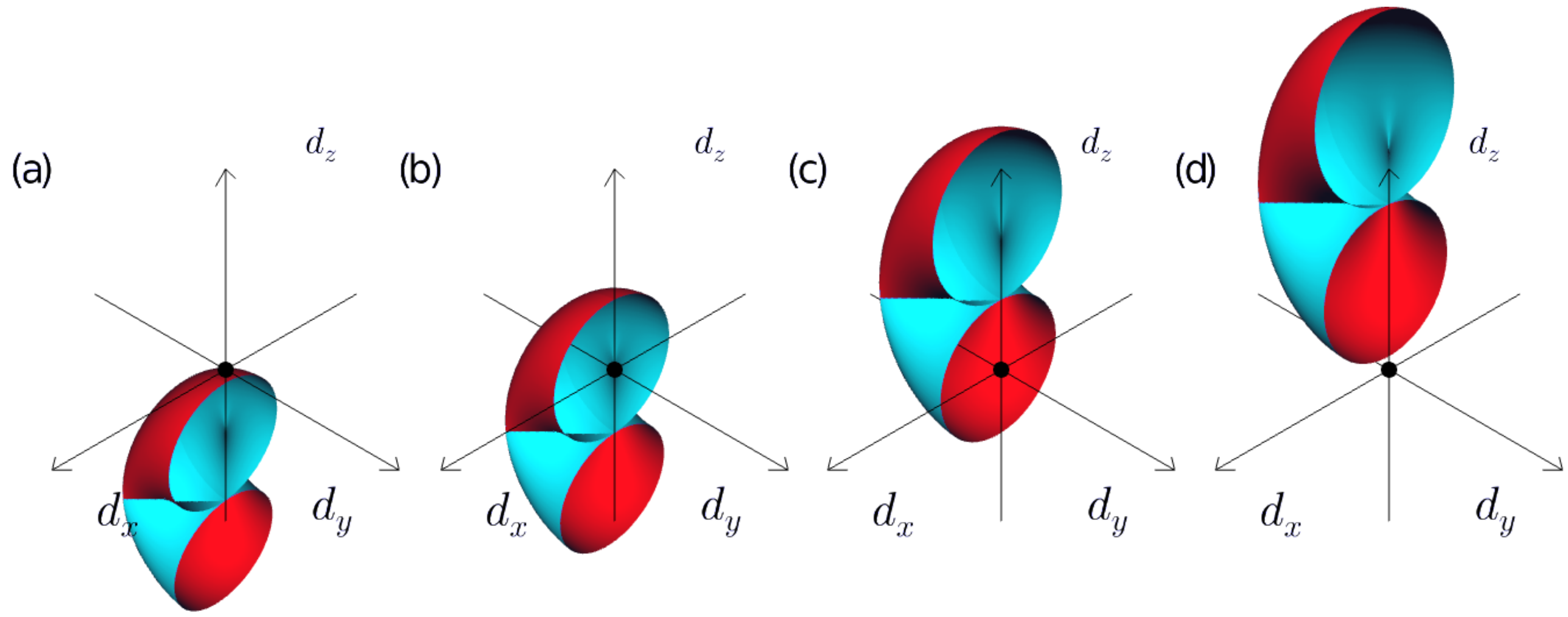
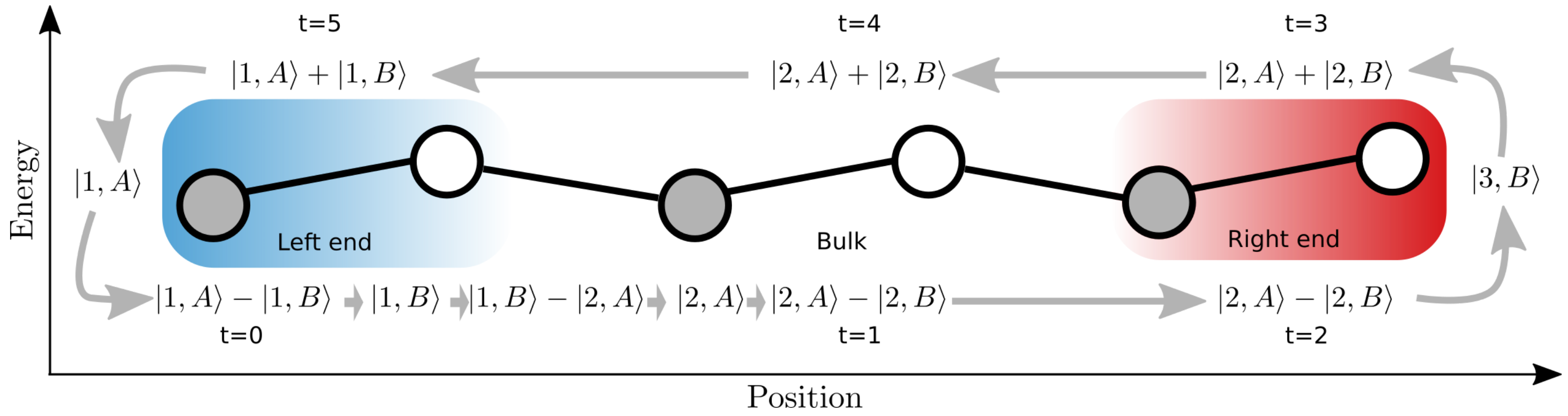
THE QWZ model

- Required:Thouless pumping
- New theory tool: Promoting time t → quasimomentum k
- Main results: Edge states in two-dimensional systems
  Bulk Chern number predicts edge states
  Topological protection
- Toy model: Qi-Wu-Zhang
  obtained from Thouless pump in Rice-Mele by promoting t → k



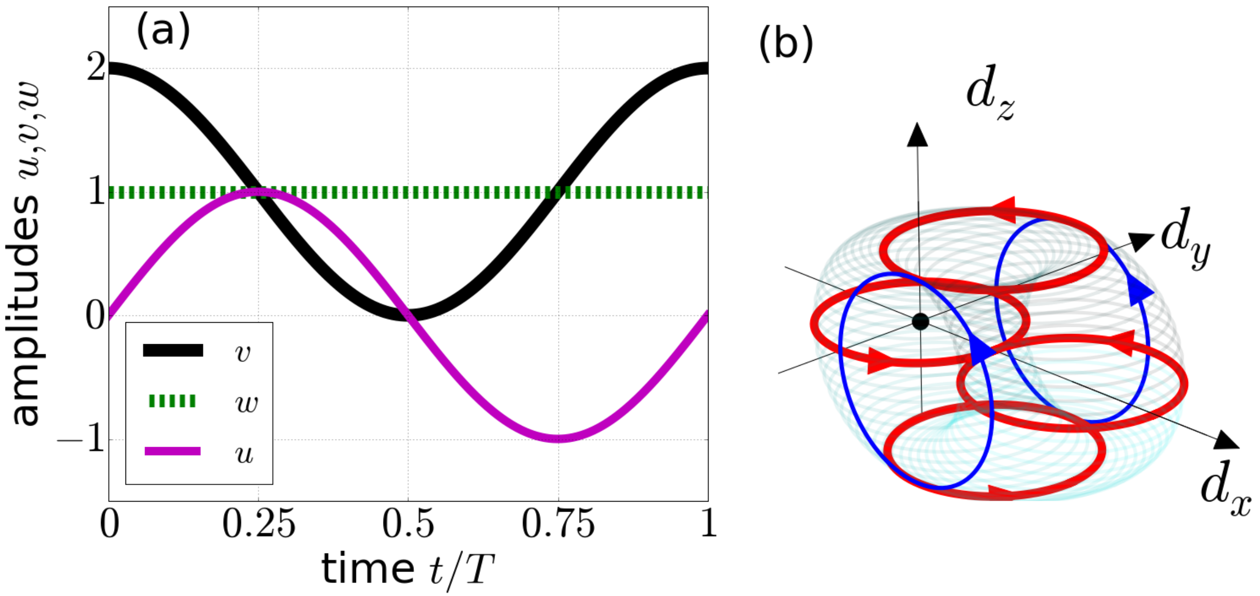
Reminder 1: Thouless pump sequence, Rice-Mele



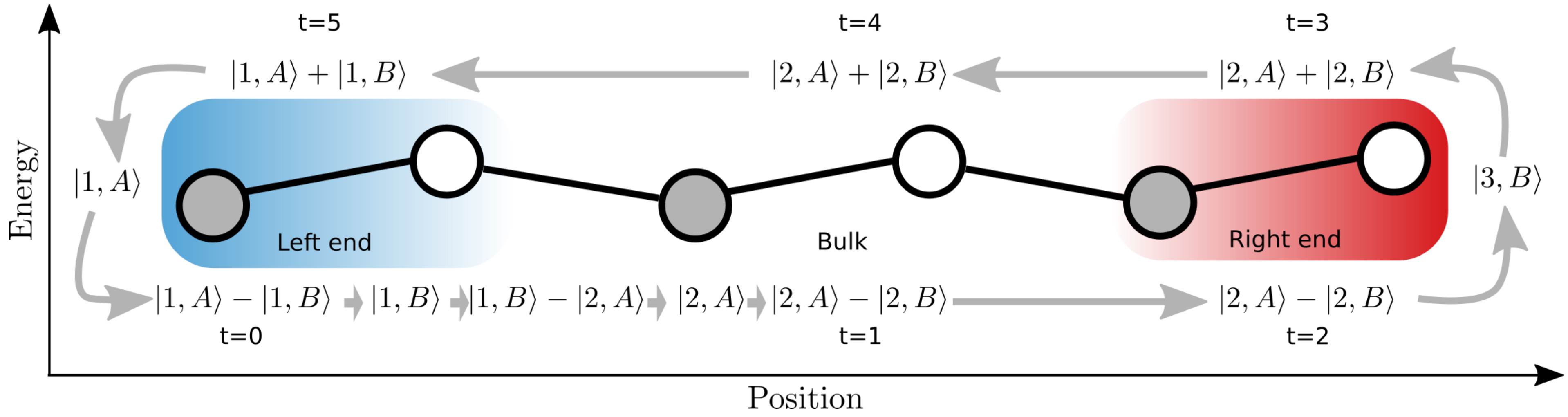
Pump charge along a dimerized chain using sublattice potential:

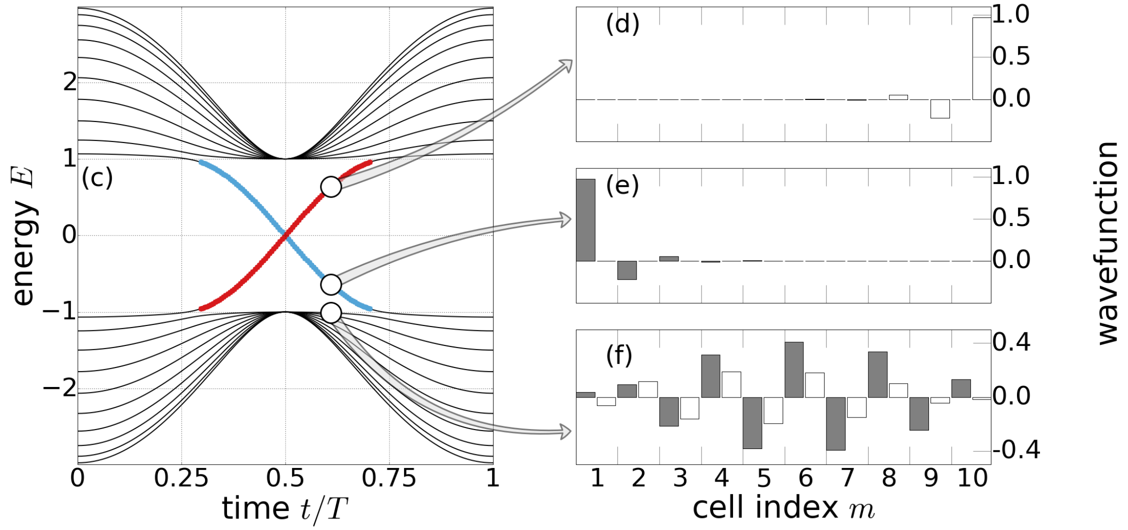
$$\hat{H}(k,t) = \mathbf{d}(k,t)\cdot\hat{\sigma} = (v(t) + w\cos(t))\,\hat{\sigma}_x + w(t)\sin(k)\hat{\sigma}_y + u(t)\hat{\sigma}_z$$

$$u(t) = \sin(\Omega t)$$
$$v(t) = \bar{v} + \cos(\Omega t)$$
$$w(t) = 1$$



Reminder 2: Protected Edge States in Thouless pump



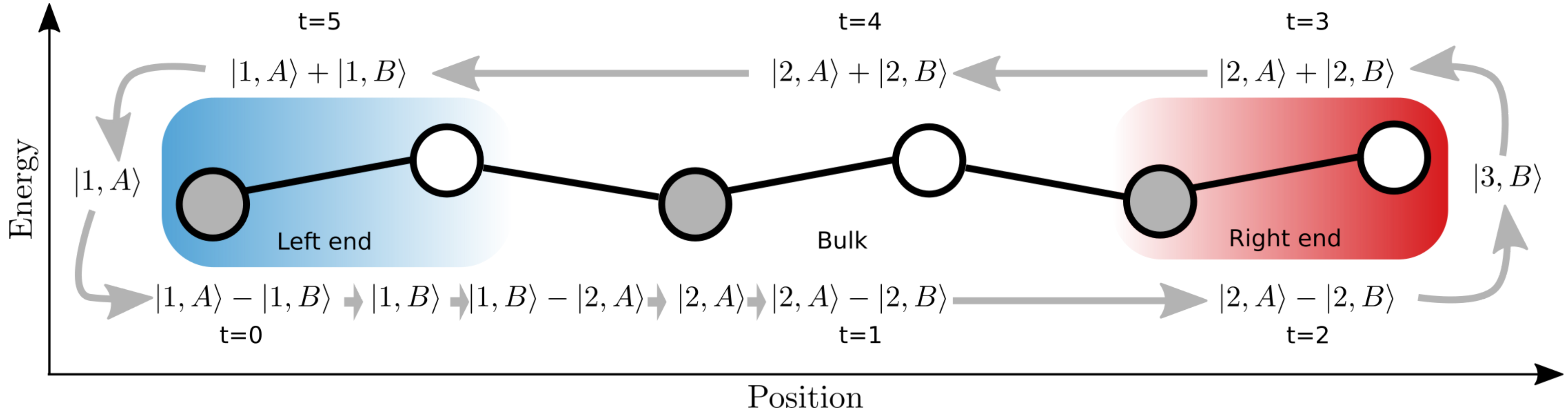


Topologically protected = robust:

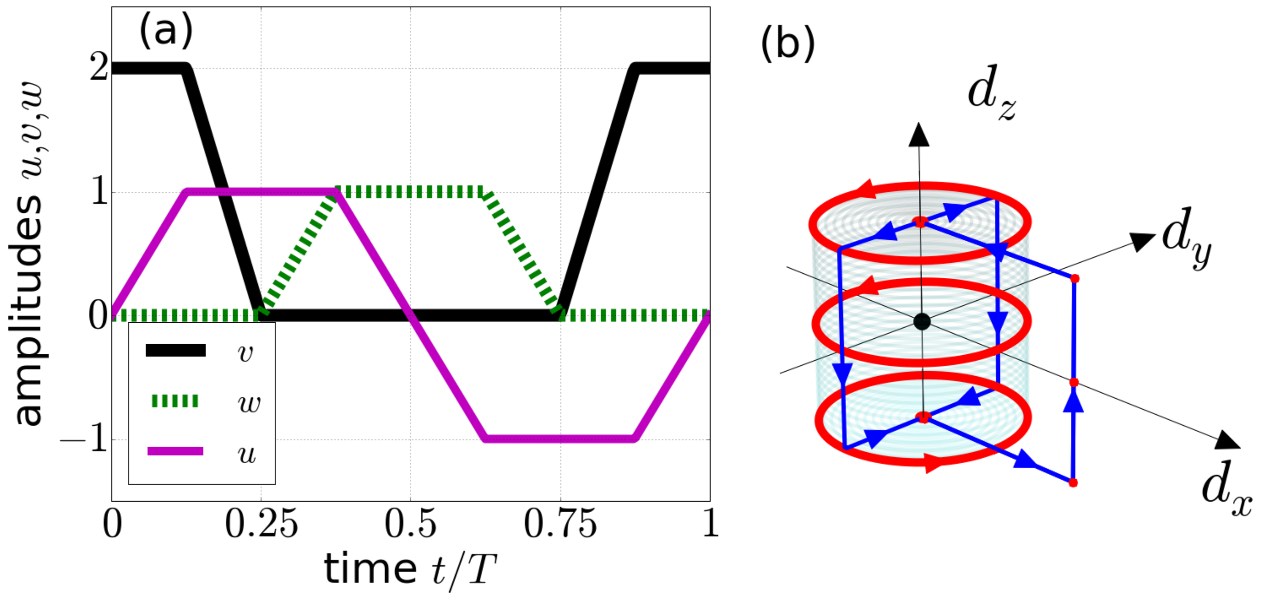
- Time - Periodic drive
- No long range hopping

1. spectrum time-periodic
2. spectrum continuous
3. bulk gap separates two edges
4. → no direct coupling,
5. → crossing, not anticrossing

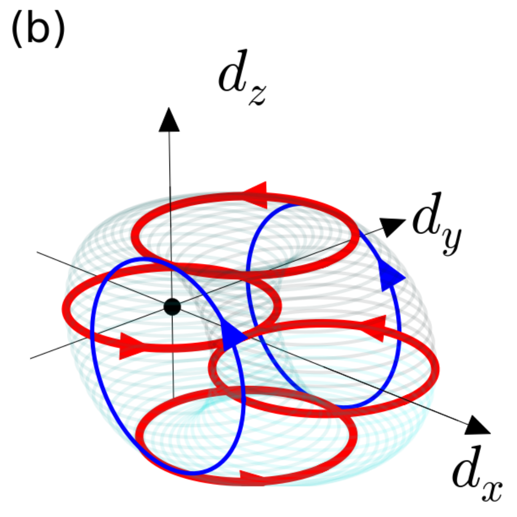
Reminder 3: Thouless pump in the bulk in d-space:
times origin in torus = # charge pumped = Chern



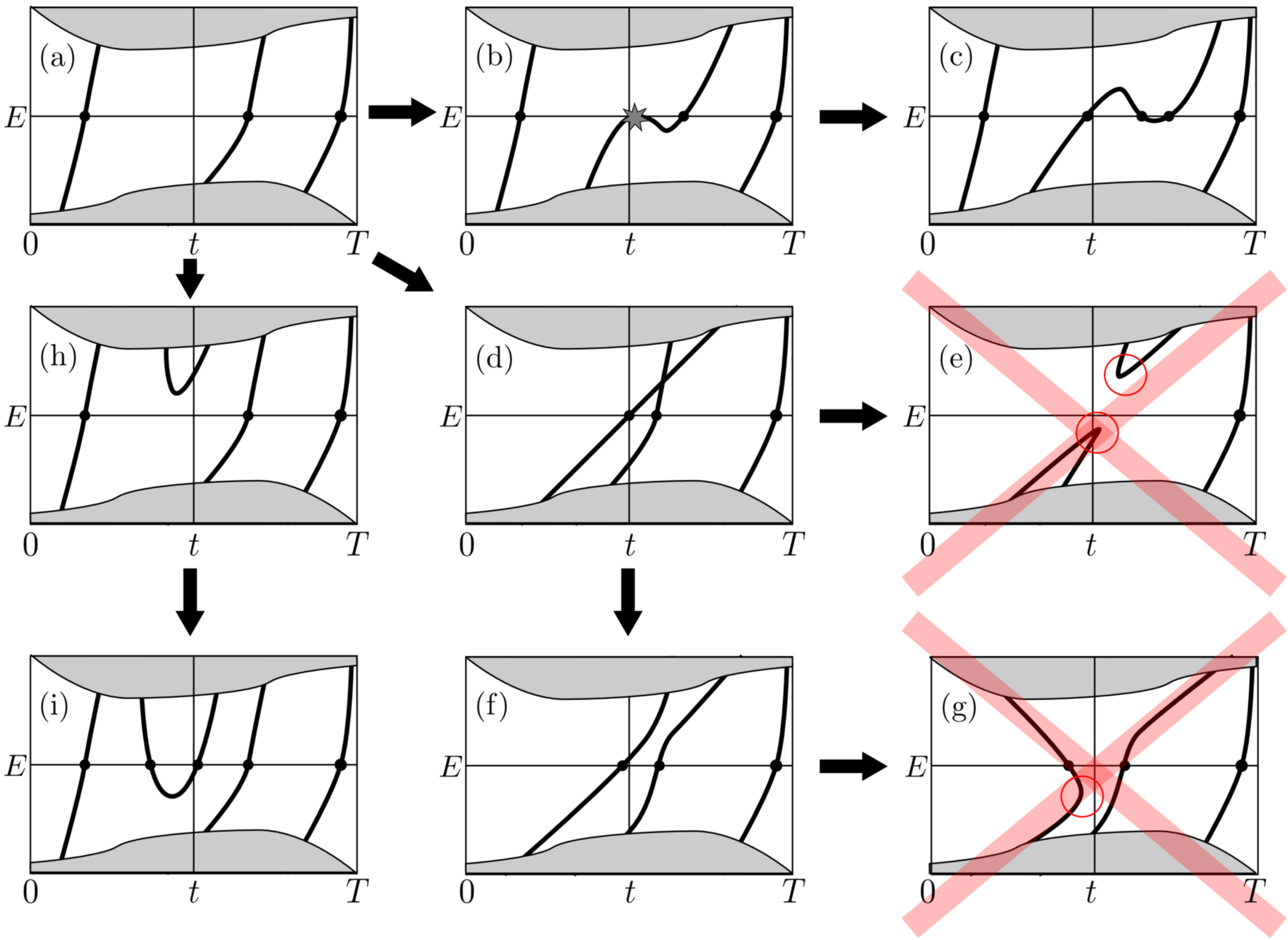
control freak sequence:



smooth sequence:



Reminder 4: Net number of charge pumped up in energy at an edge is protected against continuous deformations



New material: From Thouless pump to Chern insulator

Promote time t → wavenumber k
1D time-periodic Rice-Mele → 2D Qi-Wu-Zhang

$$\hat{H}_{\mathrm{RM}}(k,t) = \sin(k)\hat{\sigma}_y + \sin(\Omega t)\hat{\sigma}_z + \left(\bar{v} + \cos(k) + \cos(\Omega t)\right)\hat{\sigma}_z$$

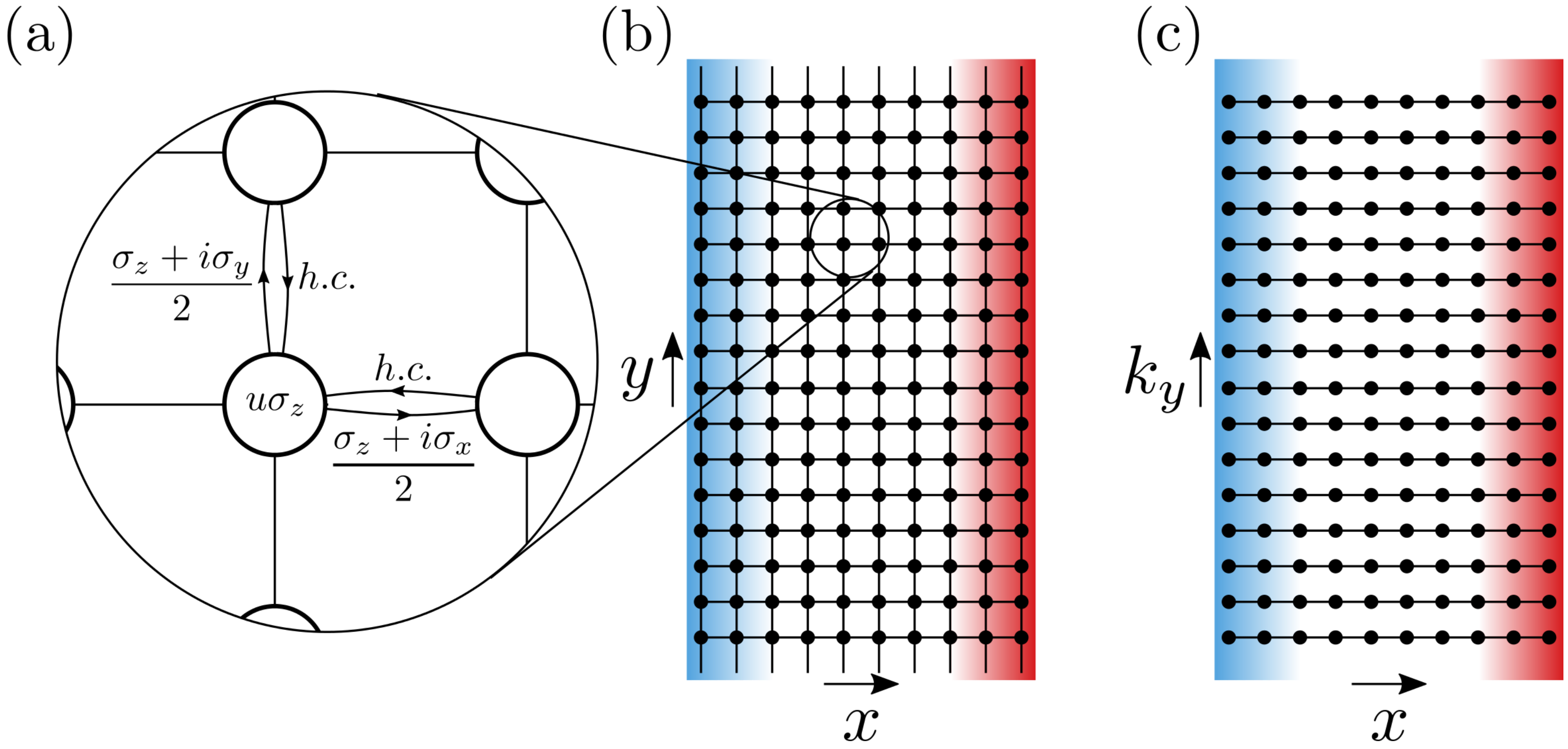
$$\boxed{\Omega t \to k_y} \qquad \hat{\sigma}_y \to \hat{\sigma}_x$$

$$k \to k_x \qquad \hat{\sigma}_z \to \hat{\sigma}_y$$

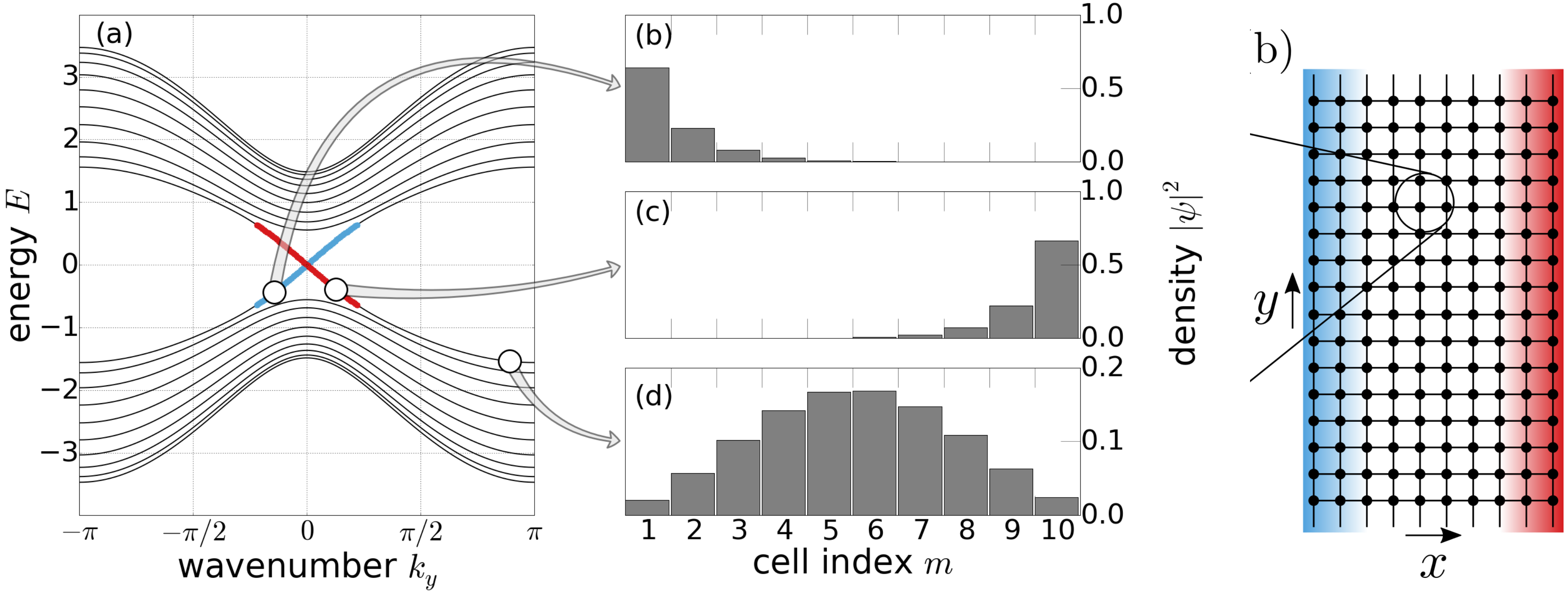
$$\bar{v} \to u \qquad \hat{\sigma}_x \to \hat{\sigma}_z$$

$$\hat{H}_{\mathrm{QWZ}}(k_x,k_y) = \sin(k_x)\hat{\sigma}_x + \sin(k_y)\hat{\sigma}_y + \left(\bar{v} + \cos(k_x) + \cos(k_y)\right)\hat{\sigma}_z$$

Promote time t → wavenumber k
1D time-periodic Rice-Mele → 2D Qi-Wu-Zhang



Edge states rising/falling in Thouless pump
→ unidirectional edge modes in Chern insulators



Topologically protected = robust:

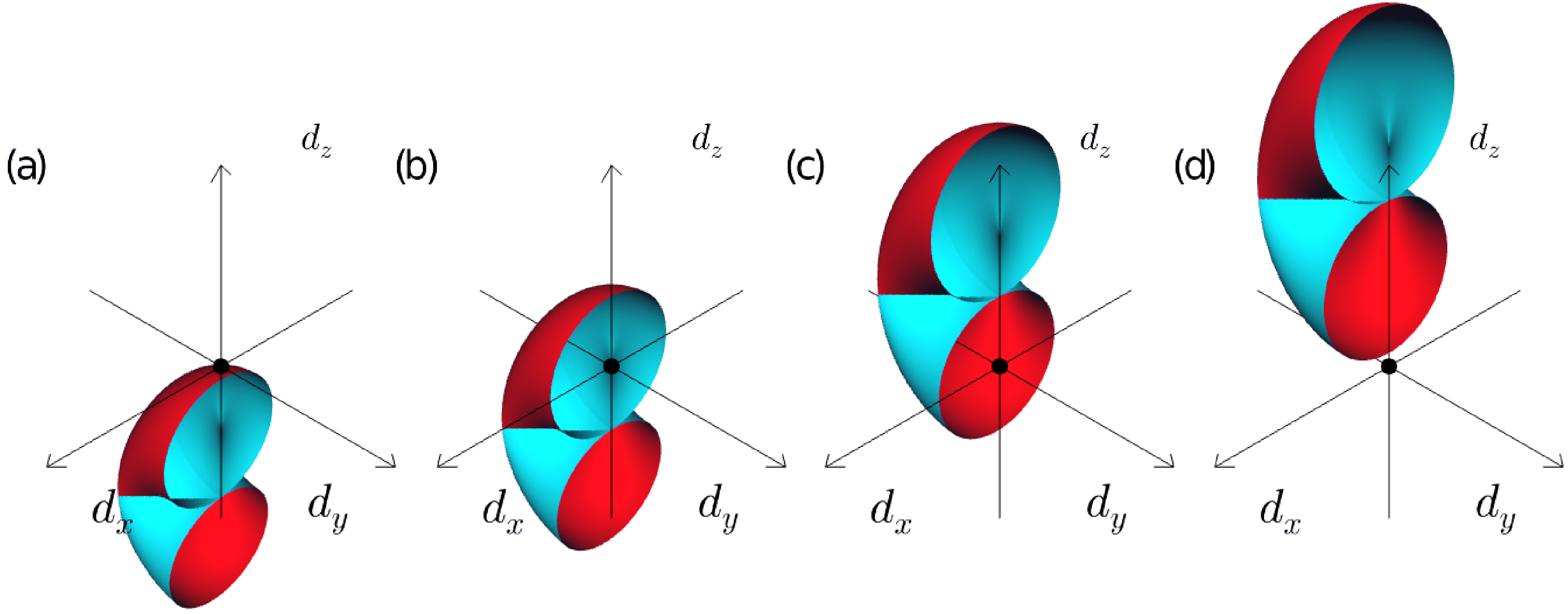
- No long range hopping

→ spectrum periodic & smooth

→ bulk gap separates two edges → no direct coupling → crossing, not anticrossing

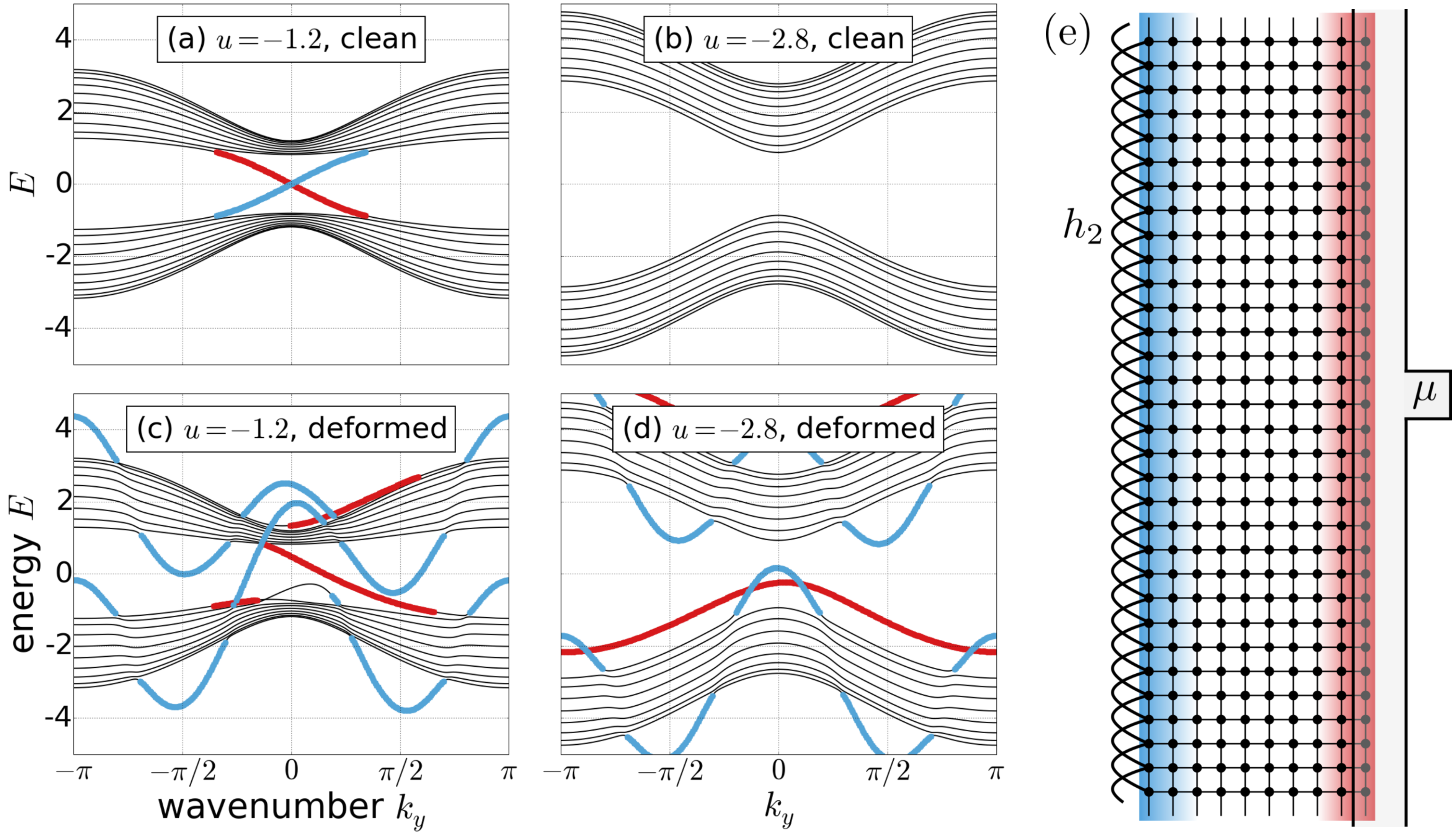
Presence, net # of edge state modes seen in bulk:
times origin in torus = # edge state modes = Chern

u<-2 | -2<u<0 | 0<u<2 | u>2

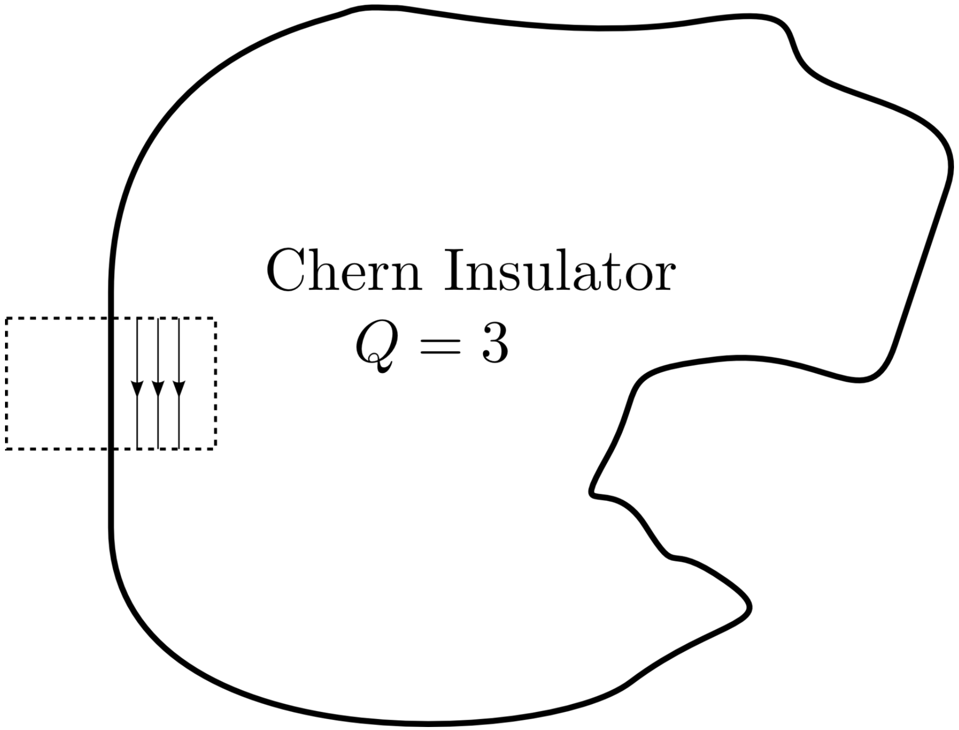


C=0 | C=-1 | C=1 | C=0

Net number of clockwise-propagating edge state modes in the gap is protected against continuous deformations

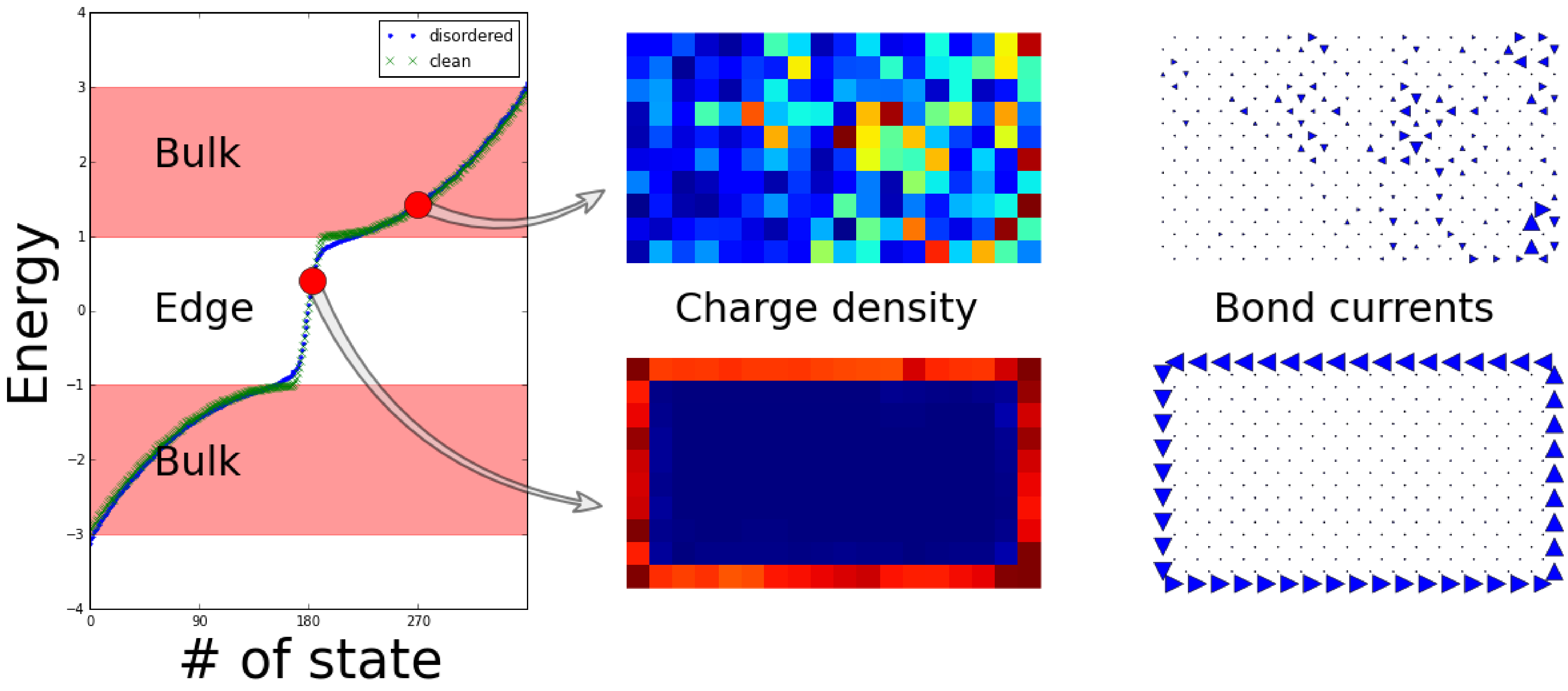


Net edge states at some section of edge → edge states all around (unitarity → particles cannot accumulate)



Topologically protected = robust against:
- Arbitrary disorder on edges
- Some disorder in bulk (interesting variation on Anderson localization)

Net edge states at some section of edge → edge states all around (unitarity → particles cannot accumulate)



Summary: Chern Insulators have robust edge states predicted by bulk Chern

- Required:Thouless pumping (ensure edge states, Chern #)
- New theory tool: Promoting time t → quasimomentum k
- Main results: Edge states in two-dimensional systems

  Bulk Chern number predicts edge states

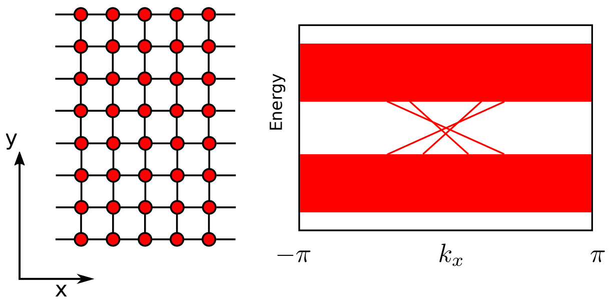
  Topological protection due to no backscattering

  Robust against disorder (large edge, small bulk)
- Toy model: Qi-Wu-Zhang

  Tune Chern number by onsite magnetic field u (-2, 0, 2)

$$\hat{H}_{\mathrm{QWZ}}(k_x, k_y) = \sin(k_x)\hat{\sigma}_x + \sin(k_y)\hat{\sigma}_y + (\bar{v} + \cos(k_x) + \cos(k_y))\,\hat{\sigma}_z$$

Consider the spectrum of an infinite translationally invariant ribbon.
Based on the depicted spectrum what is the Chern number of the bulk?



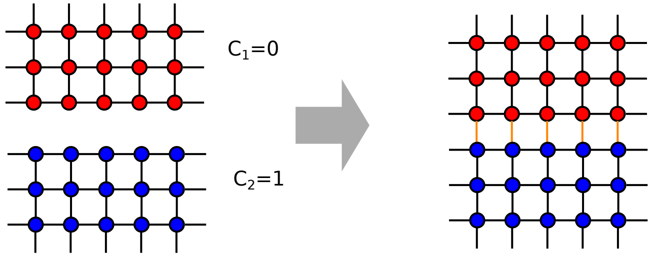
a) 0

b) 2

c) neither

d) could be either

Consider a lattice model (e. g. QWZ) glue two copies of it with different Chern numbers together $C_1=0$, $C_2=1$. On the edge of the two regions ...



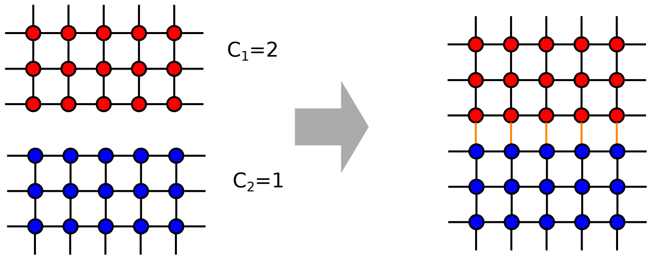
a) there is a localized state, but it is not topologically protected.

b) there is a topologically protected state.

c) an infinitezimally small coupling destroys the edge state.

d) there might be topologically protected edge states but their number is undetermined.

Consider a lattice model (e. g. QWZ) glue two copies of it with different Chern numbers together C_1=2, C_2=1. On the edge of the two regions ...



a) there is a localized state, but it is not topologically protected.

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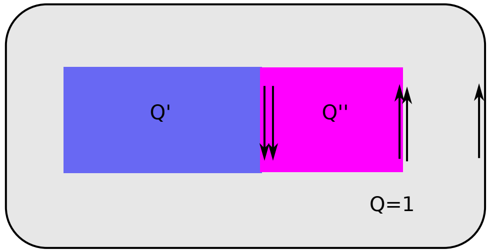
We marked topologically protected edgestates at three edges.
There could be more on the edges that are not marked.

What is the Chern number Q' of the blue region?



a) Q'=0

b) Q'=1

c) There can not be such a configuration

d) Q' is indetermined (not enough information)

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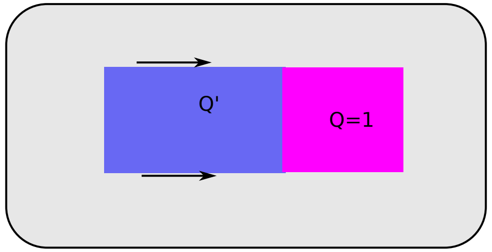

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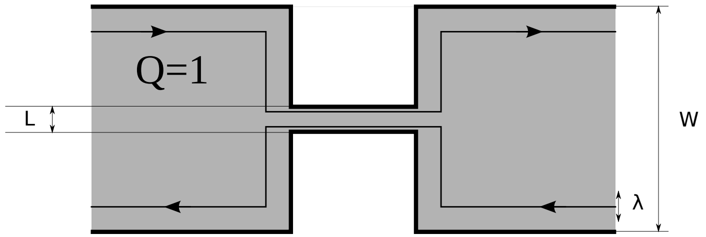
Under what condition do you expect that an electron arriving in an edge state will be perfectly transmitted through this constriction?
λ is the penetration depth of edge states towards the bulk.



a) $W \gg L$ and $W \gg \lambda$

b) Chern number is nonzero $\Longrightarrow$ edge states are protected independent of the shape of the system.

c) $W \gg L$ and $L \gg \lambda$

d) $W \gg \lambda$

Modify the QWZ model: change hopping along x to next nearest neighbor hopping.
How does this change the number of edge states along x=N_x and along y=N_y?
The number of edge states...

$$H = (u + \cos 2k_x + \cos k_y)\sigma_z + \sin 2k_x \sigma_x + \sin k_y \sigma_y$$

y

x

a) ... along x does not change but their velocity does.
$\Longrightarrow$ N_y is also unchanged.

b) ... along y doubles
$\Longrightarrow$ N_x also doubles.

c) ... increases along x, but N_y is unchanged.

d) ... doubles along y, but N_x is unchanged.

The QWZ model has spin dependent hopping amplitudes:

$$H_{QWZ} = u\sigma_z + \sin k_x \sigma_x + \sin k_y \sigma_y + (\cos k_x + \cos k_y)\sigma_z$$

Consider a simplified model:

$$H = u\sigma_z + \sin k_x \sigma_x + \sin k_y \sigma_y + v(\cos k_x + \cos k_y)\sigma_0$$

Which parameter tunes the Chern number of the simplified system?
Assume the system to be an insulator.

a) v

b) u

c) This model cannot be an insulator

d) The Chern number must always be 0

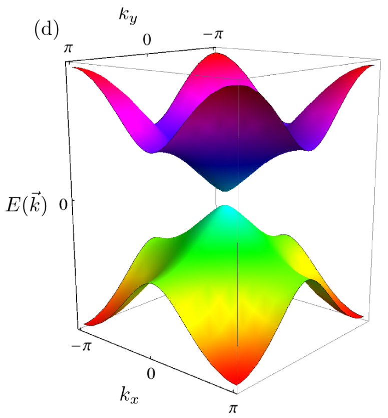
What is the Chern number of the valence band depicted in the picture?

a) 0

b) 1

c) The Chern number is defined for the model not for the valence band!

d) Can not be decided.



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c) ... increases along x, but N_y is unchanged.

d) ... doubles along y, but N_x is unchanged.

We fold a lattice model with a Chern number of +1 to the shape of a Moebius strip. The edge states on two opposite edges...

A) Propagate in the opposite direction

B) Propagate in the same direction

C) Direction of propagation depends on position along the edge

D) Do not exist any more (are gapped out)