Poor man's topological quantum gate based on the Su-Schrieffer-Heeger model

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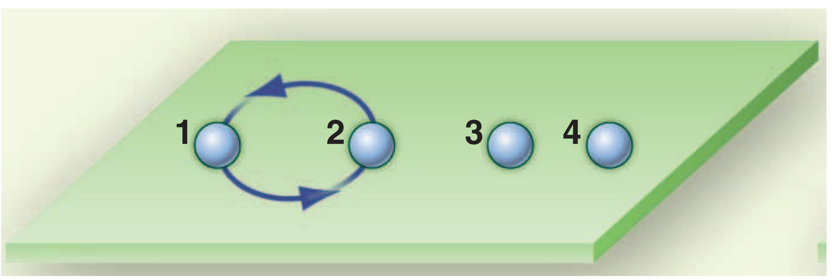
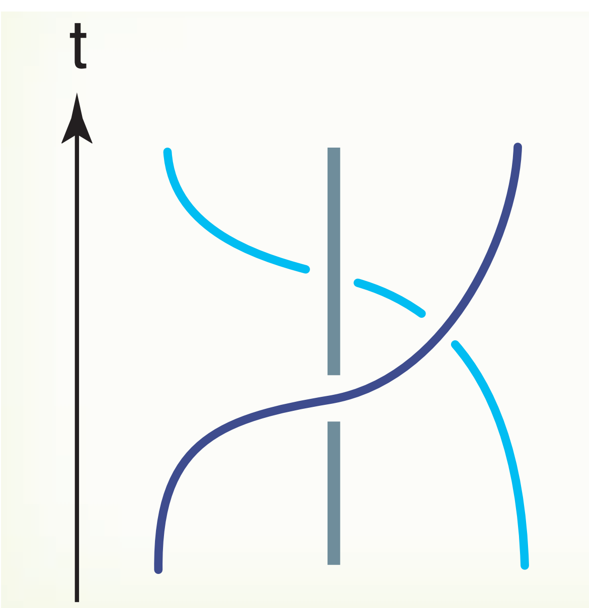
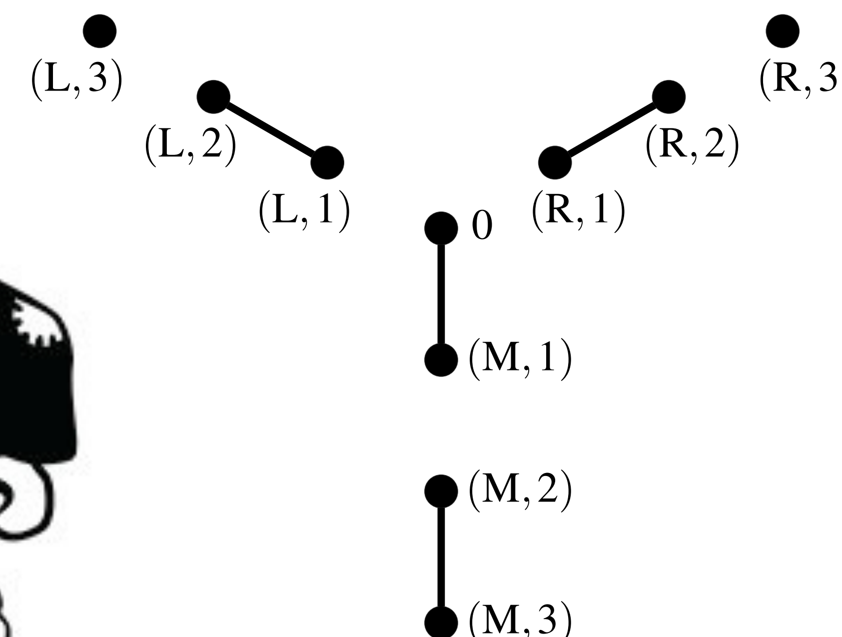
H = . . . complicated . . .





SSH Y-junction



with: Péter Boross (Wigner RCP), János Asbóth (Wigner RCP), Gábor Széchenyi (Eötvös), László Oroszlány (Eötvös)

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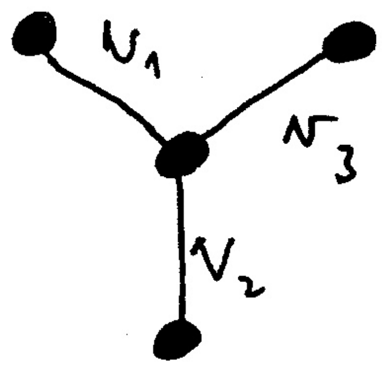
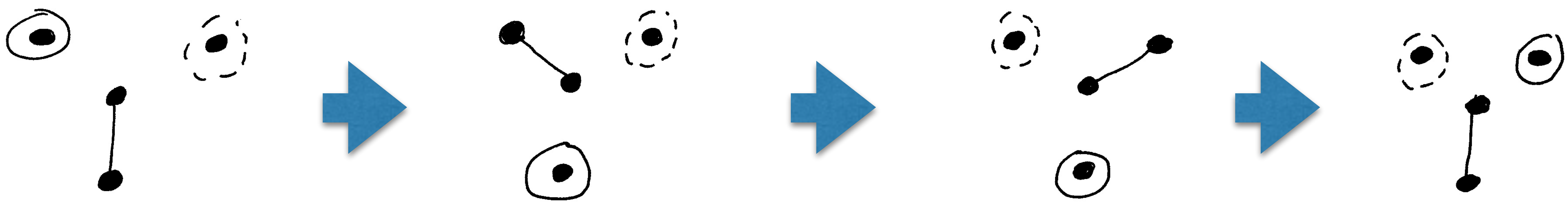
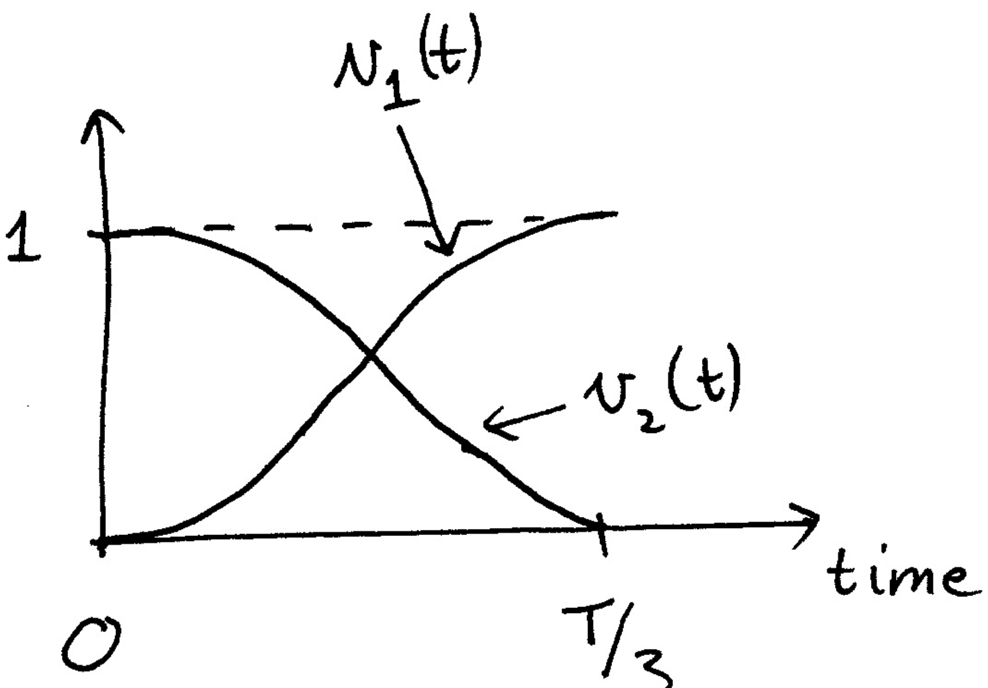
MOMENTUM OF INNOVATION

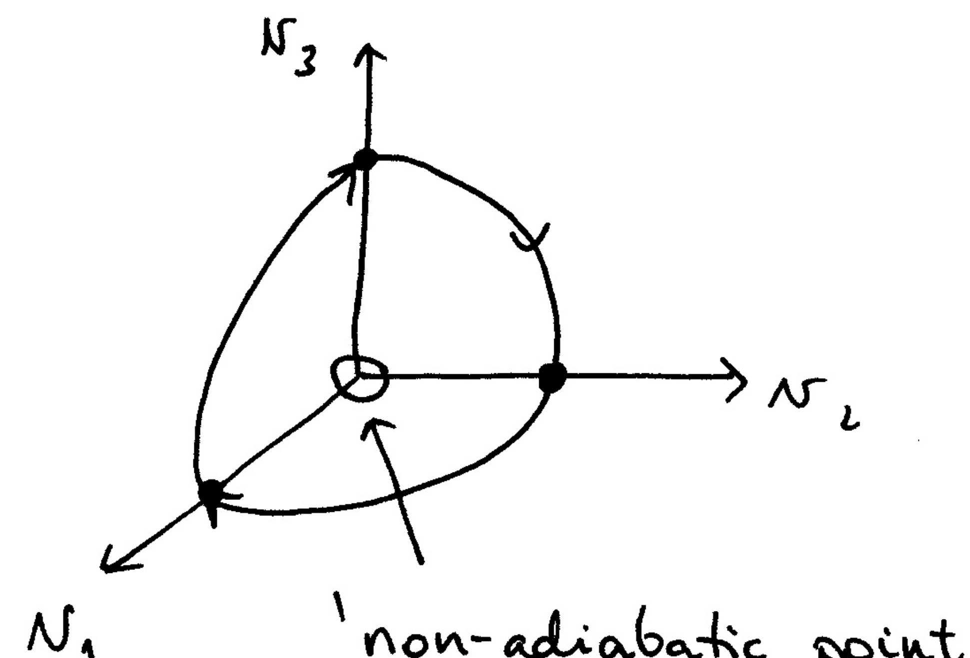
‘Topological` quantum gate: a simple example



$$H(t) = \begin{pmatrix} 0 & 0 & 0 & v_1(t) \\ 0 & 0 & 0 & v_2(t) \\ 0 & 0 & 0 & v_3(t) \\ v_1(t) & v_2(t) & v_3(t) & 0 \end{pmatrix}$$



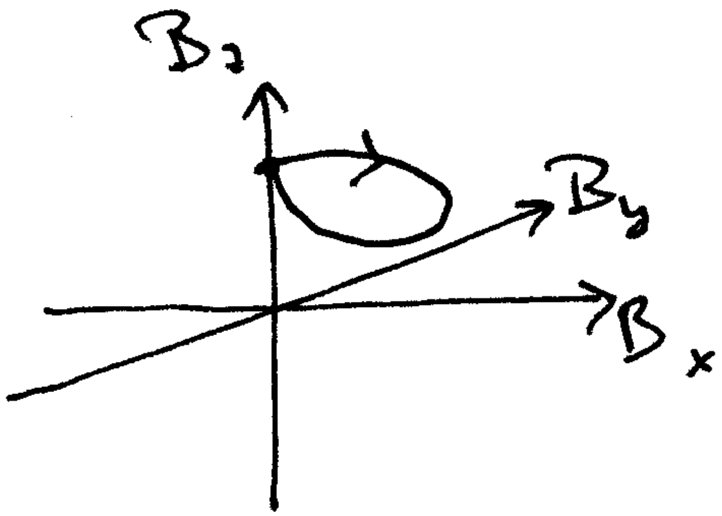




Cyclic adiabatic deformation of the Hamiltonian …

geometrical Berry phase

Berry 1984



$$H = -\underline{B}(t)\cdot\underline{S}$$

$$|\uparrow\rangle \longmapsto e^{i\varphi_p}|\uparrow\rangle$$

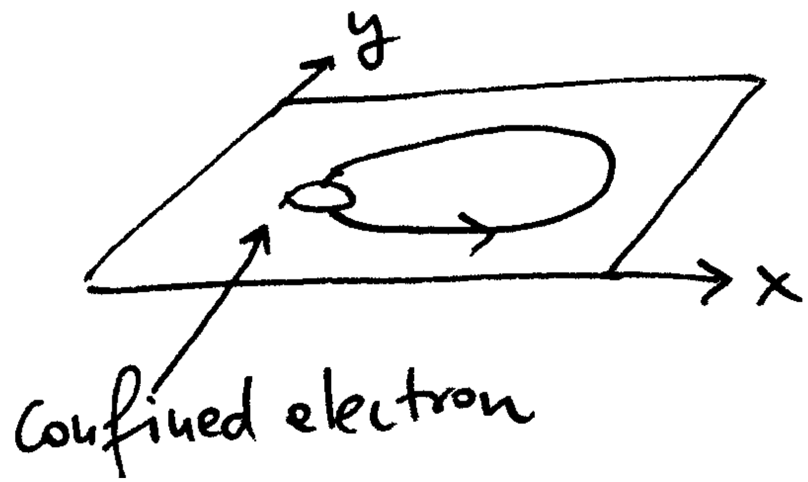
φ_p : path-dependent

robust against **timing** errors

geometrical non-Abelian Berry phase

Wilczek & Zee PRL 1984

San-Jose et al. PRB 2008



$$H = K + V(\underline{r},t) + H_{spin\text{-}orbit}$$

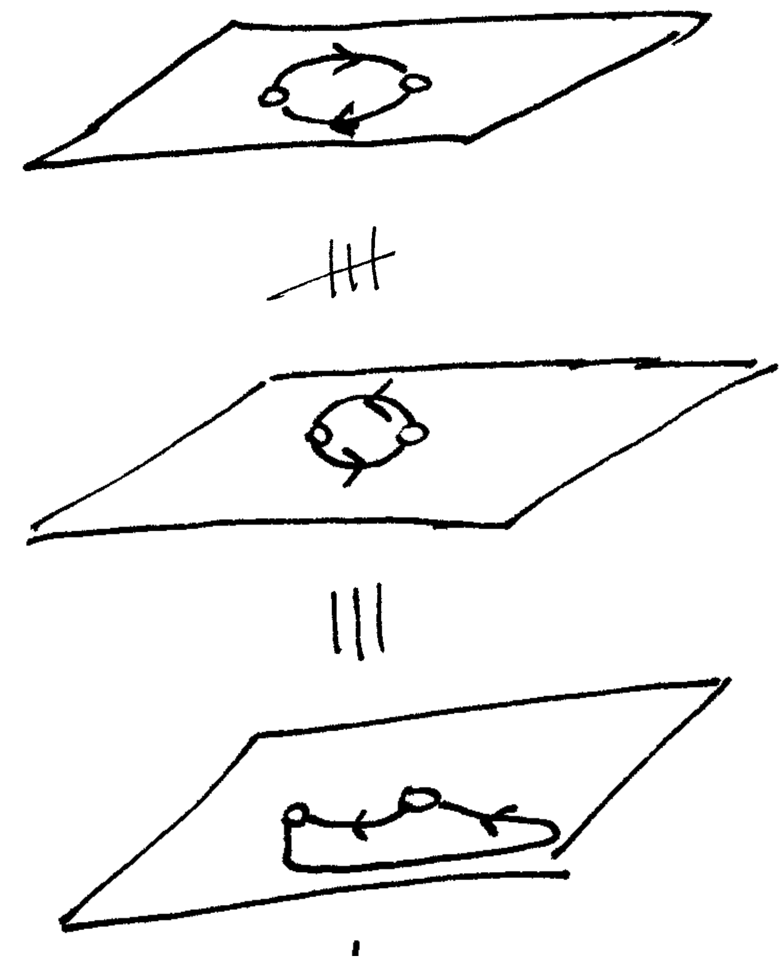
$$|\psi\rangle \longmapsto U_p|\psi\rangle$$

U_p: path-dependent 2x2 unitary

robust against **timing** errors

topological non-Abelian Berry phase

Nayak et al. RMP 2008



H = … complicated …

U_{top} : not path-dependent

robust against **timing** and **path** errors

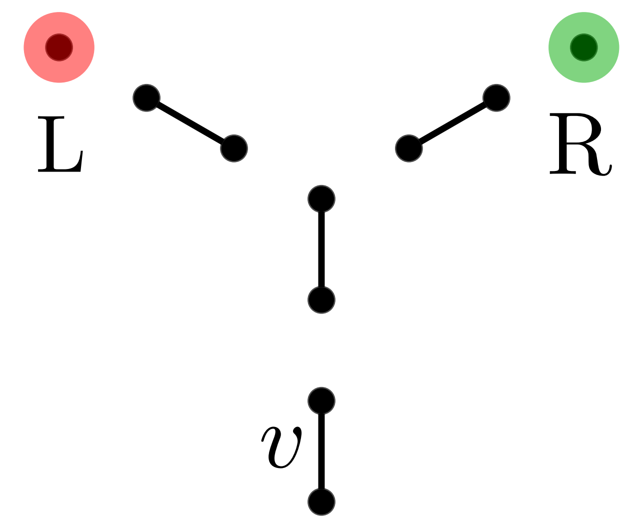
… might be useful for quantum computing

Topologically protected quantum gates: a simpler example

Boross et al., PRB 2019

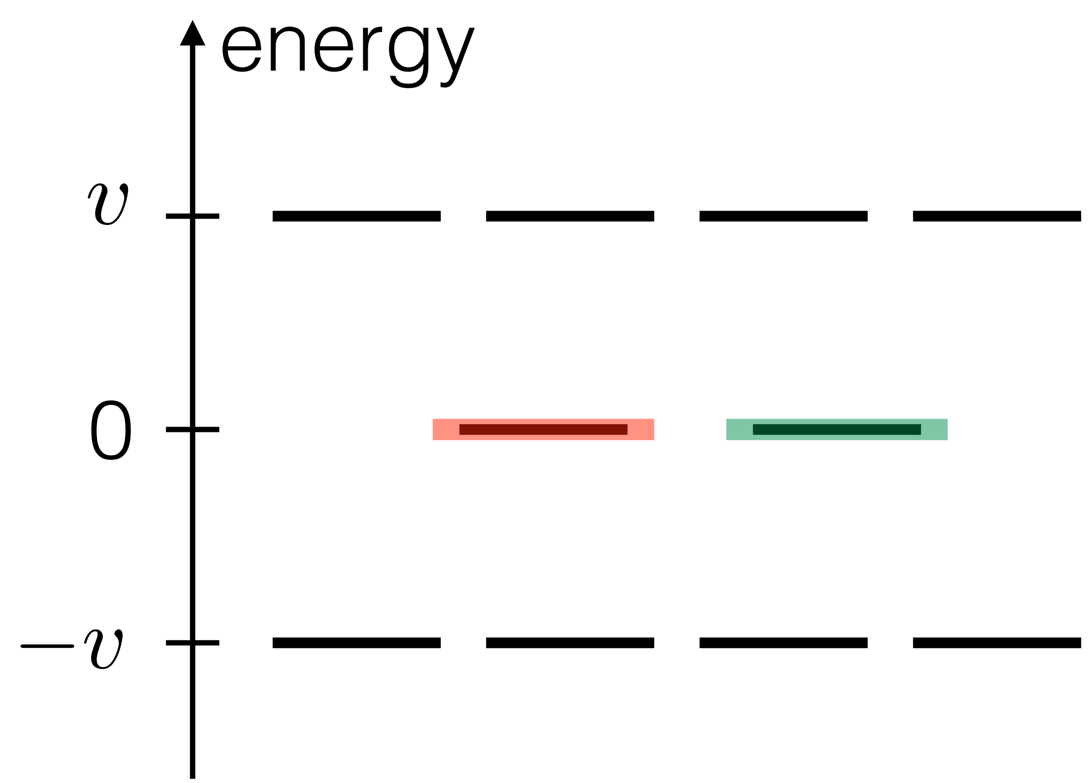
Klinovaja & Loss PRL 2013
Iadecola et al. PRL 2017

Setup: Single particle in a Y-junction of 1D Su-Schrieffer-Heeger chains



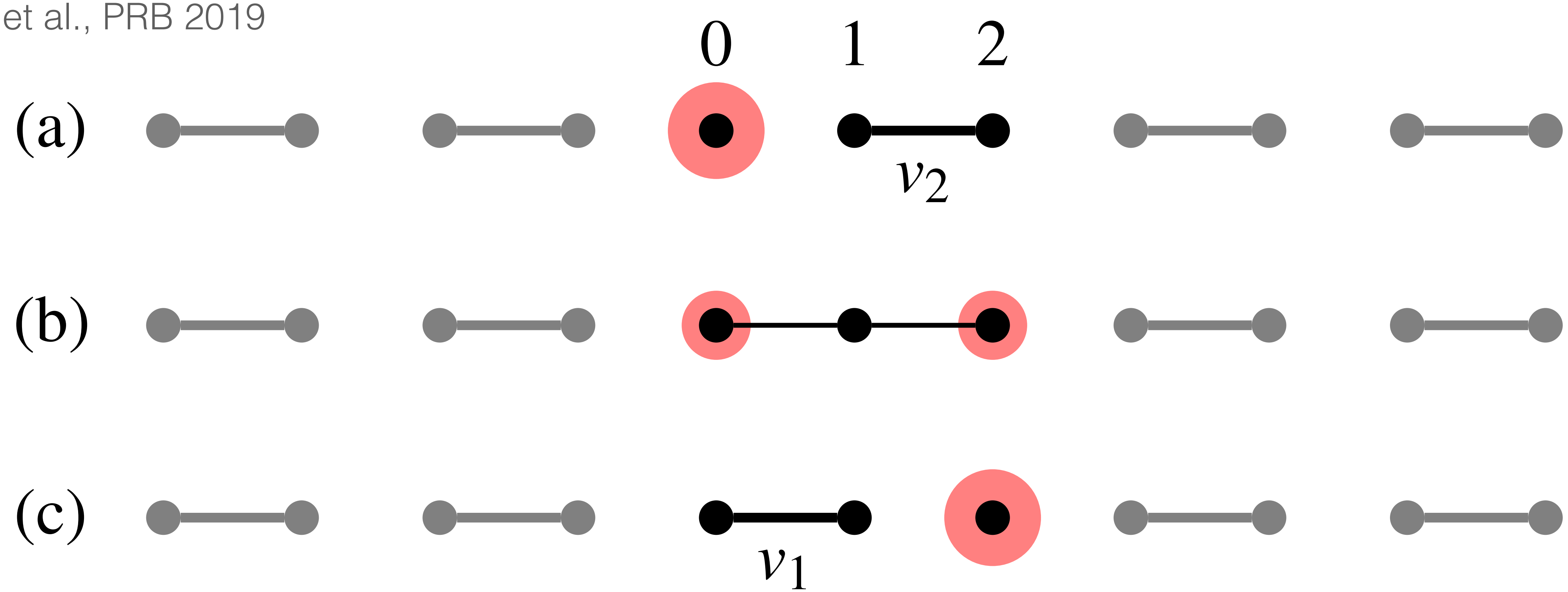
Chain length: $N_c = 3$
Dimension of Hilbert space: 10

1. defects (domain walls)
2. 2x degenerate **zero-energy subspace**
3. adiabatic 'braiding' or 'exchange' of defects: Y-gate in the zero-energy subspace $\mathrm{L} \Rightarrow \mathrm{R}$, $\mathrm{R} \Rightarrow -\mathrm{L}$
4. slower braiding $\Rightarrow$ better gate
5. longer chains $\Rightarrow$ better gate
6. does not require a perfect wire
7. does require **chiral** & **time-reversal symmetry**



Moving a defect in the Su-Schrieffer-Heeger chain

Boross et al., PRB 2019



1. defects (domain walls)
2. 2x degenerate **zero-energy subspace**
3. adiabatic 'braiding' or 'exchange' of defects:
   Y-gate in the zero-energy subspace
   L ⇒ R, R ⇒ −L
4. slower braiding ⇒ better gate
5. longer chains ⇒ better gate
6. does not require a perfect wire
7. does require **chiral** & **time-reversal symmetry**

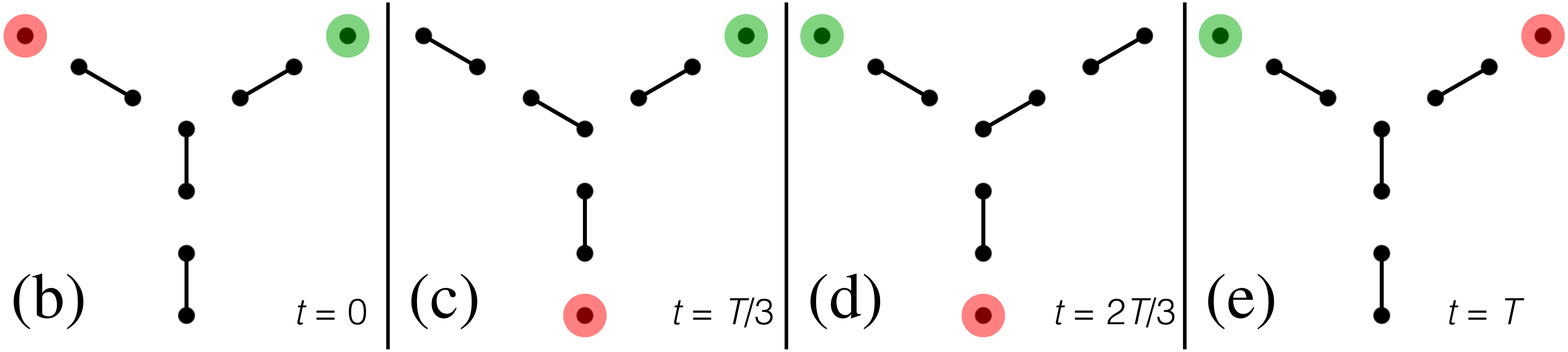
$$H = v_1 |1\rangle\langle 0| + v_2 |2\rangle\langle 1| + h.c.$$

S. Barišić, Phys. Rev. B **5**, 932 (1972).
W. P. Su, J. R. Schrieffer, and A. J. Heeger, Phys. Rev. Lett. **42**, 1698 (1979).
J. K. Asbóth, L. Oroszlány, and A. Pályi, *A Short Course on Topological Insulators* (Spinger, 2016).

Topologically protected quantum gates: a simpler example

Boross et al., PRB 2019

Setup: Single particle in a Y-junction of 1D Su-Schrieffer-Heeger chains



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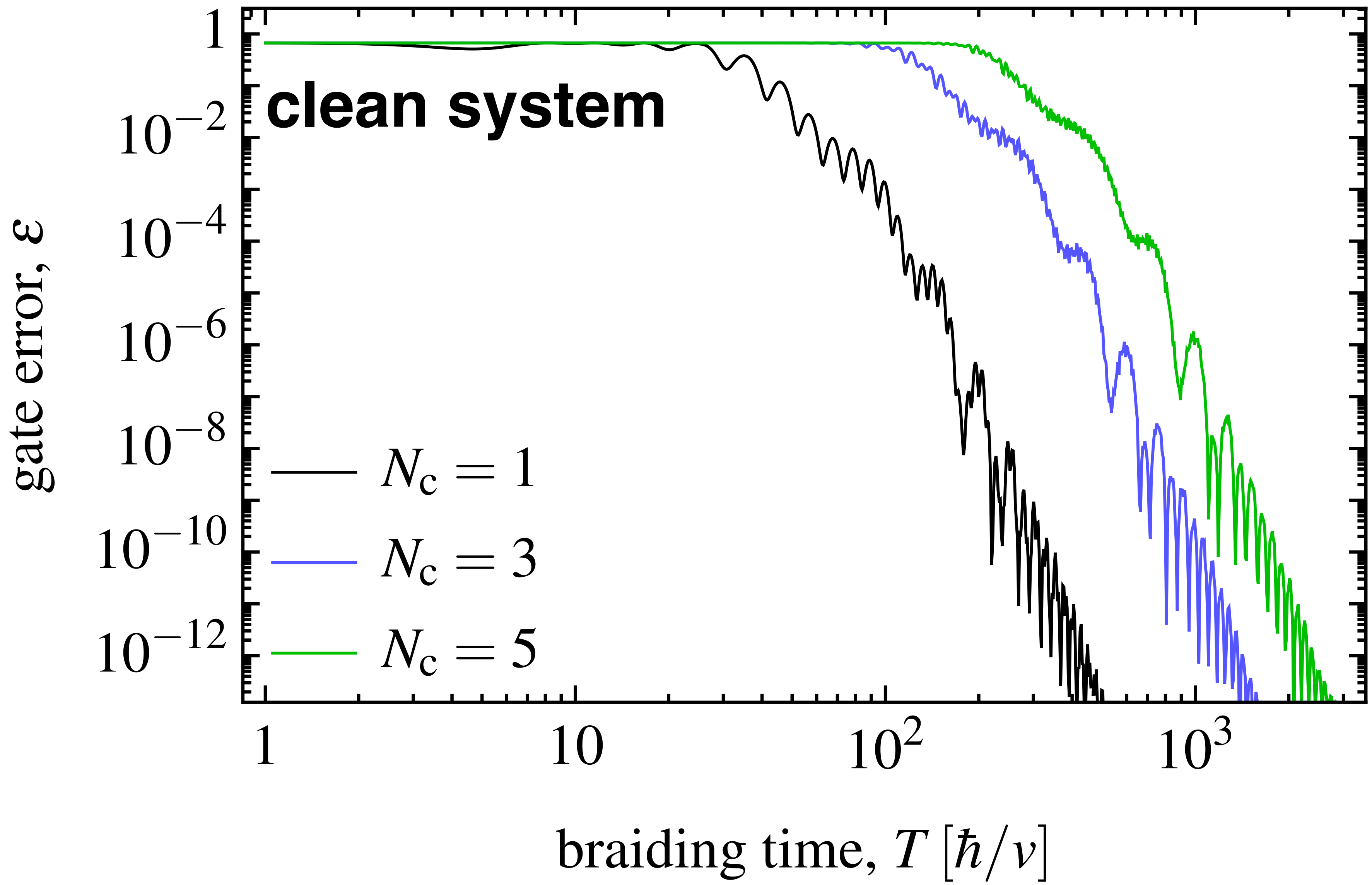
$$Y = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\sigma_y$$

A *Y* gate can be performed by exchange of 2 defects

Topologically protected quantum gates: a simpler example

Boross et al., PRB 2019



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How to characterize the error?

fidelity for a given initial state

$$f(\psi) = \left| \langle \psi | U_{\mathrm{id}}^{\dagger} U | \psi \rangle \right|^2$$

average fidelity

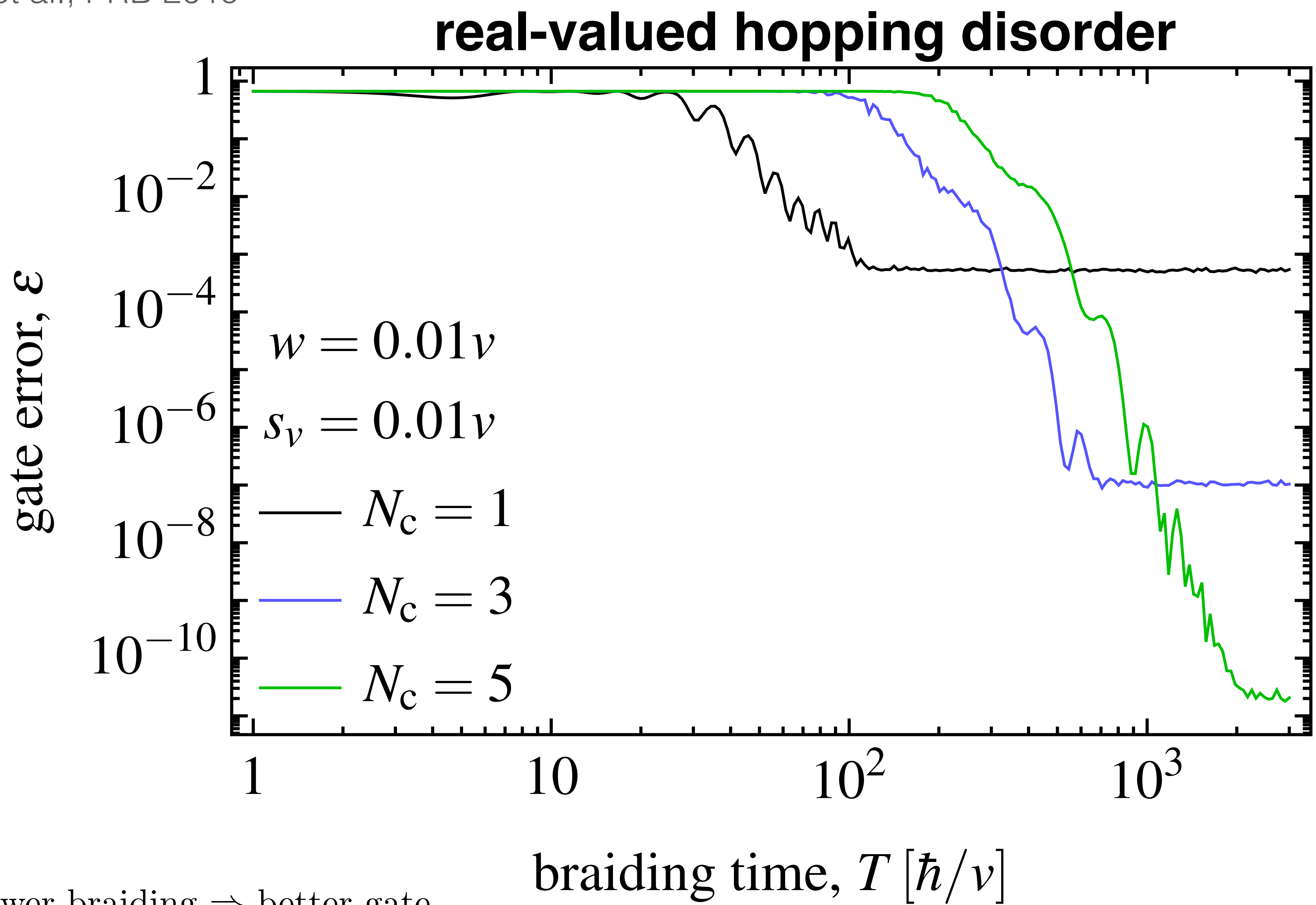
$$F = \int_{\text{Bloch-sphere surface}} d\psi \, f(\psi)$$

error

$$\varepsilon(T) = 1 - F(T).$$

Topologically protected quantum gates: a simpler example

Boross et al., PRB 2019



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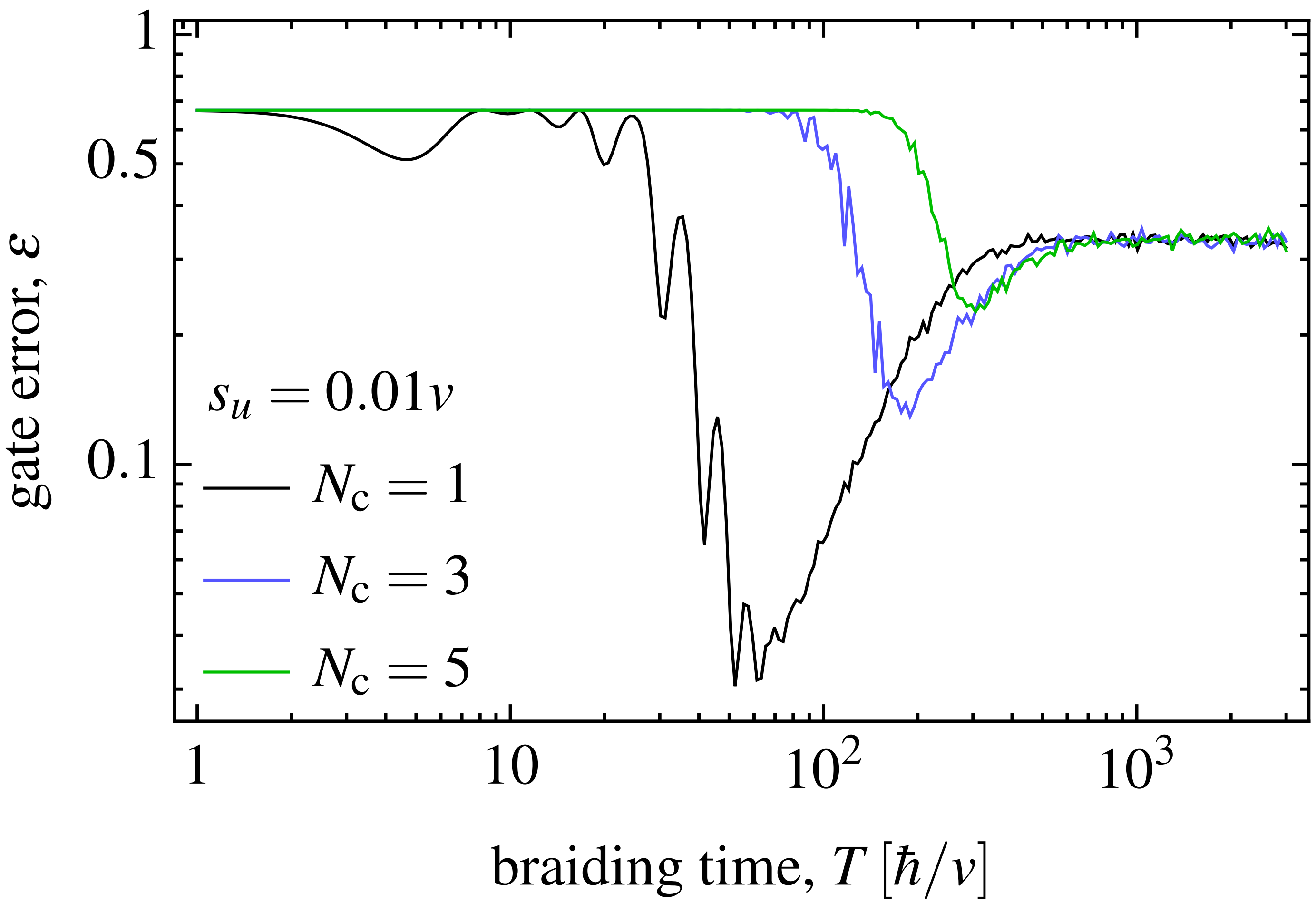
Y gate is robust against real-valued hopping disorder

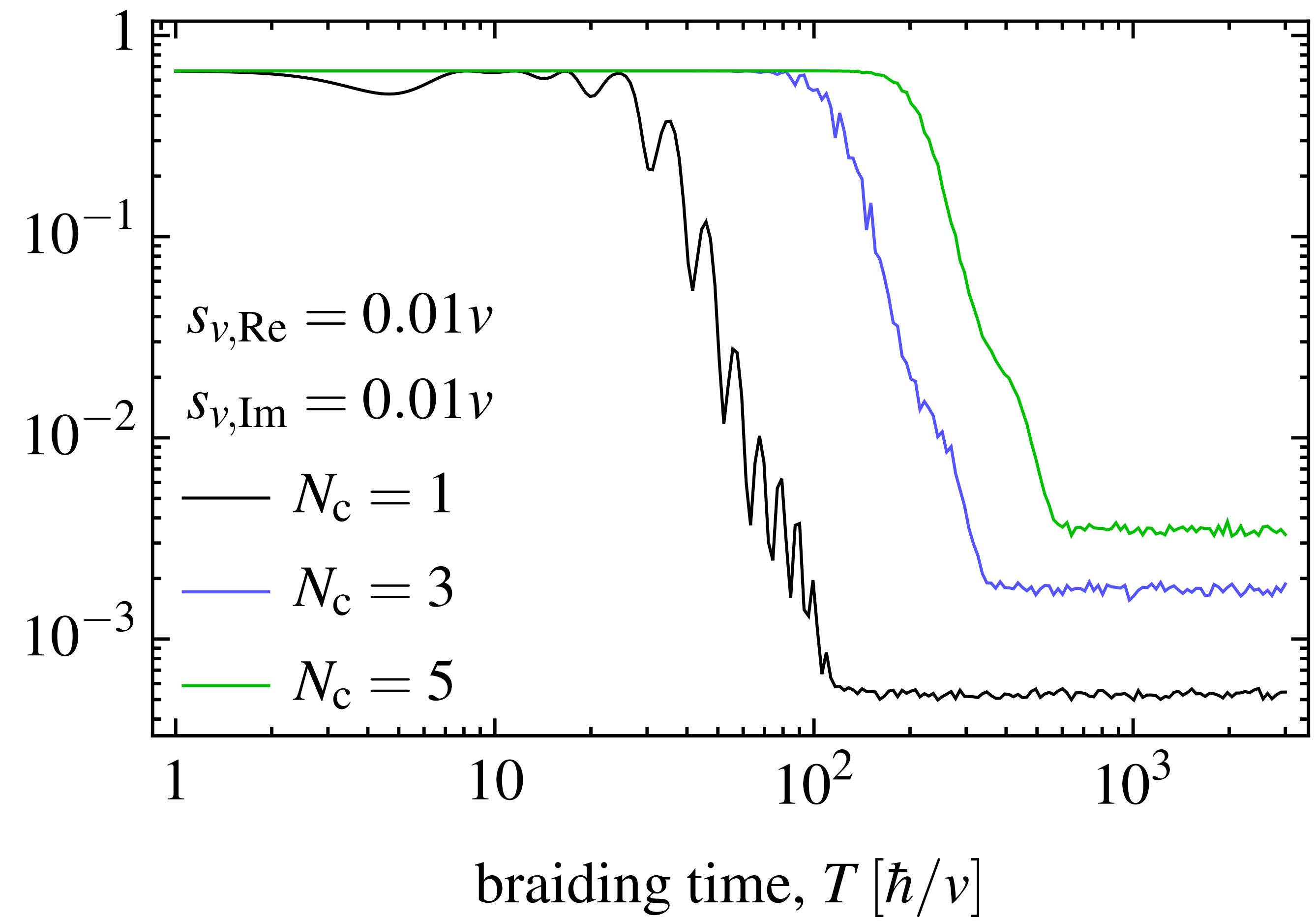
Topologically protected quantum gates: a simpler example

Boross et al., PRB 2019

on-site disorder



complex-valued hopping disorder



4. slower braiding ⇒ better gate

5. longer chains ⇒ better gate

6. does not require a perfect wire

7. does require **chiral** & **time-reversal symmetry**

Y gate is NOT robust against
(1) on-site disorder
(2) complex-valued hopping disorder

Does the SSH Y-junction provide a practical route toward topological quantum computing?

Don’t think so.

(1) Hard to imagine a physical system where hopping disorder is strong and on-site disorder is weak. (e.g., quantum-dot array is certainly not like that)

(2) Set of gates is very limited.

Control questions

1. For chain length $N_c = 7$, what is the dimension of the Hamiltonian?
2. Is the time-reversal symmetry of our model fermionic or bosonic?
3. How would you quantify the “precision” or “accuracy” of a quantum-logical operation (gate)?
4. Does our quantum gate work if the lengths of the 3 chains forming the Y junction are different?
5. Do you think our quantum gate offers a route toward universal topological quantum computing? Why?
6. List 3 characteristics of a topological quantum gate.
7. Explain similarities and differences between the three types of adiabatic quantum dynamics, characterized by the Berry phase, the non-Abelian Berry phase, and the topological Berry phase.