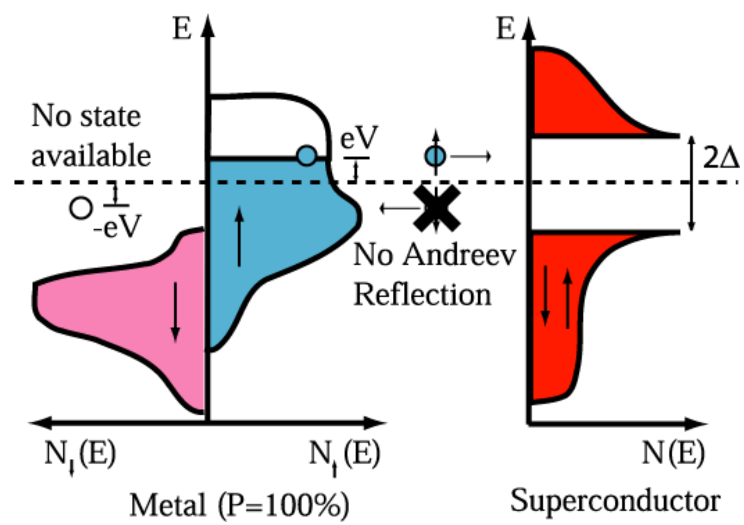
For a normal metal with P=0 an incoming electron is Andreev reflected, thus for each incoming e^- a charge of 2e is transmitted, $G_{NS}=2G_N$

In a half-metal (P=1) Andreev reflection is prohibited, $G_{FS}=0$



The fit of the I-V curves tells the spin-polarization!

As a first approx.: $G_{FS}(V=0)=2(1-P_C)G_N$



Inclusion of spin polarization in the BTK theory

Spin polarization on the N side can be considered as a sum of fully polarized and unpolarized currents:

$$I = I_\uparrow + I_\downarrow = \underbrace{2I_\downarrow}_{I_{unpol}} + \underbrace{(I_\uparrow - I_\downarrow)}_{I_{pol}}$$

$$G(V,T,P_C,Z) = (1-P_C)G_{unpol}(V,T,Z) + P_C G_{pol}(V,T,Z)$$

For the unpolarized current the original BTK result is used.
In the polarized current the Andreev reflection is suppressed, $A \to \widetilde{A} = 0$

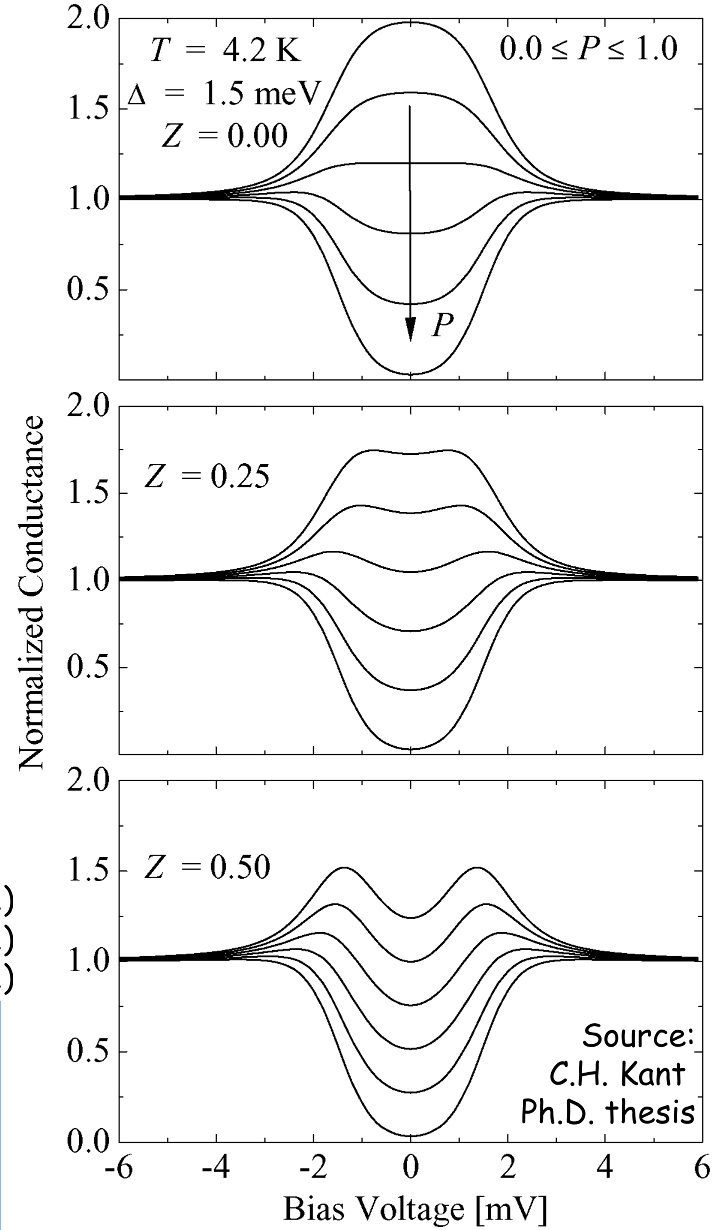
The probability for the normal reflection is rescaled to preserve current conservation: $B \to \widetilde{B}$

Assumption: the ratio for the normal reflection and quasiparticle transmission is independent of spin-polarization:

$$\frac{R_n}{T_n} = \frac{B}{1-A-B} = \frac{\widetilde{B}}{1-\widetilde{B}} \Rightarrow \widetilde{B} = \frac{B}{1-A}$$

For more details see: G. J. Strijkers et al. Phys. Rev. B **63**, 104510 (2001)
I. I. Mazin et al. J. Appl. Phys. **89**, 7576 (2001)
Y. Ji etal. Phys. Rev. B **64**, 224425 (2001)

$$\frac{G_{NS}}{G_{NN}} = -P_C(1-Z^2)\int\left[1-\widetilde{B}(E)\right]f'(E-eV)\mathrm{d}E - (1-P_C)(1+Z^2)\int\left[1+A(E)-B(E)\right]f'(E-eV)\mathrm{d}E$$



Source: C.H. Kant Ph.D. thesis

First measurements: tip-sample approach

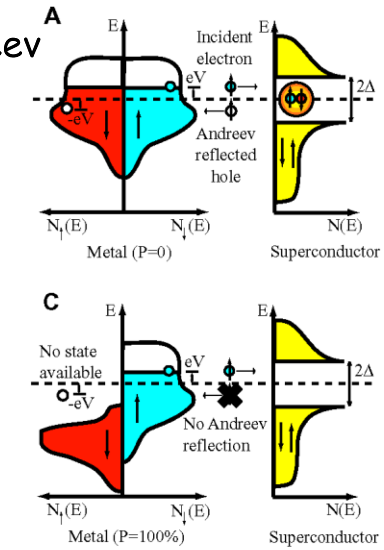
R. J. Soulen Jr., J. M. Byers,* M. S. Osofsky, B. Nadgorny, T. Ambrose, S. F. Cheng, P. R. Broussard, C. T. Tanaka, J. Nowak, J. S. Moodera, A. Barry, J. M. D. Coey, Science **282**, 85 (1998)

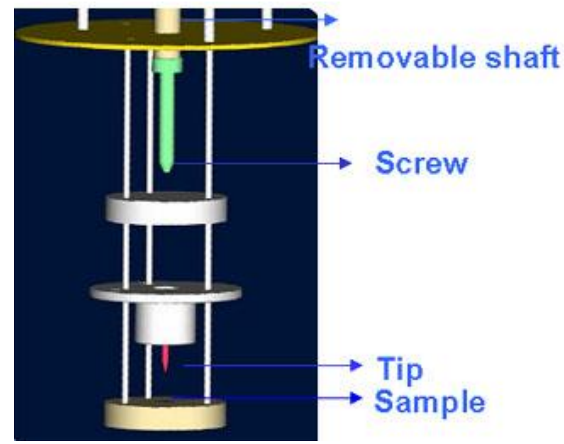
One of the first studies demonstrating the Andreev spectroscopy technique for various ferromagnetic metals.

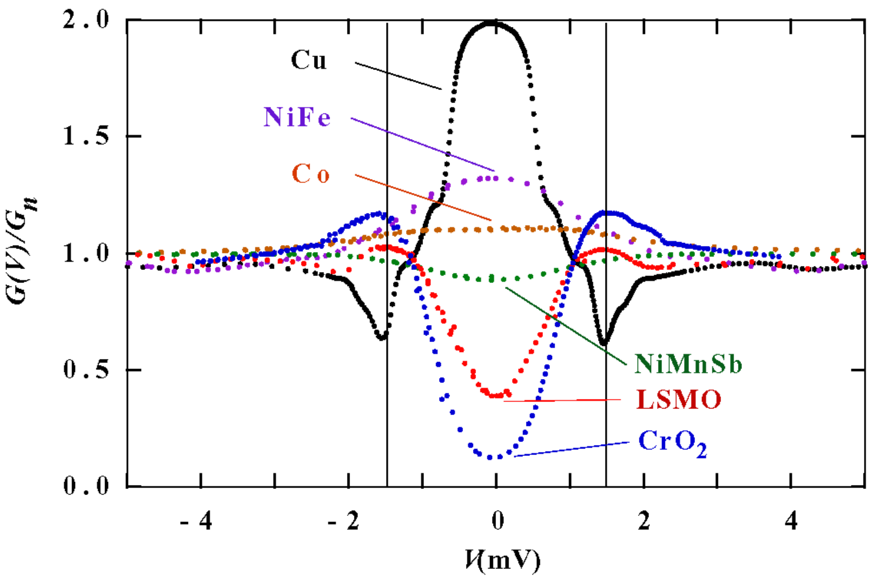
The spin-polarization is determined by the simple formula:

$$G_{FS}(V=0)=2(1-P_C)G_N$$

Probably rather large diffusive contacts were studied, as the BTK theory would not give good fit to the curves.



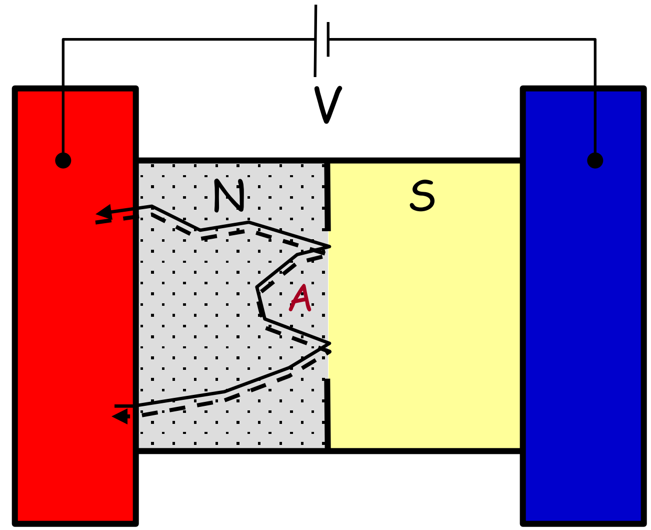




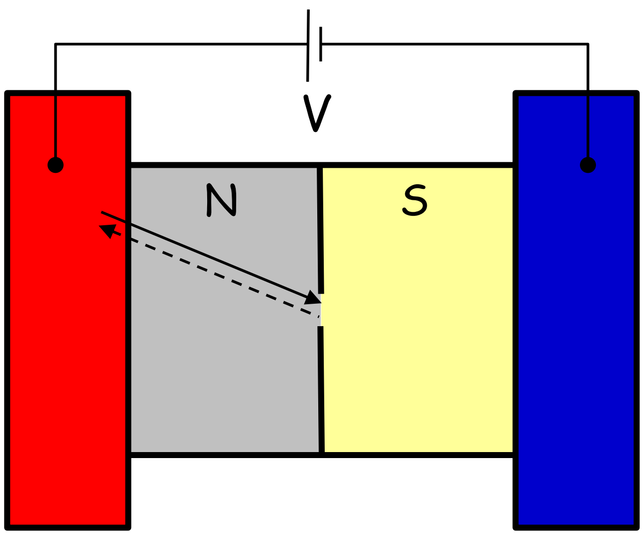
Material studied	Point	Base	N	P_T (%)	P_C (%)
NiFe	Nb	$Ni_{0.8}Fe_{0.2}$ film	14	25 ± 2	37 ± 5
Co	Nb	Co foil	7	35 ± 3	42 ± 2
Fe	Ta	Fe film	12	40 ± 2	45 ± 2
	Fe	Ta foil	14		46 ± 2
	Nb	Fe film	4		42 ± 2
	Fe	V crystal	10		45 ± 2
Ni	Nb	Ni foil	4	23 ± 3	46.5 ± 1
	Nb	Ni film	5		43 ± 2
	Ta	Ni film	8		44 ± 4
NiMnSb	Nb	NiMnSb film	9	–	58 ± 2.3
LSMO	Nb	$La_{0.7}Sr_{0.3}MnO_3$ film	14	–	78 ± 4.0
CrO_2	Nb	CrO_2 film	9	–	90 ± 3.6

Proximity effect (why shall we use ballistic contacts?)

Proximity effect: the Andreev reflection introduces superconducting correlations at the normal side. The Andreev reflected hole is travelling on the time-reversed path of the incoming electron, thus the electron and the hole form phase-conjugated pairs.



Ballistic contact:



In a ballistic contact the reflected hole travels back to the reservoir, where it thermalizes. The incoming states at the NS interface all have the distribution of the left electrode, and no superconducting correlations are present.

Diffusive contact:

In a diffusive contact an electron and the Andreev reflected hole can bounce back and forth on the same trajectory between different points of the contact, causing a coherent superosition.

An energy difference, ΔE destroys the phase coherence after a time:

$$\tau \sim \frac{\hbar}{\Delta E}$$

Thus the coherence length is: $\xi_N = \sqrt{D\tau} = \sqrt{\frac{\hbar D}{\Delta E}}$

The phase coherence can be destroyed by magnetic field, temperature and applied voltage

$$\xi_N = \sqrt{\frac{\hbar D}{kT}} \qquad \xi_N = \sqrt{\frac{\hbar D}{eV}}$$

+ the inelastic diffusive length:

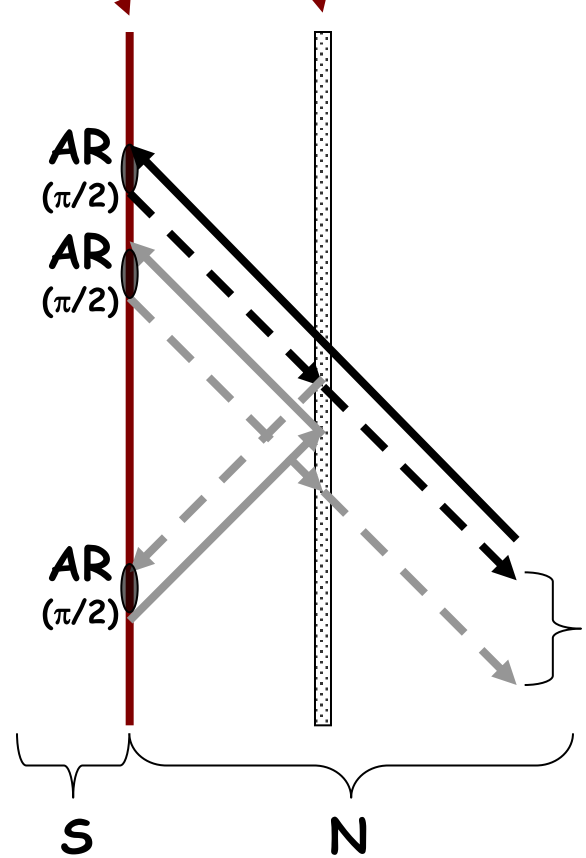
$$\xi_{\text{in}} = \sqrt{D\tau_{\text{in}}}$$

Proximity effects 1.: reentrance

C.W.J. Beenakker, cond-mat/9909293, T.M. Klapwijk, Journal of Superconductivity **17**, 593 (2004), C.W.J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997)

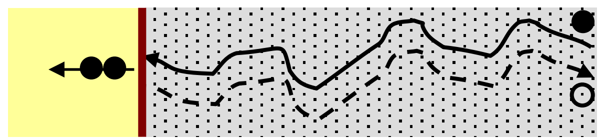
The diffusive region is modelled by a single barrier with transmission t_D

NS interface (Z=0)

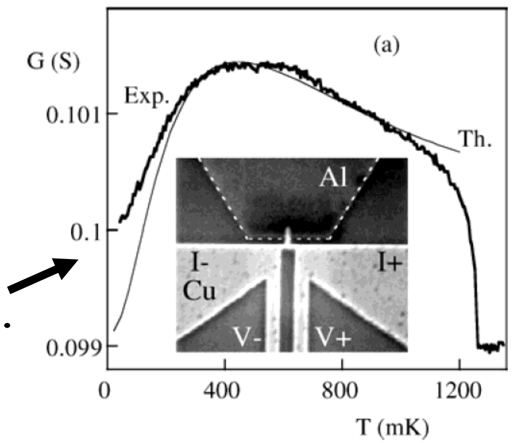


π phase shift, destructive interference!

In a diffusive contact, the incoming electron reaches the interface through a lot of scatterings, however the Andreev reflected hole comes back on the time-reversed path, thus **a fully phase coherent NS junction is expected to be completely transparent, $G_{NS}=2G_N$**



The experiments, however show, that **the conductance** increases below the T_c, but it **drops at low enough temperature.** (H. Courtois et al., Superlattices and Microstructures **25**, 721 (1999))

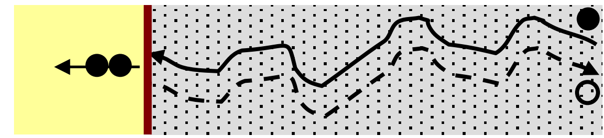


The incoming electron acquires a phase ϕ, whereas the Andeev reflected hole on the time-reversed path acqires a phase $-\phi$, but **the Andreev reflection causes a phase shift of $\pi/2$**, thus the net phase between the two paths is π!

At low enough temperature the coherence length increases, and the destructive interference becomes important.
It can be shown, that at T=0 $G_{NS}=G_N$!

Proximity effects 1.: reentrance

C.W.J. Beenakker, cond-mat/9909293, T.M. Klapwijk, Journal of Superconductivity **17**, 593 (2004), C.W.J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997)



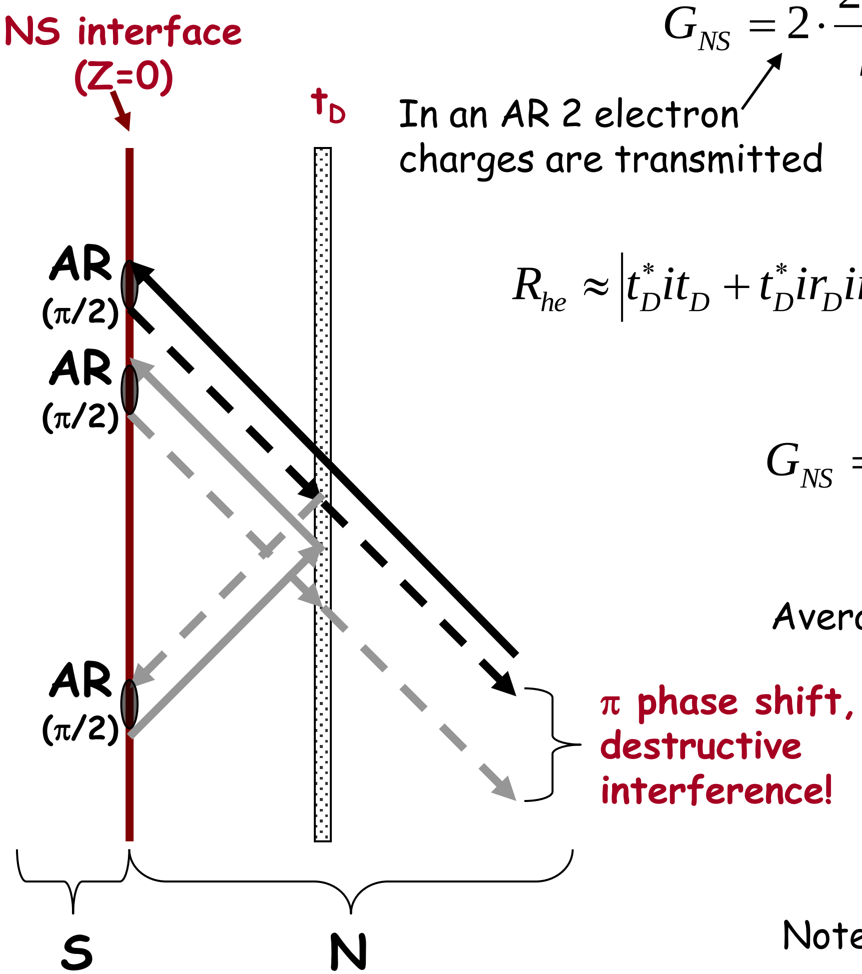
The zero-bias conductance of an NS junction:

$$G_{NS} = 2 \cdot \frac{2e^2}{h} N R_{he}$$

In an AR 2 electron charges are transmitted

Number of channels

The probability that an incoming e^- is Andreev reflected as a hole



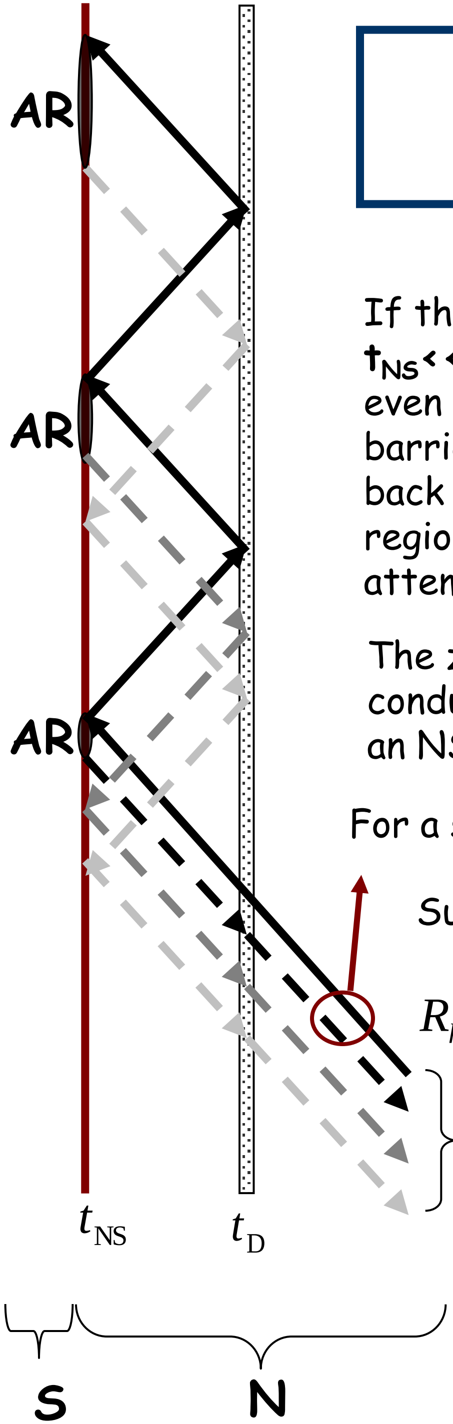
$$R_{he} \approx \left| t_D^* i t_D + t_D^* i r_D i r_D^* i t_D + \ldots \right|^2 = \left| \frac{i t_D^* t_D}{1 - i i r_D^* r_D} \right|^2 = \frac{T_D^2}{\left(1 + R_D\right)^2} = \frac{T_D^2}{\left(2 - T_D\right)^2}$$

$$G_{NS} = 2 \cdot \frac{2e^2}{h} \sum_n \frac{T_n^2}{\left(2 - T_n\right)^2} \overset{!}{<} 2G_N = 2 \cdot \frac{2e^2}{h} \sum_n T_n$$

Averaging with random matrix theory:

$$G_{NS} = 2 \cdot \frac{2e^2}{h} \left\langle \sum_n \frac{T_n^2}{\left(2 - T_n\right)^2} \right\rangle = G_N$$

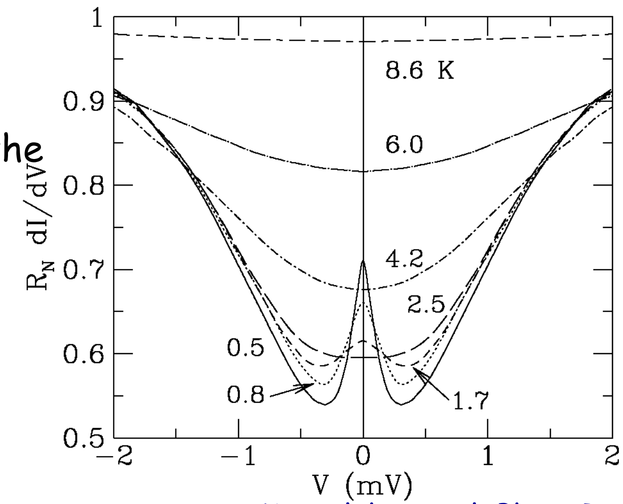
Note: without the phase shift of „i" $G_{NS} = 2G_N$ would come!



Proximity eff. 2.: reflectionless tunneling

T.M. Klapwijk, Journal of Superconductivity **17**, 593 (2004), C.W.J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997)

If the NS interface has small transmission, $\mathbf{t_{NS}<<1}$, the amplitude of Andreev reflection is even smaller, $r_A \sim t^2_{NS}$ (2 electron charges cross the barrier). However, the electron can be reflected back to the NS interface by the disordered region several times, thus it can repeatedly attempt the Andreev reflection.



A. Kastalsky et al. Phys. Rev. Lett. **67**, 3026 (1991)

The zero-bias conductance of an NS junction:

$$G_{NS} = 2 \cdot \frac{2e^2}{h} N R_{he}$$

For a single process: $R_{he} = \left| t_D^* r_A t_D \right|^2 = R_A T_D^2 \approx T_{NS}^2 T_D^2$

Summing up the multiple attempts: (the phase is same for all!)

The conductance is considerably larger!

$$R_{he} \approx \left| t_D^* r_A t_D + t_D^* r_{NS}^* r_D^* r_A r_D r_{NS} t_D + \ldots \right|^2 = \left| \frac{t_D^* r_A t_D}{1 - r_{NS}^* r_{NS} r_D^* r_D} \right|^2 \approx \frac{T_{NS}^2 T_D^2}{\left(T_{NS} + T_D - T_{NS} T_D \right)^2}$$

General statement: (Beenakker, Rev. Mod. Phys. **69**, 731 (1997))

$$R_{NS}^{\text{class}} = \frac{h}{2e^2 N} \left(g_N^{-1} + 2 T_{NS}^{-2} \right)$$

$$R_N^{\text{class}} = \frac{h}{2e^2 N} \left(g_N^{-1} + T_{NS}^{-1} \right)$$

$$T_{NS} >> g_N$$

$$R_{NS}(B=0, V=0) \approx R_N^{\text{class}}$$

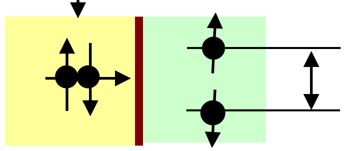
$$R_{NS}(B>0, V=0) \approx R_{NS}^{\text{class}}$$

Proximity effects 3.: ferromagnetic electrode

A.I. Buzdin, Rev. Mod. Phys. **77** 935 (2005)

A Cooper pair in a superconductor consists of two electrons with opposite spins and momenta. In a ferromagnet the up-spin electron decreases its energy by $\mu_b H_{eff}$, while the downspin electron energy increases by the same value. To compensate this energy variation, the up-spin electron increases its kinetic energy, while the down-spin electron decreases its kinetic energy.

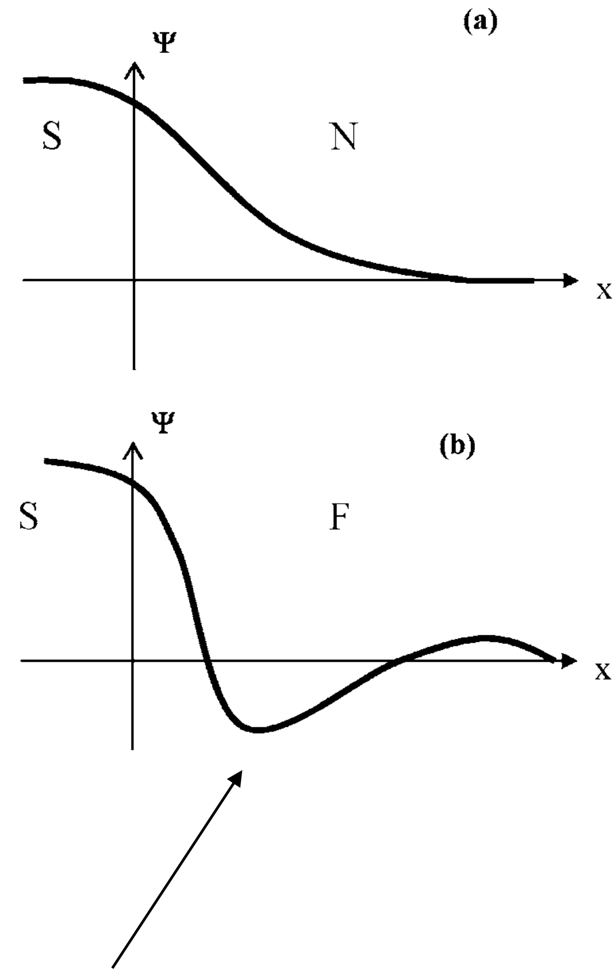
$k_F, -k_F$ $\quad$ $k_F + \dfrac{\mu_B H_{eff}}{\hbar v_F}$



$2\mu_B H_{eff}$

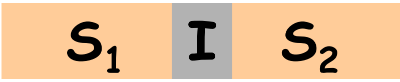
$-k_F + \dfrac{\mu_B H_{eff}}{\hbar v_F}$

$$\delta k = \frac{2\mu_B H_{eff}}{\hbar v_F}, \quad \lambda_{osc} = \frac{\pi\hbar v_F}{\mu_B H_{eff}}$$



Oscillation of the order parameter

Josephson effect (traditional approach)



$$\psi_1 = \sqrt{\rho_1} e^{i\phi_1} \qquad \psi_2 = \sqrt{\rho_2} e^{i\phi_2}$$

Macroscopic wave functions. $|\psi|^2 \sim$ particle density (ρ)
+ phase difference ($\delta = \phi_2 - \phi_1$)

We apply a voltage of eV on the junction!

$$i\hbar \frac{d\psi_1}{dt} = \frac{2eV}{2}\psi_1 + T\psi_2 \;\Rightarrow\; i\hbar\left(\frac{1}{2\sqrt{\rho_1}}\dot{\rho}_1 e^{i\phi_1} + \sqrt{\rho_1} e^{i\phi_1} i\dot{\phi}_1\right) = \frac{2eV}{2}\sqrt{\rho_1} e^{i\phi_1} + T\sqrt{\rho_2} e^{i\phi_2}$$

$$i\hbar \frac{d\psi_2}{dt} = -\frac{2eV}{2}\psi_2 + T\psi_1 \;\Rightarrow\; \ldots$$

Dividing by $e^{i\phi 1}$ (or $e^{i\phi 2}$) and writing the equations separately for the real and imaginary part:

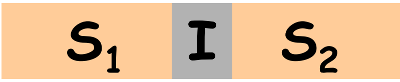
$$\dot{\rho}_1 = \frac{2T}{\hbar}\sqrt{\rho_1\rho_2}\sin\delta, \quad \dot{\rho}_2 = -\frac{2T}{\hbar}\sqrt{\rho_1\rho_2}\sin\delta$$

$$\dot{\phi}_1 = -\frac{T}{\hbar}\sqrt{\frac{\rho_2}{\rho_1}}\cos\delta - \frac{2eV}{2\hbar}, \quad \dot{\phi}_2 = -\frac{T}{\hbar}\sqrt{\frac{\rho_1}{\rho_2}}\cos\delta + \frac{2eV}{2\hbar}$$

The current is proportional to $d\rho_1/dt = -d\rho_2/dt$: $\quad I = I_0 \sin\delta$

Subtracting the equations for the phase: $\quad \dot{\delta} = \frac{2eV}{\hbar} \;\Rightarrow\; \delta(t) = \delta_0 + \frac{2e}{\hbar}\int V(t)\,dt$

$$\left.\begin{aligned} I &= I_0 \sin\delta \\ \dot{\delta} &= \frac{2eV}{\hbar} \;\Rightarrow\; \delta(t) = \delta_0 + \frac{2e}{\hbar}\int V(t)\,dt \end{aligned}\right\} \text{Josephson equations}$$



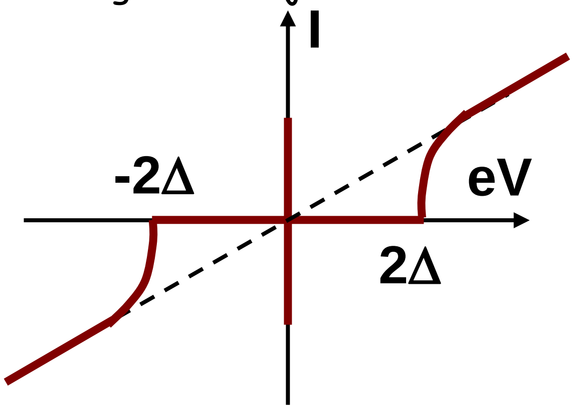
Josephson effect (traditional approach)

$$\delta = \phi_2 - \phi_1$$

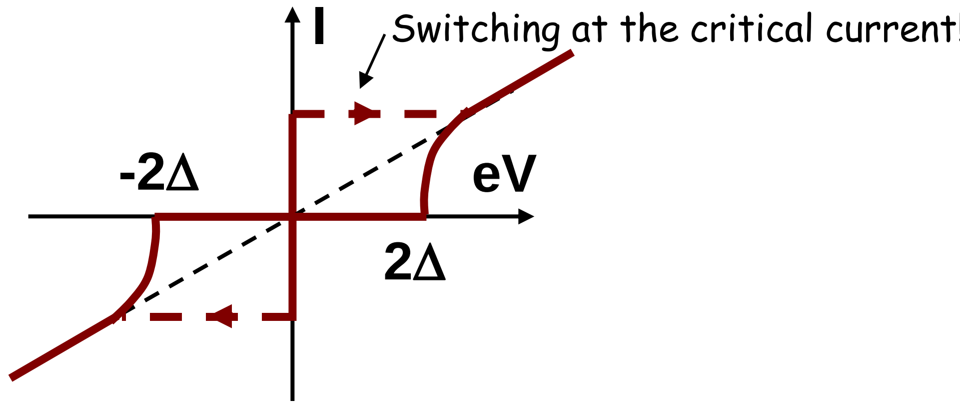
Applying a constant bias voltage: $I = I_0 \sin\left(\delta_0 + \frac{2eVt}{\hbar}\right)$ An AC current with $\omega = 2eV/\hbar$ is flowing. The DC current averages to zero.

At zero bias voltage a maximal suppercurrent of I_0 can flow between the two sides!

Voltage biased junction:



Current biased junction:



Let us superimpose an AC (microwave) voltage on a DC voltage!

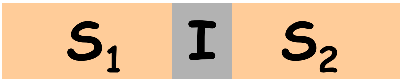
$$V(t) = V_0 + V_1 \cos \omega t \;\Rightarrow\; \delta(t) = \delta_0 + \frac{2eV_0}{\hbar} t + \frac{2e}{\hbar}\frac{V_1}{\omega}\sin \omega t$$

$$I = I_0 \sin\left(\delta_0 + \frac{2eV_0}{\hbar} t + \frac{2e}{\hbar}\frac{V_1}{\omega}\sin \omega t\right) \approx I = I_0 \sin\left(\delta_0 + \frac{2eV_0}{\hbar} t\right) + I_0 \frac{2e}{\hbar}\frac{V_1}{\omega}\sin(\omega t)\cos\left(\delta_0 + \frac{2eV_0}{\hbar} t\right) =$$

$$= I_0 \sin\left(\delta_0 + \frac{2eV_0}{\hbar} t\right) + I_0 \frac{2e}{\hbar}\frac{V_1}{2\omega}\left(\sin\left(\omega t - \delta_0 - \frac{2eV_0}{\hbar} t_0\right) + \sin\left(\omega t + \delta_0 + \frac{2eV_0}{\hbar} t_0\right)\right)$$

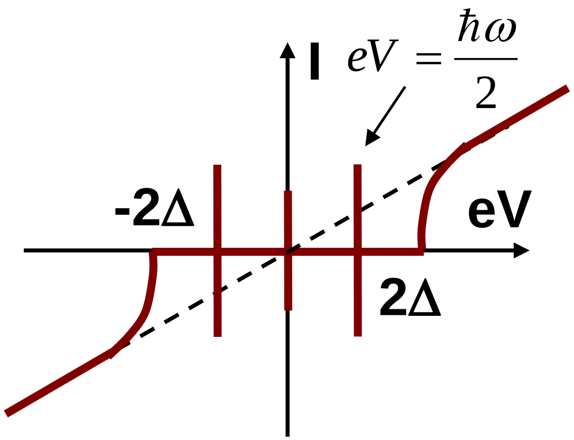
At $V_0 = \pm\hbar\omega/2e$ a DC current will flow!

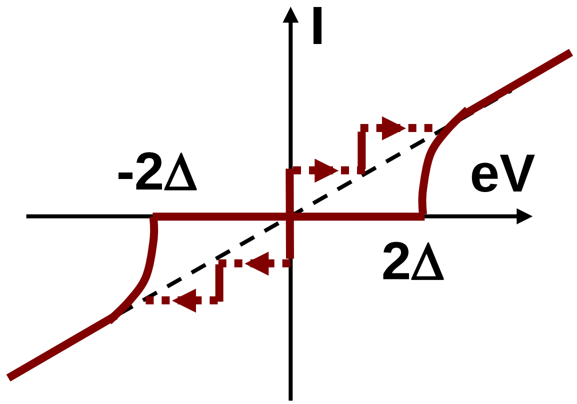
If we would expand in higher order, we would get that DC current can flow at: $V_0 = \pm\frac{\hbar\omega \cdot n}{2e}$, where n is an integer



Shapiro steps, SQUID

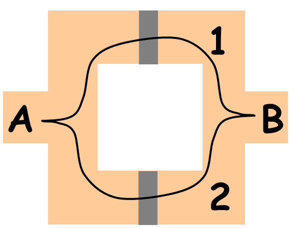
Shapiro resonances at voltage and current bias:





Superconducting quantum interferometer device (SQUID):

Two Josephson junctions in parallel in a "loop" geometry. The loop encloses a magnetic flux of Φ



The superconductor have a well-defined phase at very position. -> The pase difference between A and B is constant for all trajectories.

$$(\phi_B - \phi_A)_1 = \delta_1 + \frac{2e}{\hbar}\int_1 \mathbf{A}\mathbf{ds} \quad = \quad (\phi_B - \phi_A)_2 = \delta_2 + \frac{2e}{\hbar}\int_2 \mathbf{A}\mathbf{ds}$$

$$\Rightarrow \delta_2 - \delta_1 = \frac{2e}{\hbar}\oint \mathbf{A}\mathbf{ds} = \frac{2e}{\hbar}\Phi = 2\cdot 2\pi\frac{\Phi}{\Phi_0}$$

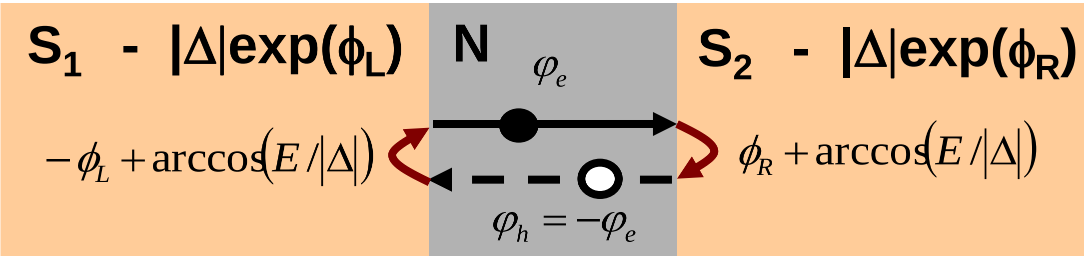
Let us take: $\delta_1 = \delta_0 + \frac{e}{\hbar}\Phi, \ \ \delta_2 = \delta_0 - \frac{e}{\hbar}\Phi$

$$I = I_1 + I_2 = I_0\left[\sin(\delta_0 + 2\Phi/\hbar) + \sin(\delta_0 - 2\Phi/\hbar)\right] = 2I_0 \sin\delta_0 \cos(e\Phi/\hbar)$$

The maximal value of the critical current is tuned by the magnetic flux: $I_{max} = 2I_0\left|\cos(e\Phi/\hbar)\right|$

Andreev bound states in a short SNS junction with phase bias

Single channel contact with perfect transmission:



The electron-hole par can bounce back and forth in the normal region.

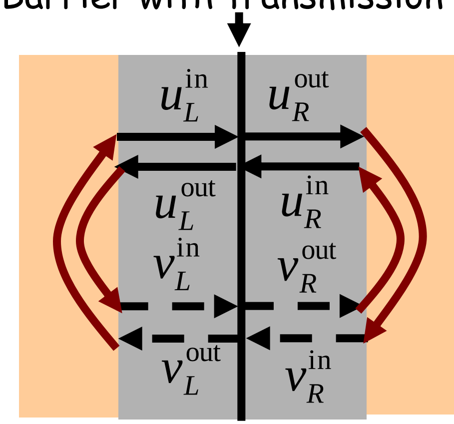
If the aquired phase is n2π (constructive interference) a bound state is formed.

The condition for the bound state: $\phi_R - \phi_L + 2\arccos(E/|\Delta|) = n2\pi \Rightarrow$ $\boxed{E_\pm = \pm|\Delta|\cos(\delta/2)}$

$\delta = \phi_R - \phi_L$

Single channel contact with arbitrary transmission:

Barrier with transmission τ



Normal scattering for the electrons:

$$\begin{pmatrix} u_L^{out} \\ u_R^{out} \end{pmatrix} = \overbrace{\begin{pmatrix} -ir & t \\ t & -ir \end{pmatrix}}^{\underline{\underline{S}}} \begin{pmatrix} u_L^{in} \\ u_R^{in} \end{pmatrix}, \quad t, r \text{ are real}, \quad t^2 = \tau,\ r^2 = 1-\tau$$

Normal scattering for holes:

$$\begin{pmatrix} v_L^{out} \\ v_R^{out} \end{pmatrix} = \underline{\underline{S}}^* \begin{pmatrix} u_L^{in} \\ u_R^{in} \end{pmatrix}$$

Andreev reflection at the left:

$$\begin{pmatrix} v_L^{in} \\ u_L^{in} \end{pmatrix} = \begin{pmatrix} a(E,\phi_L) & 0 \\ 0 & a(E,-\phi_L) \end{pmatrix} \begin{pmatrix} u_L^{out} \\ v_L^{out} \end{pmatrix},$$

Andreev reflection at the right:

$$\begin{pmatrix} v_R^{in} \\ u_R^{in} \end{pmatrix} = \begin{pmatrix} a(E,\phi_R) & 0 \\ 0 & a(E,-\phi_R) \end{pmatrix} \begin{pmatrix} u_R^{out} \\ v_R^{out} \end{pmatrix}$$

These can be combined to: $\begin{pmatrix} u_L^{in} \\ u_R^{in} \end{pmatrix} = \underline{\underline{M}} \begin{pmatrix} u_L^{in} \\ u_R^{in} \end{pmatrix}$, $\underline{\underline{M}} = \begin{pmatrix} a(E,-\phi_L) & 0 \\ 0 & a(E,-\phi_R) \end{pmatrix} \underline{\underline{S}}^* \begin{pmatrix} a(E,\phi_L) & 0 \\ 0 & a(E,\phi_R) \end{pmatrix} \underline{\underline{S}}$

The condition to get nonzero solution: $\det(\underline{\underline{M}} - \underline{\underline{1}}) = 0 \Rightarrow$ $\boxed{E_\pm = \pm|\Delta|\sqrt{1-\tau\sin^2(\delta/2)}}$

Current - phase relation (V=0, phase biased junction)

Reminder: calculation of the current in a 1D conductor: $I = \frac{e}{L}\sum_{k,\sigma} v_{k,\sigma} f(\varepsilon_{k,\sigma})$

In an Andreev bound state a singly occupied energy state is considered, which carries 2e charge: $\Rightarrow \; I = \frac{2e}{L} v$

The velocity can be obtained as: $v = \partial E / \partial p$ As $E = f(p - 2eA) \Rightarrow \partial E / \partial p = -(\partial E / \partial A)/2e$

$$\Rightarrow \; I = -\frac{1}{L}\frac{\partial E}{\partial A}$$

However, A and the superconducting phase, ϕ are not invariant for a gauge transformation. The energy must be a function of a gauge invariant function of A and ϕ.

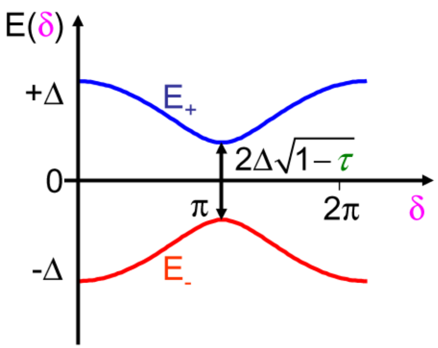
$$A' = A + \nabla\chi \;\Rightarrow\; \phi' = \phi + 2e\chi/\hbar \qquad \Rightarrow\; \nabla\phi - 2eA/\hbar$$

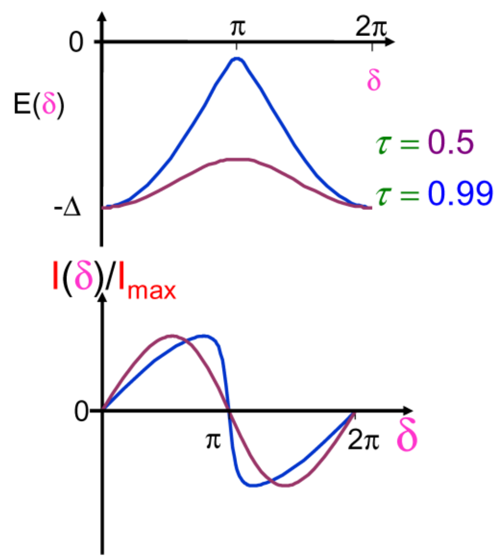
is the proper gauge invariant quantity

$$E = g(\nabla\phi - 2eA/\hbar) \;\Rightarrow\; \partial E/\partial A = -\frac{2e}{\hbar}\frac{\partial E}{\partial \underbrace{\nabla\phi}_{(\phi_R - \phi_L)/L}}$$

$$\boxed{\Rightarrow \; I(\delta) = \frac{2e}{\hbar}\frac{\partial E}{\partial \delta}}$$

$$\boxed{E_{\pm} = \pm|\Delta|\sqrt{1 - \tau \sin^2(\delta/2)}}$$





Current phase relation for the occupied (negative energy) Andreev bound state:

$$\boxed{I(\delta) = \frac{e|\Delta|}{2\hbar}\frac{\tau \sin\delta}{\sqrt{1 - \tau \sin^2(\delta/2)}}}$$

For a tunnel junction ($\tau \ll 1$):

$$\boxed{I(\delta) = \frac{e|\Delta|}{2\hbar}\tau \sin\delta}$$

We get back the Josephson relation!

Finite bias voltage

At finite DC bias voltage the phasedifference is linearly increasing by time: $\delta(t) = \delta_0 + \frac{2e}{\hbar} V \cdot t$

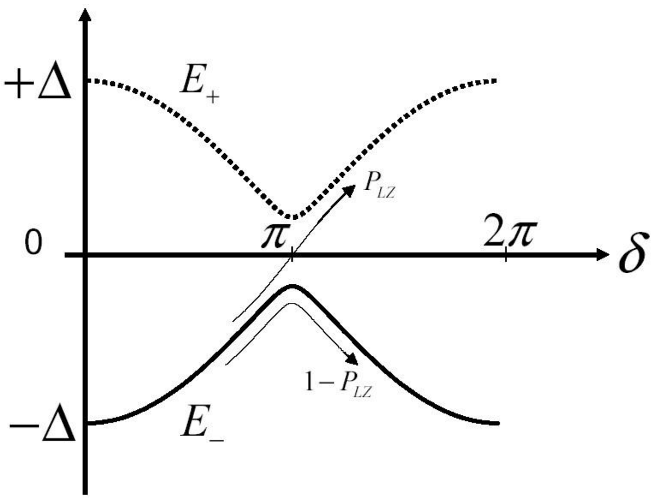
If we plug it to the current phase relationship of the occupied Anddreev bound statewe get an alternating current averagong out to zero.

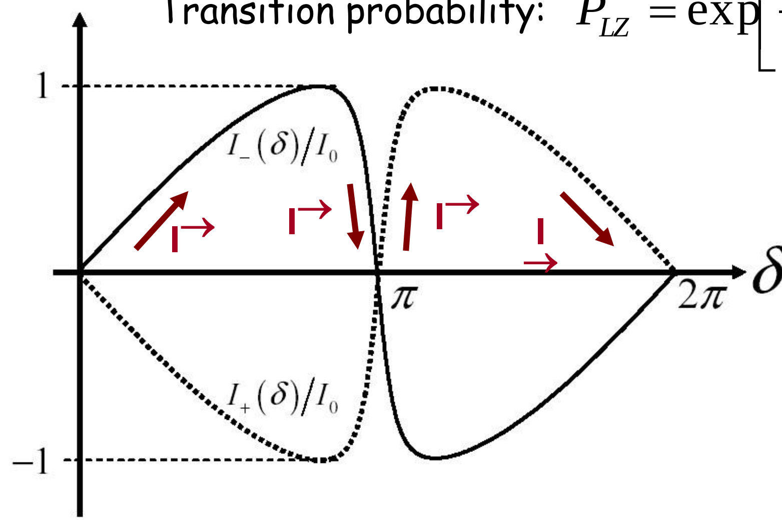
$$I^-(\delta(t)) = \frac{e|\Delta|}{2\hbar} \frac{\tau \sin\delta(t)}{\sqrt{1-\tau\sin^2(\delta(t)/2)}} \Rightarrow \left\langle I^-(\delta) \right\rangle_\delta = 0$$

I is an odd function of the phase

However, at finite bias there is a finite probability for transition to the excited Andreev bound state (**Landau - Zener transition**).

Transition probability: $P_{LZ} = \exp\left[-\pi \frac{\Delta(1-\tau)}{eV}\right]$



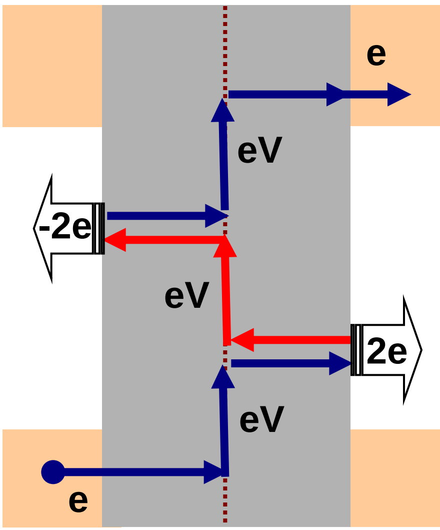


If the transition occurs the current is written as: $I^\rightarrow = \begin{cases} I^- \text{ if } 0 < \delta < \pi \\ I^+ \text{ if } \pi < \delta < 2\pi \end{cases}$

The total current is: $I(V,t) \approx (1 - P_{LZ}(V)) \cdot I^-(\delta(t)) + P_{LZ}(V) \cdot I^\rightarrow(\delta(t))$

The time averaged current is: $I_{DC} = \langle I(V,t) \rangle \approx P_{LZ}(V) \cdot \left\langle I^\rightarrow(\delta) \right\rangle_\delta = \frac{4}{\pi} \frac{e\Delta}{2\hbar} \exp\left[-\pi \frac{\Delta(1-\tau)}{eV}\right]$

A more complete descrition at finite bias : Multiple Andreev Reflections



At finite bias the electrons and holes gain or loose an energy of eV when they cross the barrier. We start a quasiparticle with $E<-\Delta$ from the left side. It gains an energy eV, but if $E+eV<\Delta$ it can only be Andreev reflected as a hole. Going backward the hole gains an energy of eV, and so on. Finally the energy will increase above $+\Delta$ and a quasiparticle can leave to the electrodes.

The amplitude of the incoming quasiparticle:

$$J(E)=\sqrt{1-\left|r_{A(E,0)}\right|^2}$$

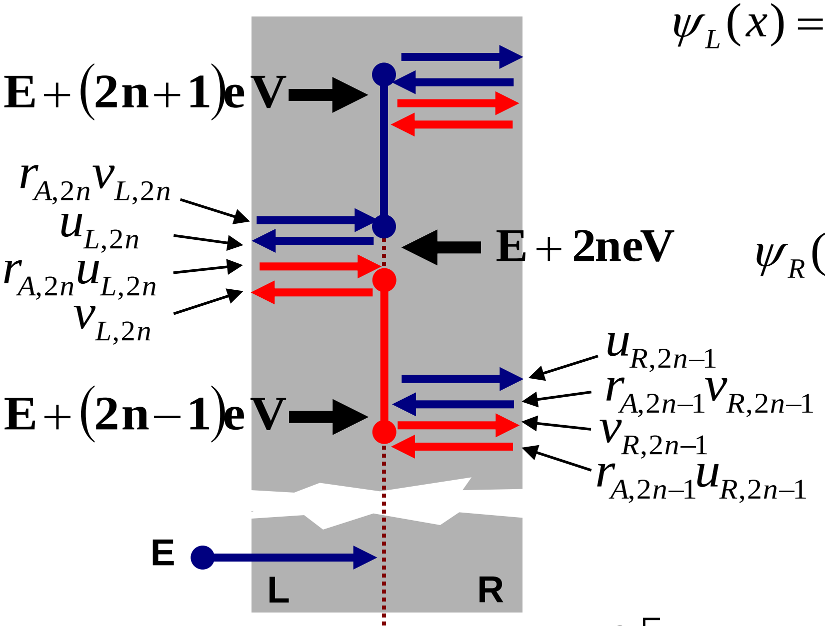
The Andreev reflection amplitude: (ϕ=0, it is included in the timedependence!)

$$r_{A,2n}=r_A(\varepsilon=E+2neV,\phi=0)$$

Wavefunctions at the left and right side of the barrier:

$$\psi_L(x)=\begin{pmatrix}\sum_n\left(\left(r_{A,2n}v_{L,2n}+J(E)\delta_{n,0}\right)e^{ik_ex}+u_{L,2n}e^{-ik_ex}\right)e^{-i(E+2neV)t/\hbar}\\ \sum_n\left(r_{A,2n}u_{L,2n}e^{-ik_hx}+v_{L,2n}e^{ik_hx}\right)e^{-i(E+2neV)t/\hbar}\end{pmatrix}$$

$$\psi_R(x)=\begin{pmatrix}\sum_n\left(r_{A,2n+1}v_{R,2n+1}e^{-ik_ex}+u_{R,2n+1}e^{ik_ex}\right)e^{-i(E+(2n+1)eV)t/\hbar}\\ \sum_n\left(r_{A,2n+1}u_{R,2n+1}e^{ik_hx}+v_{R,2n+1}e^{-ik_hx}\right)e^{-i(E+(2n+1)eV)t/\hbar}\end{pmatrix}$$



Scattering on the barrier:

$$\begin{pmatrix}u_{L,2n}\\ u_{R,2n+1}\end{pmatrix}=\underline{\underline{S}}\begin{pmatrix}r_{A,2n}v_{L,2n}\\ r_{A,2n+1}v_{R,2n+1}\end{pmatrix}\quad\begin{pmatrix}v_{L,2n}\\ v_{R,2n-1}\end{pmatrix}=\underline{\underline{S}}^*\begin{pmatrix}r_{A,2n}u_{L,2n}\\ r_{A,2n-1}u_{R,2n-1}\end{pmatrix}$$

The DC current:

$$I_{DC}(V)=\frac{2e^2}{h}\left[V-\frac{1}{e}\int dE\left(J(E)r_{A,0}\left(v_{L,0}^*+v_{L,0}\right)+\sum_n\left(1+\left|r_{A,2n}\right|^2\right)\left(\left|v_{L,2n}\right|^2-\left|u_{L,2n}\right|^2\right)\right)\right]$$

Multiple Andreev Reflections - qualitative picture

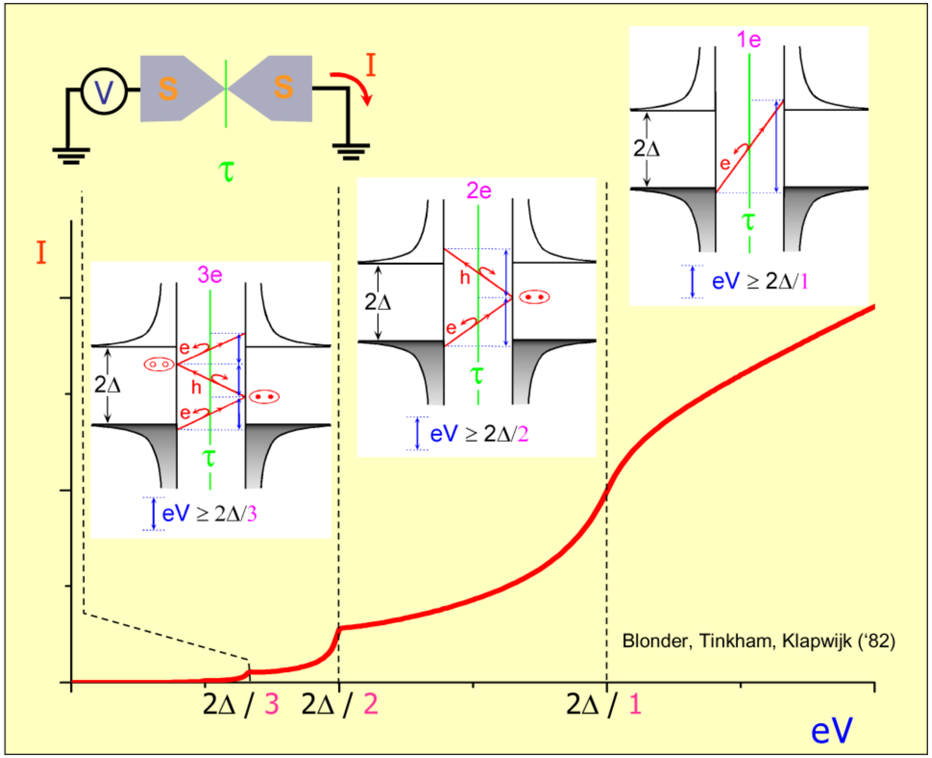
•If $eV > 2\Delta$ the quasiparticle started from the left side gains enough energy to reach an empty qausiparticle state on the right side. In this process a single electron charge is transmitted with probability τ.

•If $eV > \Delta$ single qausiparticle transmission is prohibited, but with a single Andreev reflection the charge transfer is already possible. In this process 2e charges are transmitted with probability of τ^2 (the carriers cross twice the barrier)

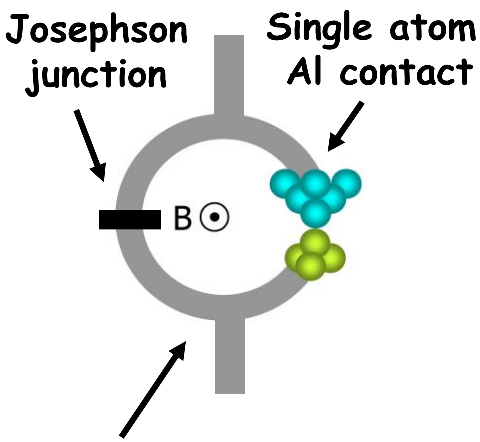
•At $eV > 2\Delta/n$ an n^{th} process with a charge transfer of ne and a probability τ^n becomes available

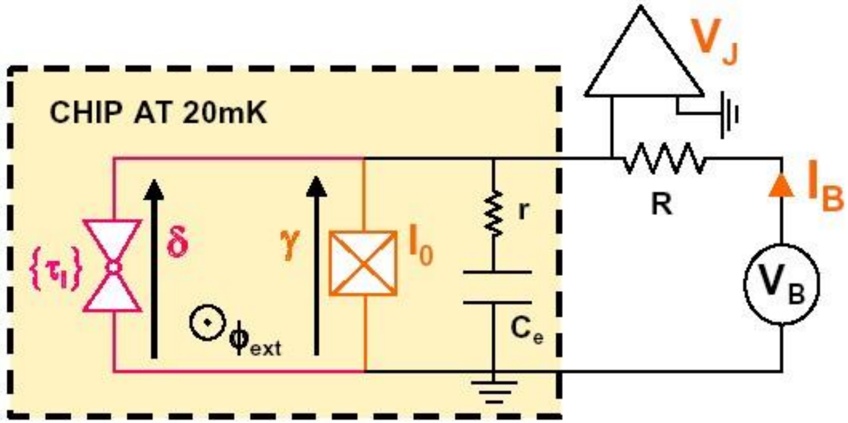
•Accordingly at the I-V curve shows singularities at $eV = 2\Delta/n$.

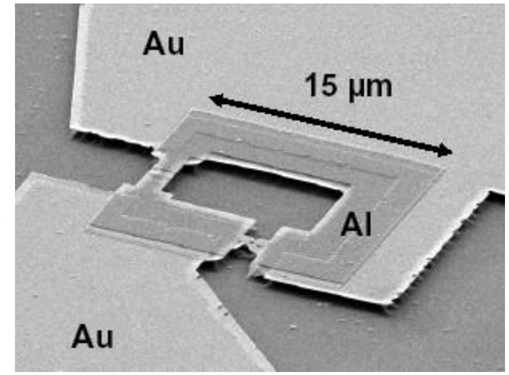
•If we have a junction with a few conductance channels all the transmission probabilities can be determined by placing the junction between superconducting electrodes and fitting the "subgap" structures in the I-V curve.



Measurement of the current-phase relation



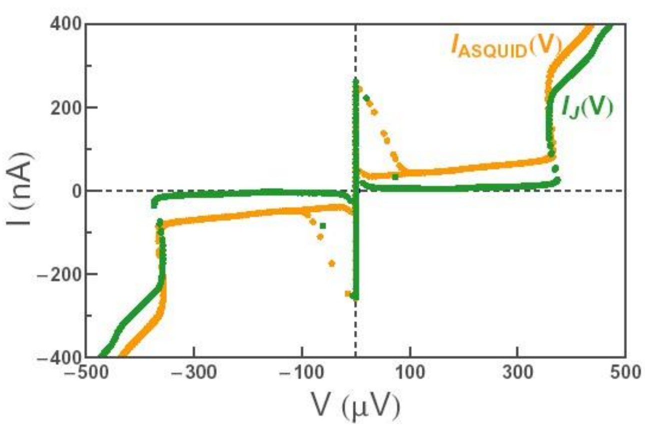


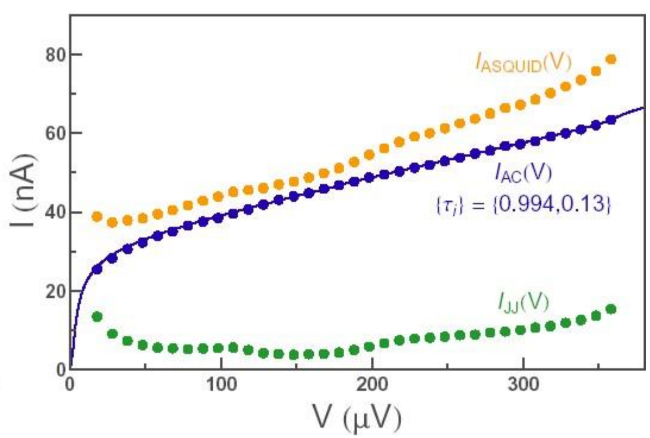


The device is an "atomic SQUID" (ASQUID), i.e. an SC atomic contact (AC) and a Josephson junction (JJ) in parallel. The critical current is much larger for the JJ than for the AC.

To know the transmission probabilities of the AC the subgap structures must be measured in the I-V curve. This is done with a voltage biased measurement. In principle the DC current of the JJ should be zero at finite Dc bias, thus the I-V curve would come purely from the AC. In reality the JJ also has structures in the I-V curve due to interferences with the environment. The I-V curve of the JJ can be separately measured by completely breaking the junction.

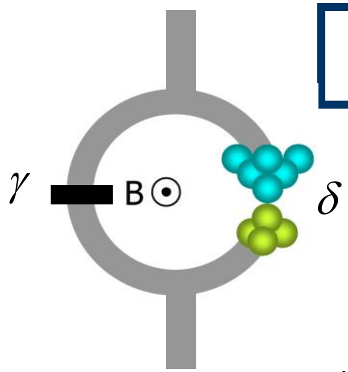
$$I_{AC}(V) = I_{ASQUID}(V) - I_{JJ}(V)$$





After subtracting the I-V curve of the JJ from that of the ASQUID the transmission probabilities of the AC can be determined.

Measurement of the current-phase relation



The phase difference on the atomic contact is: $\delta = \gamma + \underbrace{\Phi / \Phi_0}_{\varphi}$

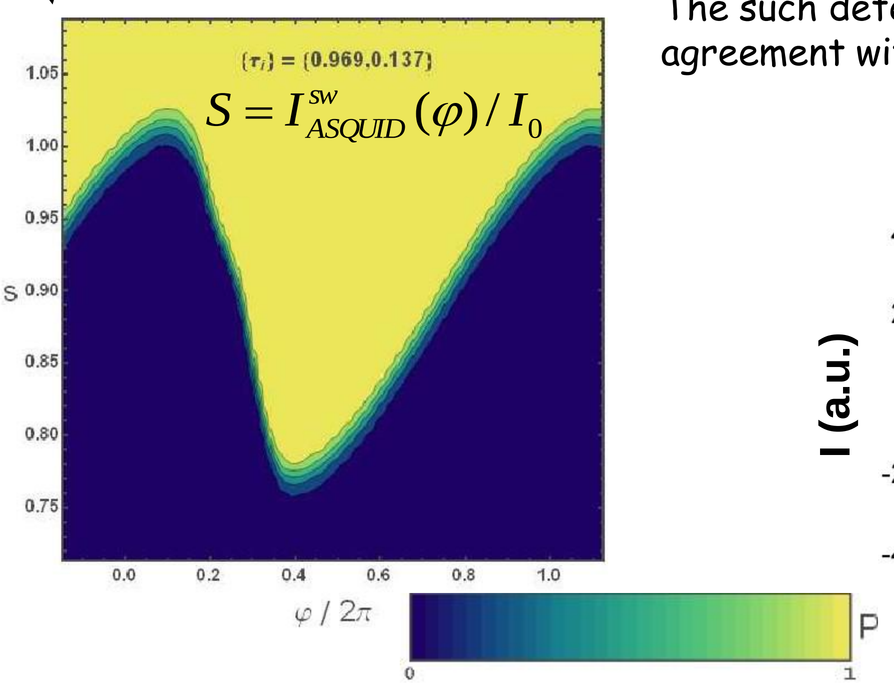
At zero temperature the JJ switches out of its zero bias state at $\gamma=\pi/2$ (at a critical current I_0), therefore the critical current of the ASQUID is: $I^0_{ASQUID}(\varphi) = I_0 + I_{AC}(\varphi + \pi/2)$

I.e. the critical current of the JJ is modulated by the current phase relation of the AC.

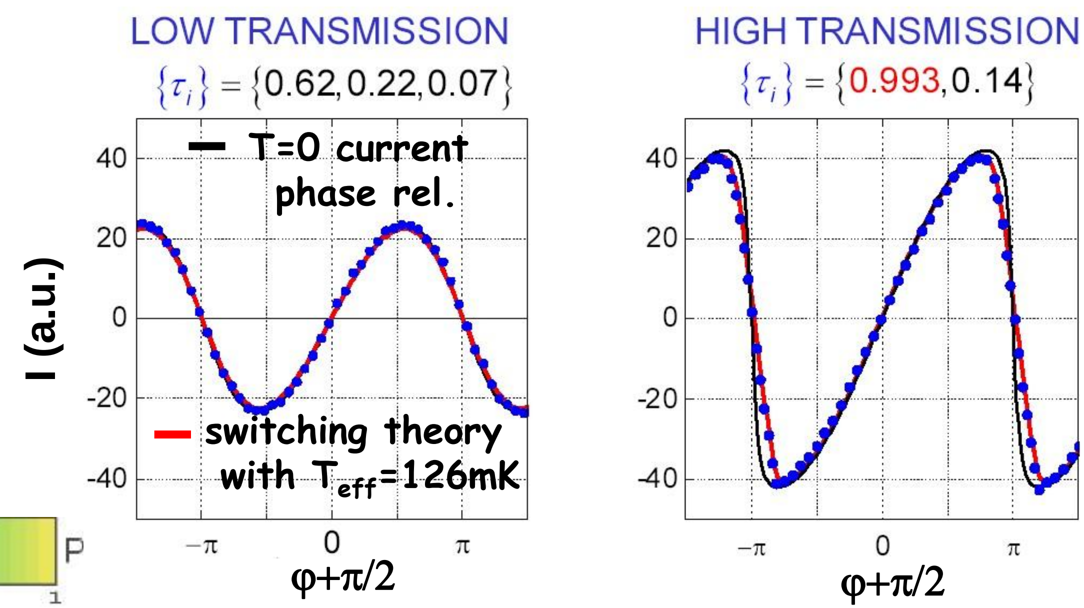
I.e. the critical current of the JJ is modulated by the current phase relation of the AC.
In experiment we measure the mean switching current instead of the critical current, which is a thermally activated stochastic variable. Short current pulses are applied on the sample and the distribution of the switching current is measured.

$$I^{sw}_{ASQUID}(\varphi) = I^{sw}_{JJ} + I_{AC}(\varphi + \pi/2)$$

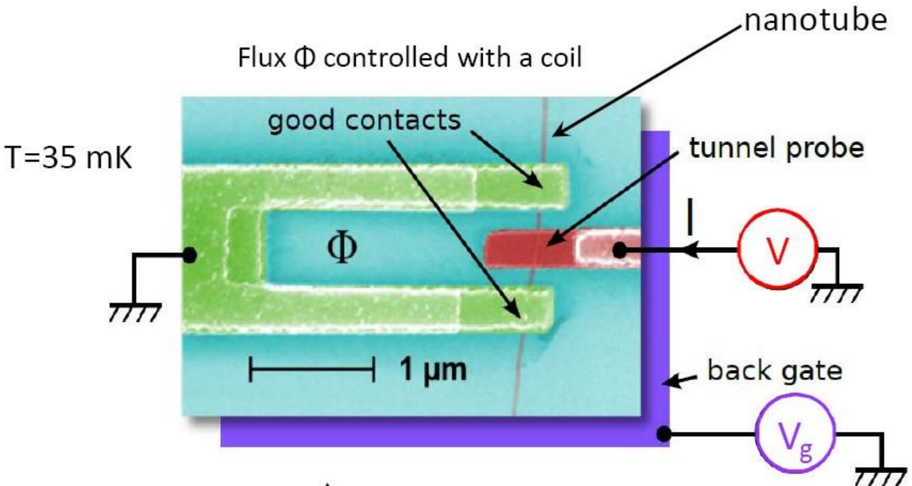
The average value of the JJ switching current can be independently measured with on aopen AC.



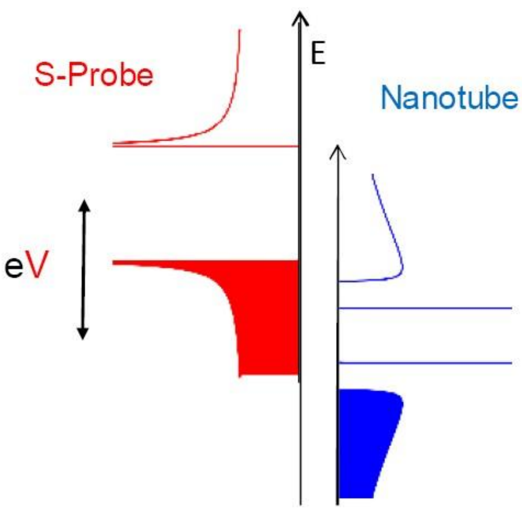
The such determined current-phase relation shows remarkable agreement with the theory:



Direct spectroscopic measurement of Andreev bound states

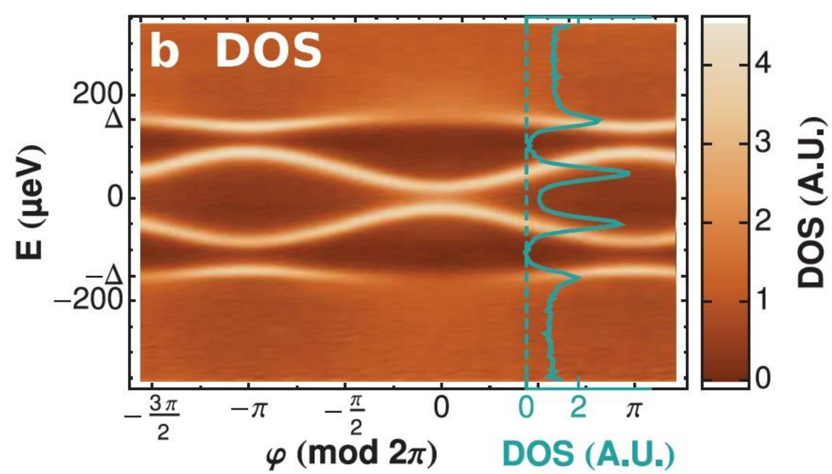


A carbon nanotube is placed in a superconducting loop as a Josephson junction. At the middle of the nanotube a superconducting tunnel probe is placed. The SC loop is grounded, and the voltage on the tunnel probe can be varied. The I-V curve builds upt form the Fermi functions, the known known density of states of the tunnel probe and the unknown DOS of the nanotube.



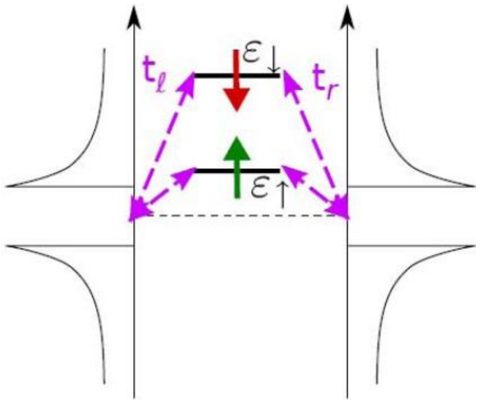
$$I(V) \propto \int \left(f_P(\varepsilon - eV) - f_{NT}(\varepsilon) \right) \rho_{NT}(\varepsilon)\, \rho_P(\varepsilon - eV)\, d\varepsilon$$

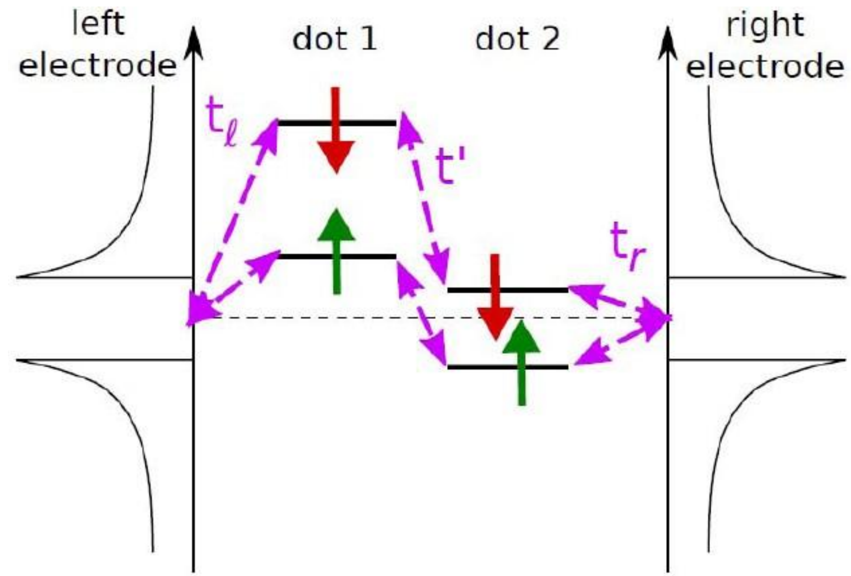
In the density of states four discrete states can be observed showing a periodic modulation with the phase difference (~flux) -> a two pairs of Andreev bound states!

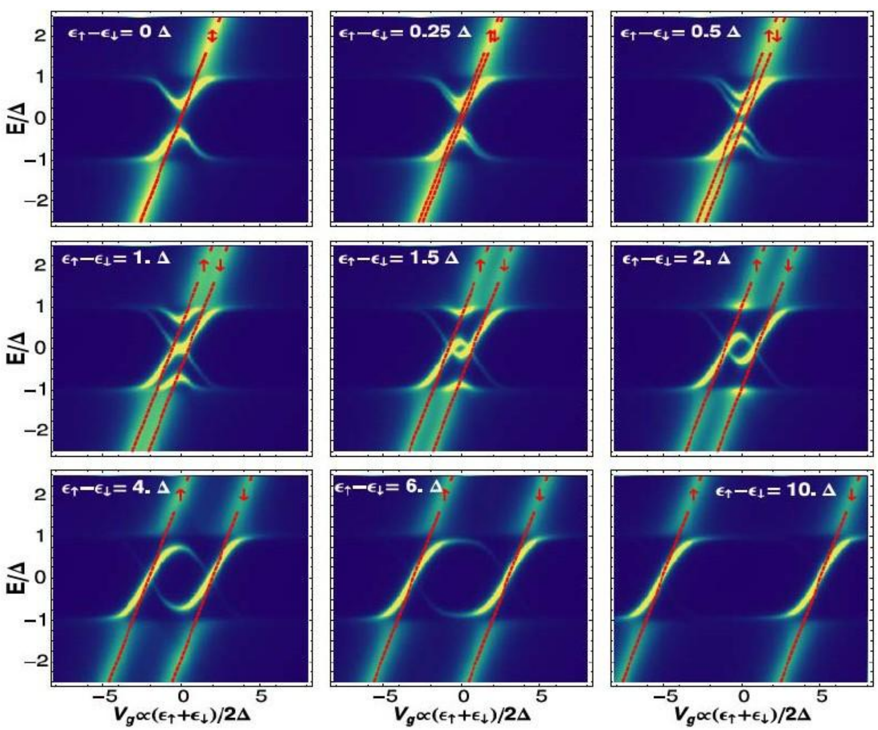


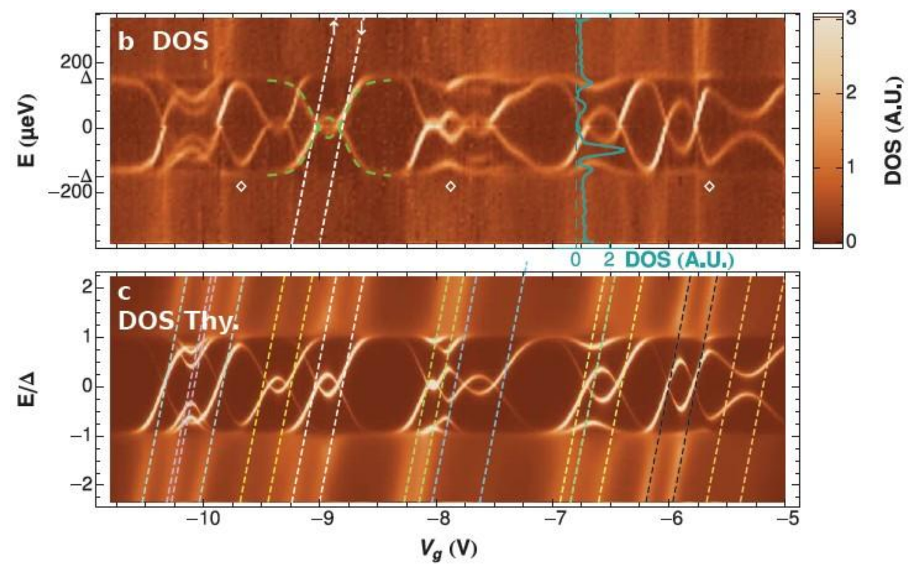
Direct spectroscopic measurement of Andreev bound states

Changing the gate voltage the energy levels in the nanotube can be tuned. Some features can be modelled as a single QDOT in the tube with two spin-split levels. Assuming double quantum dots all the observed features can be described -> new spectroscopic tool!



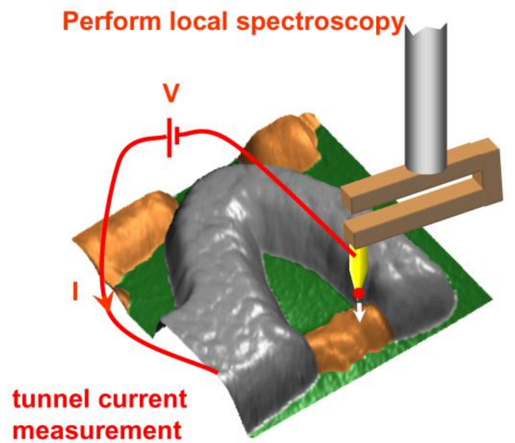




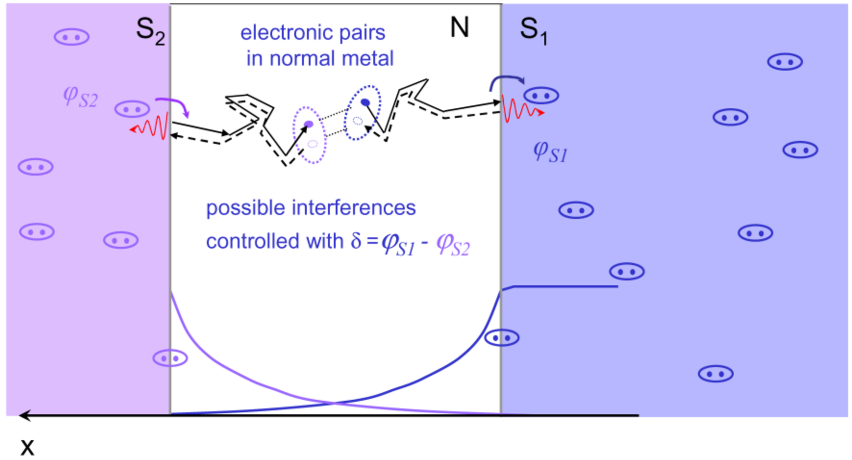


Proximity effects 4.: SNS junction

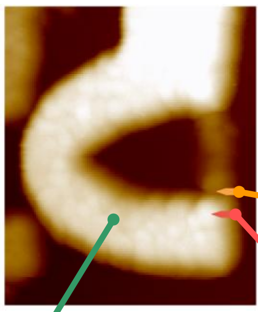
(H. le Sueur, P. Joyez, H. Pothier, C. Urbina , D. Esteve - Saclay)

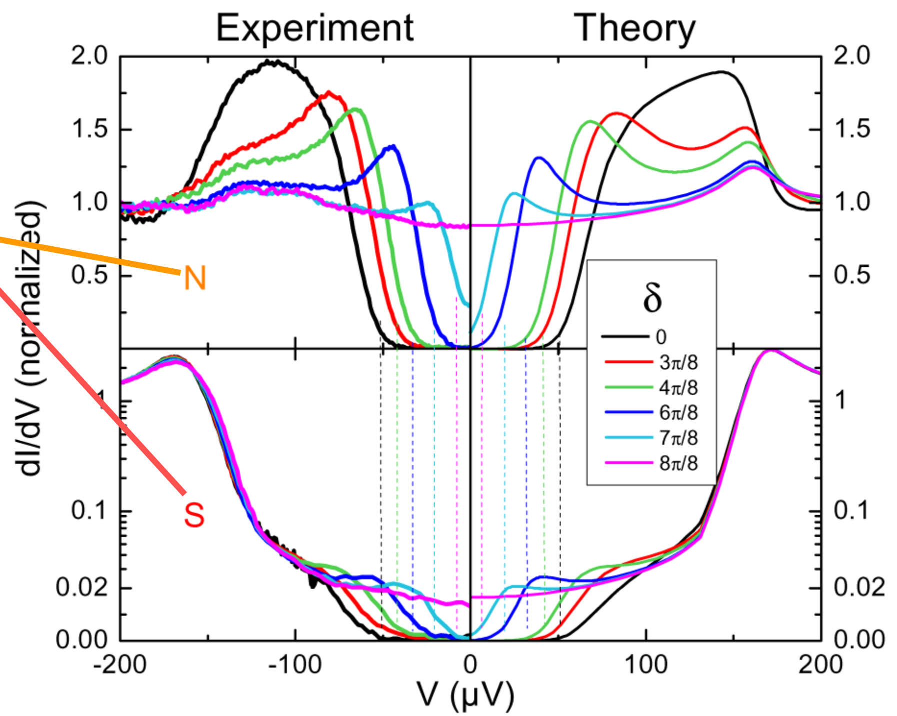


The variation of the DOS in a short wire between phase biased SC electrodes is measured with a combined STM + AFM setup.



In the normal region usually a proximity induced minigap is observed near zero bias. However, at a phase difference $\delta=\pi$ the superconducting correlations are suppressed due to interference effect.





The density of states in the bulk SC electrode: