Experimental signatures

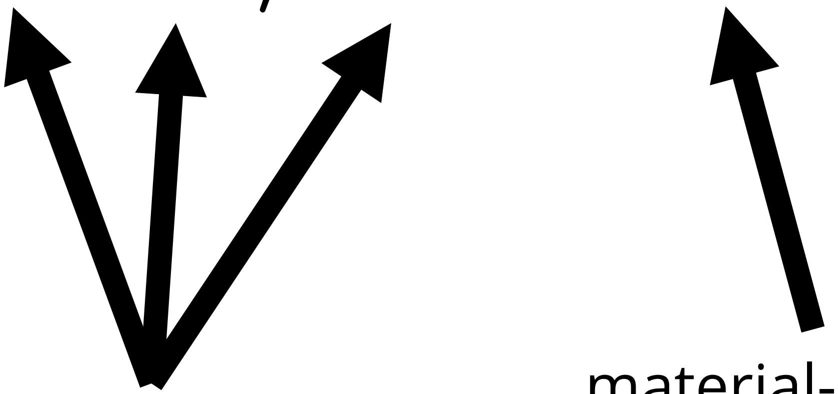
- Ohmic vs. non-ohmic conductors
- Landauer formalism
- Signatures of edgestates
- State of the art

Ohmic conductors

conductance/resistance

$$I/V = G \equiv R^{-1}$$

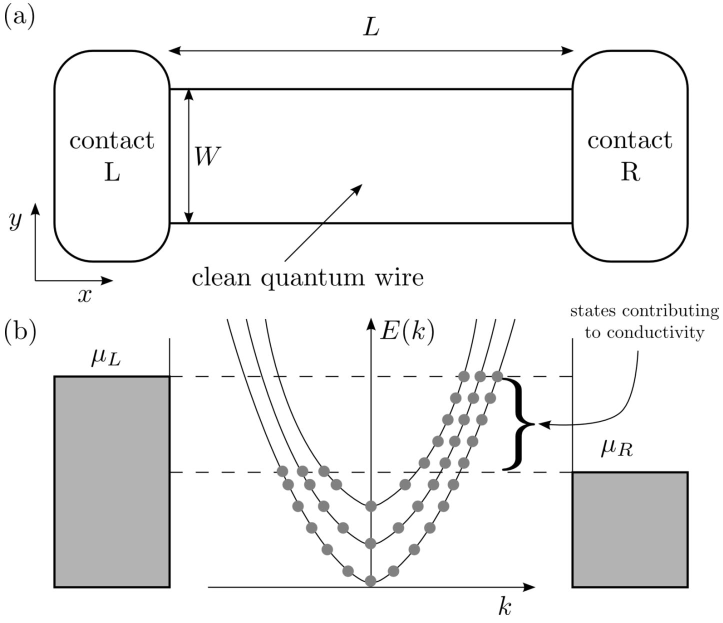
$$G_i L_i / A_i = \sigma$$



Sample specific

material-specific
conductivity

Landauer formalism

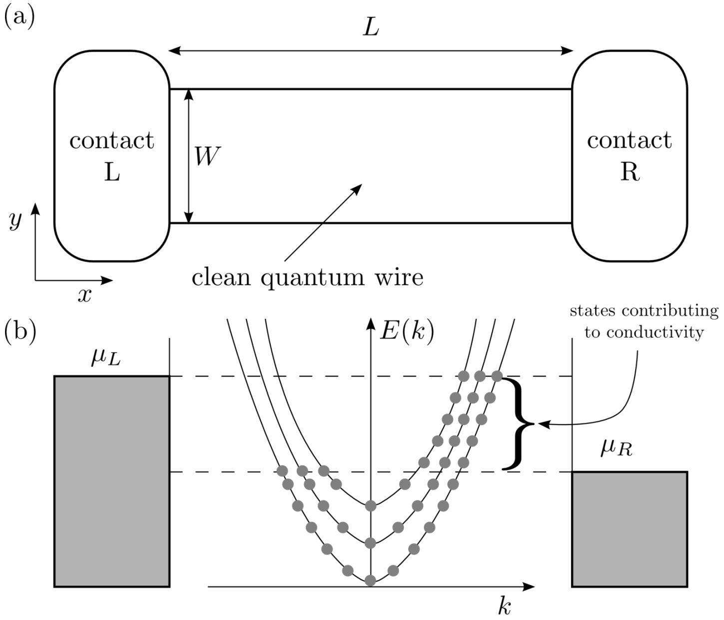


$$I = -|e| \sum_n \int_{-\pi/a}^{\pi/a} \frac{dk}{2\pi} \frac{1}{\hbar} \frac{dE_{lk}}{dk} \left[f(E_{lk} - \mu_L) - f(E_{lk} - \mu_R) \right]$$

$$I = -\frac{|e|}{h} M \int_{\mu_R}^{\mu_L} dE = -\frac{|e|}{h} (\mu_L - \mu_R) M = M \frac{e^2}{h} V$$

A clean wire can not be Ohmic!!

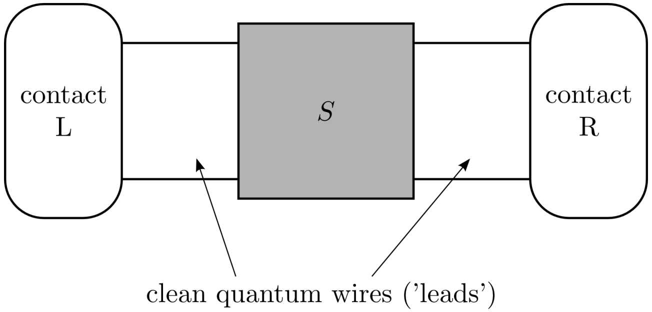
Landauer formalism



$$I = -|e| \sum_{n} \int_{-\pi/a}^{\pi/a} \frac{dk}{2\pi} \frac{1}{\hbar} \frac{dE_{lk}}{dk} \left[f(E_{lk}-\mu_L) - f(E_{lk}-\mu_R)\right]$$

$$I = -\frac{|e|}{h} M \int_{\mu_R}^{\mu_L} dE = -\frac{|e|}{h}(\mu_L-\mu_R)M = M\frac{e^2}{h}V$$

A clean wire can not be Ohmic!!



$$I = -\frac{|e|}{h} T \int_{\mu_R}^{\mu_L} dE \left[f_L(E) - f_R(E)\right] = \frac{e^2}{h} T V$$

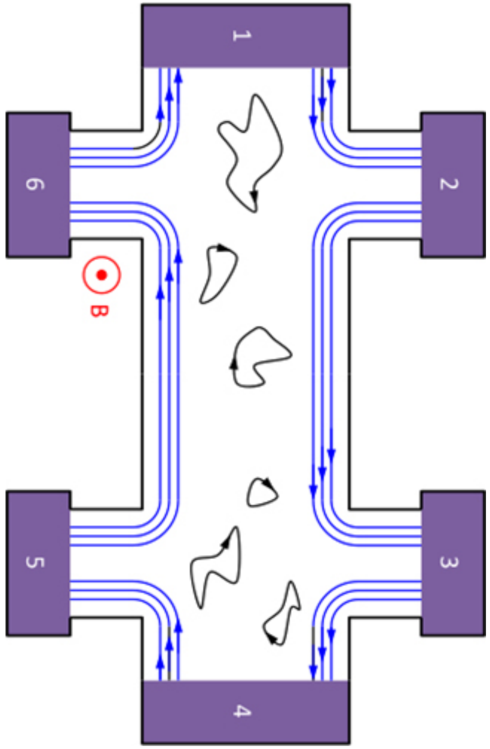
Signatures of Topological edge states in transport I: Chern Insulators

Disorder-free sample with a strip geometry

- Fermi energy lies in a band:

  the conductance is quantized and **insensitive to the length** of the sample, **grows with the width**
- Fermi energy lies in the gap:

  conductance is quantized, a **behaviour insensitive to both the length and the width** of the sample



Disordered sample with an irregular shape

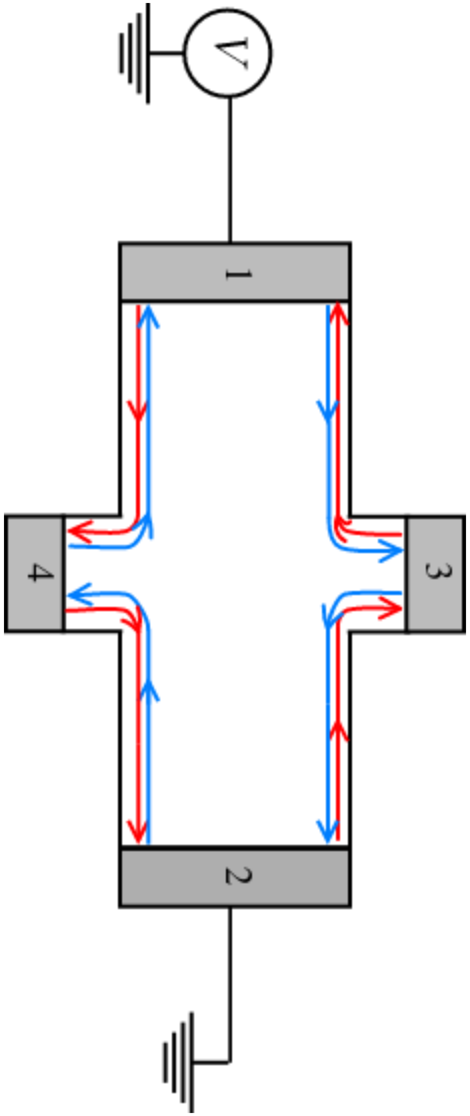
- Fermi energy lies in a band:

  Might be Ohmic. There are no protected edge states at the Fermi energy.

  Fermi energy lies in the gap:

  conductance is quantized, a **behaviour insensitive to both the length and the width** of the sample, **a hallmark of Chern Insulators.**

Signatures of Topological edge states in transport II: TRS Insulators



Disorder-free sample with a strip geometry

- Fermi energy lies in a band:

  the conductance is quantized and **insensitive to the length** of the sample, **grows with the width**
- Fermi energy lies in the gap:

  conductance is quantized to multiples of $\frac{2e^2}{h}$, since edge states come in pairs a **behaviour insensitive to both the length and the width** of the sample

Disordered sample with an irregular shape

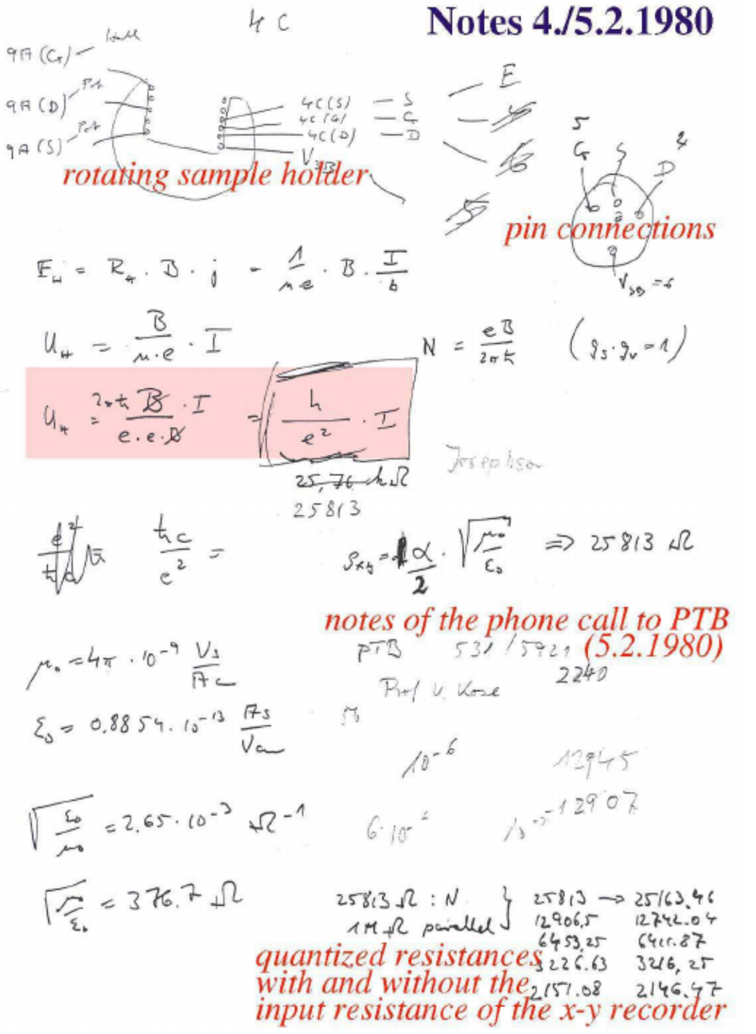
- Fermi energy lies in a band:

  Might be Ohmic.

  Fermi energy lies in the gap **and TRS is preserved**:

  conductance is quantized to $\frac{2e^2}{h}$, a **behaviour insensitive to both the length and the width** of the sample, **a hallmark of TRS Insulators.**

Quantum Hall Effect


Notes 4./5.2.1980

rotating sample holder

pin connections

$E_H = R_H \cdot B \cdot j = \frac{1}{ne} \cdot B \cdot \frac{I}{b}$

$U_H = \frac{B}{n \cdot e} \cdot I$ $\quad N = \frac{eB}{2\pi\hbar}$ $\quad (g_s \cdot g_v = 1)$

$U_H = \frac{2\pi\hbar B}{e \cdot e \cdot B} \cdot I = \boxed{\frac{h}{e^2} \cdot I}$

~~25,76~~ kΩ

25813

Josephson

$\frac{\hbar c}{e^2} =$

$\rho_{xy} = \frac{\alpha}{2} \cdot \sqrt{\frac{\mu_0}{\varepsilon_0}} \Rightarrow 25813\ \Omega$

notes of the phone call to PTB (5.2.1980)

PTB 531 / 5921

2240

Prof. v. Kose

$\mu_0 = 4\pi \cdot 10^{-9} \frac{Vs}{Acm}$

$\varepsilon_0 = 0{,}8854 \cdot 10^{-13} \frac{As}{Vcm}$

10^{-6}

12945

$\sqrt{\frac{\varepsilon_0}{\mu_0}} = 2{,}65 \cdot 10^{-3}\ \Omega^{-1}$

$6 \cdot 10^{-2}$

12907

$\sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7\ \Omega$

25813 Ω : N
1 MΩ parallel

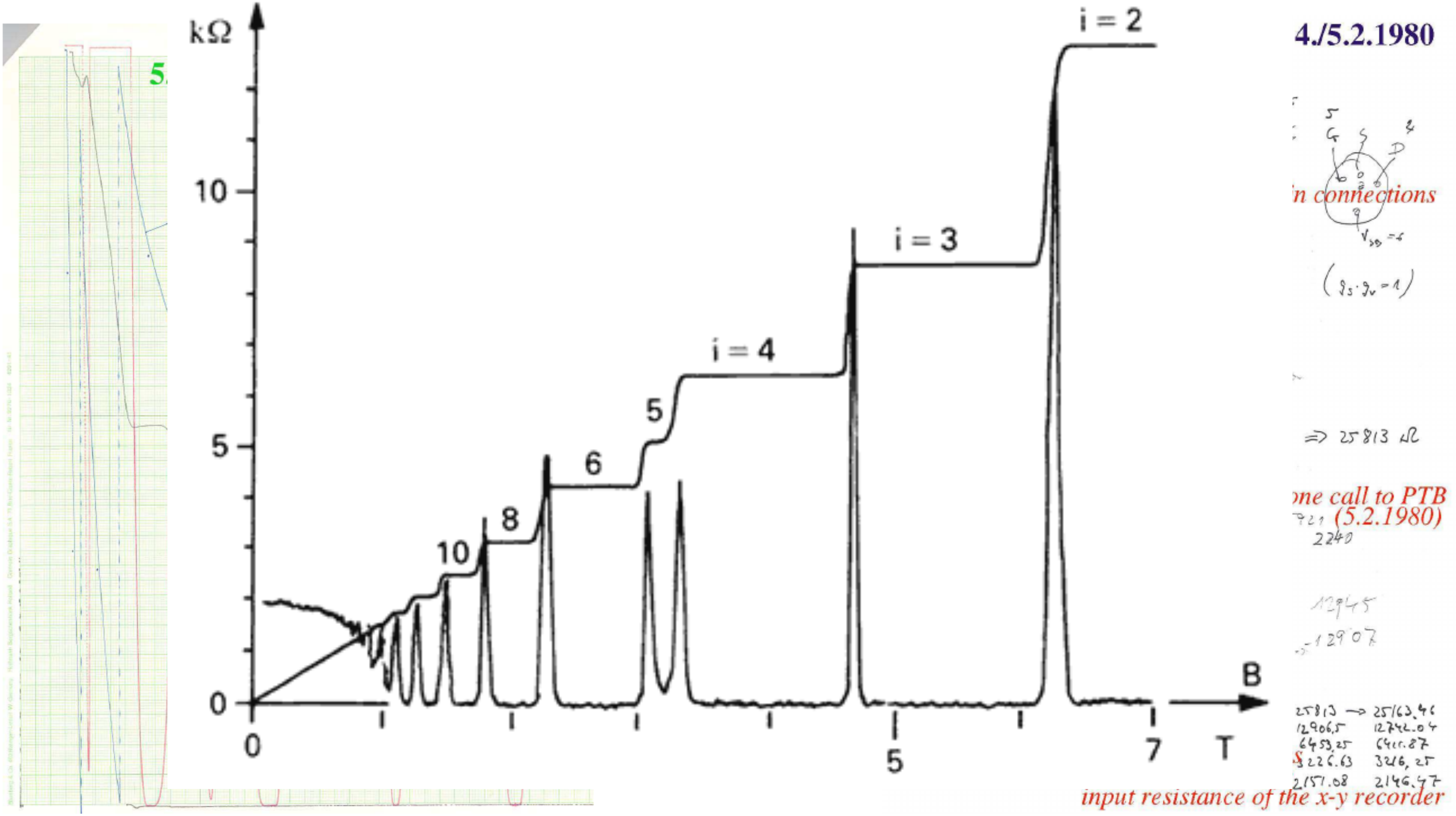
25813 →	25163.46
12906,5	12742.04
6453,25	6411.87
3226.63	3216, 25
2151.08	2146.47

quantized resistances with and without the input resistance of the x-y recorder

Klitzing, K. v., Dorda, G., Pepper, M. Phys. Rev. Lett., **45**, 494 (1980)

Klitzing, K. v. Seminaire Poincare **2**, 1 (2004)

Quantum Hall Effect



Klitzing, K. v., Dorda, G., Pepper, M. Phys. Rev. Lett., **45**, 494 (1980)

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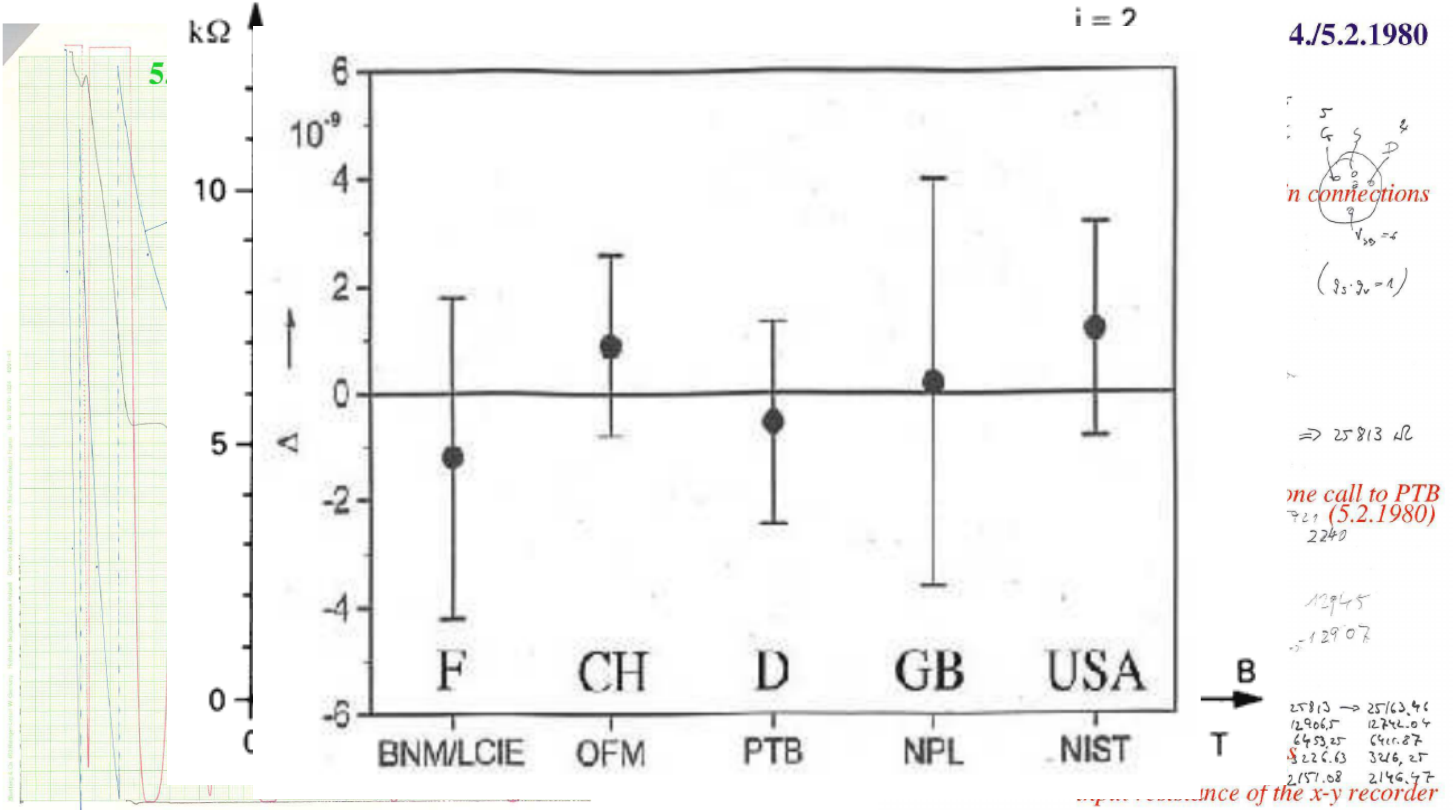
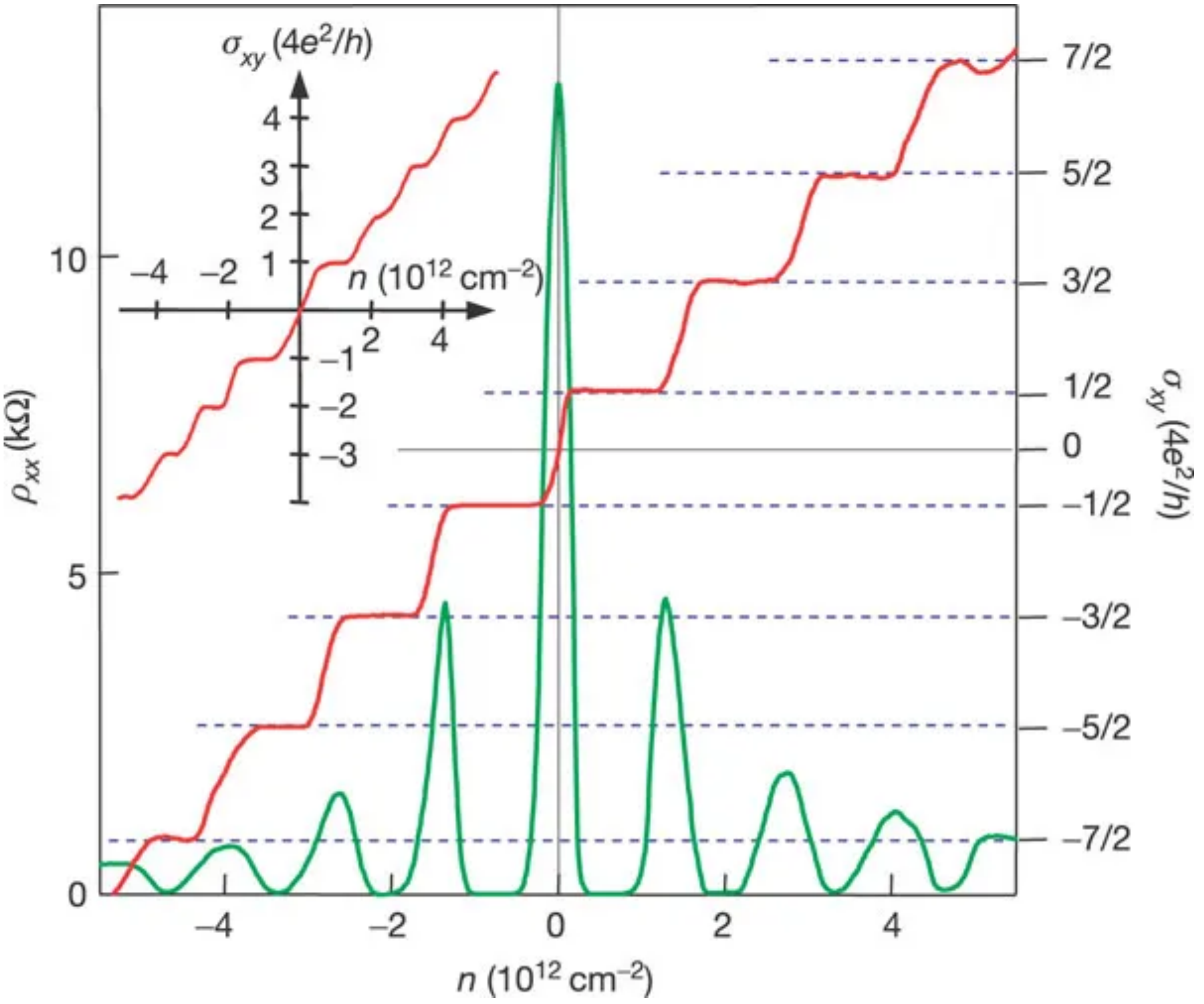
Quantum Hall Effect



Klitzing, K. v., Dorda, G., Pepper, M. Phys. Rev. Lett., **45**, 494 (1980)

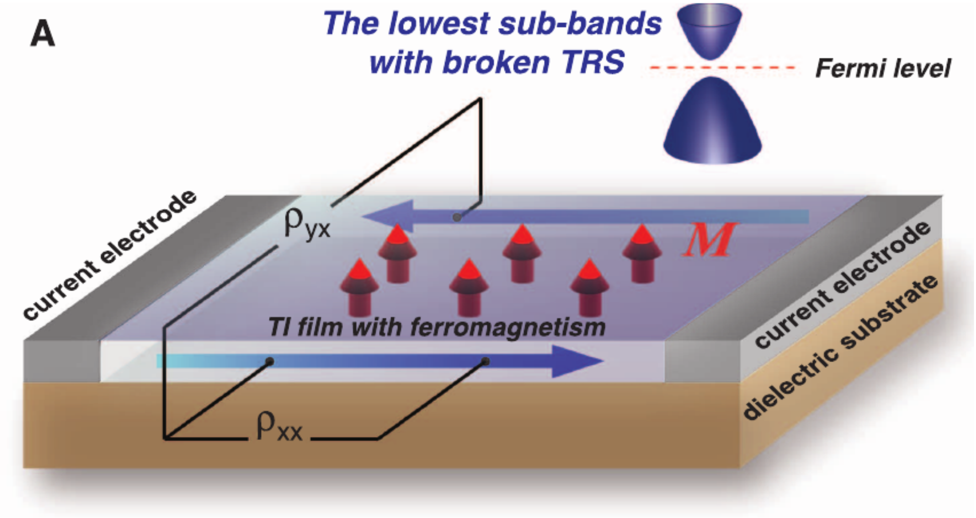
Klitzing, K. v. Seminaire Poincare **2**, 1 (2004)

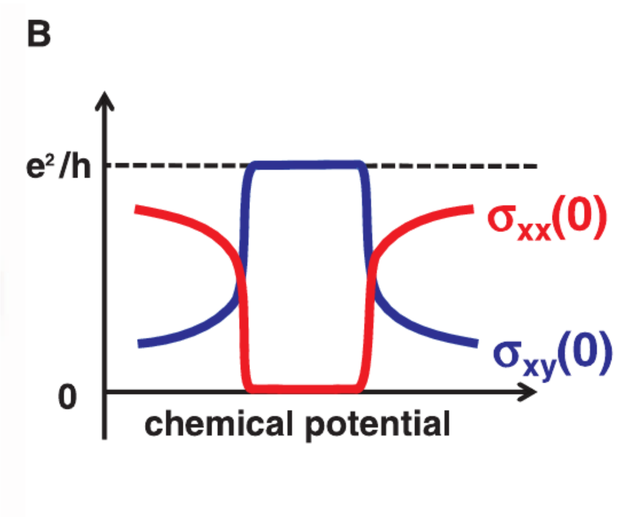
Quantum Hall Effect in graphene

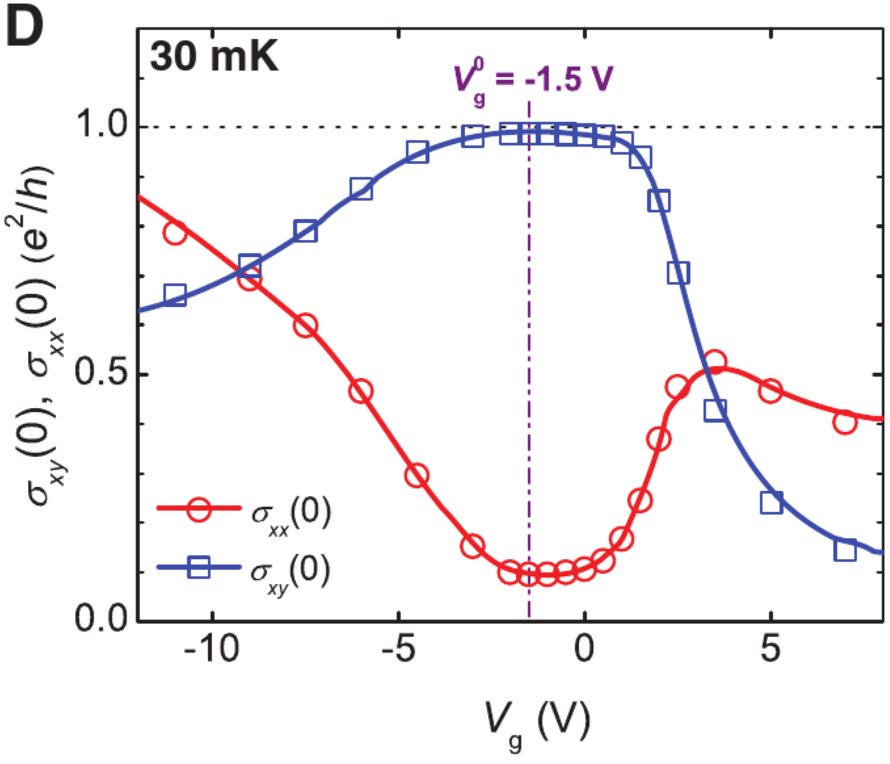


K. S. Novoselov *et al.* Nature **438**, 197 (2005)

Quantum Anomalous Hall Effect in 3D TI

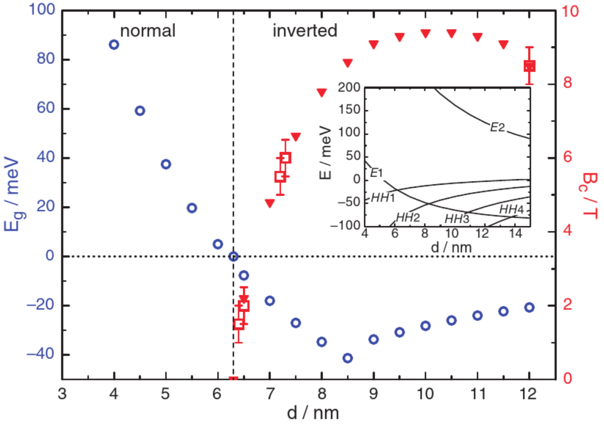


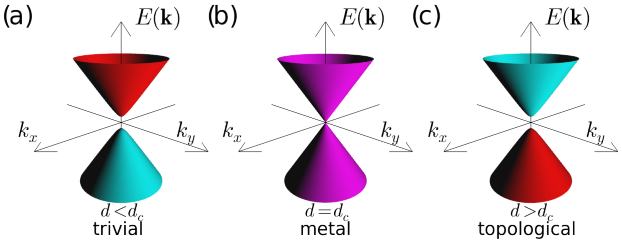


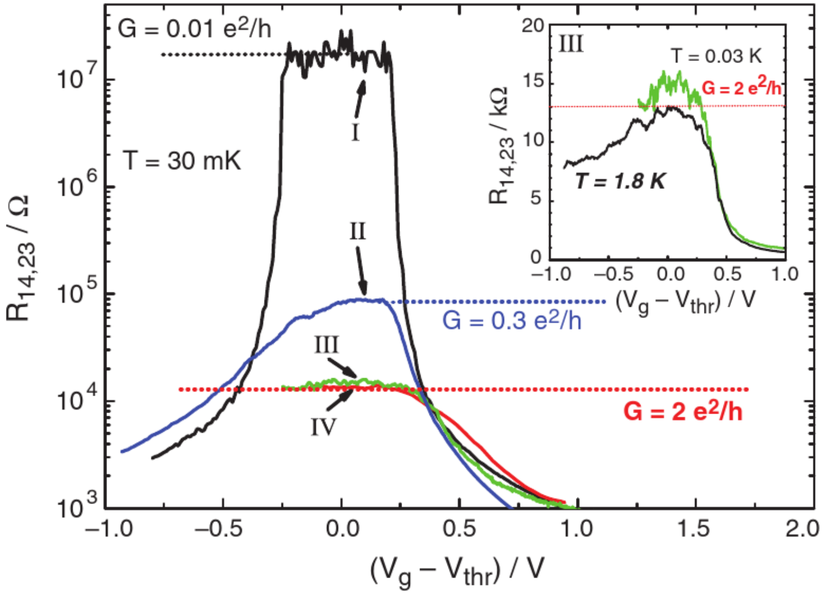


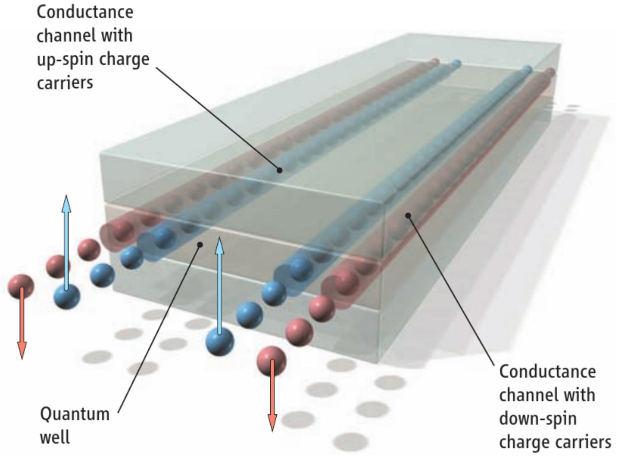
Cui-Zu Chang *et al.* Science **340**, 167(2013)

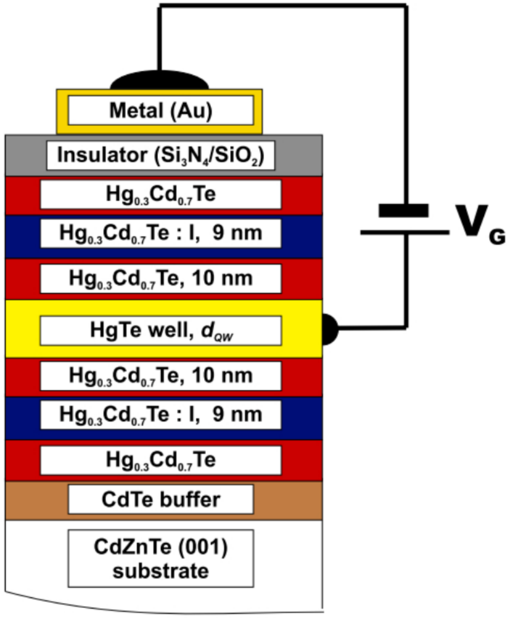
Quantum Spin Hall Effect in HgTe/HgCdTe





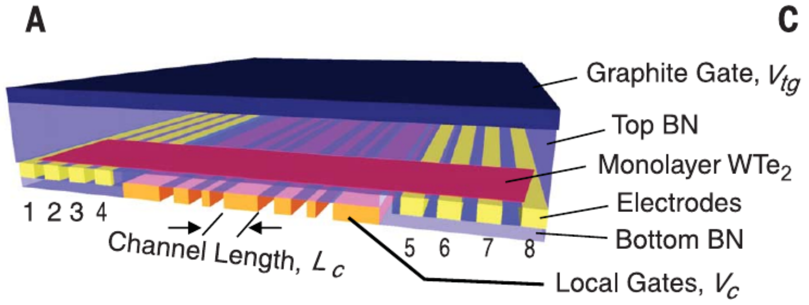


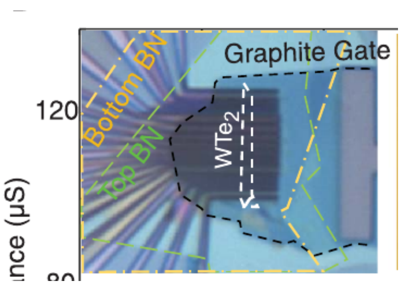


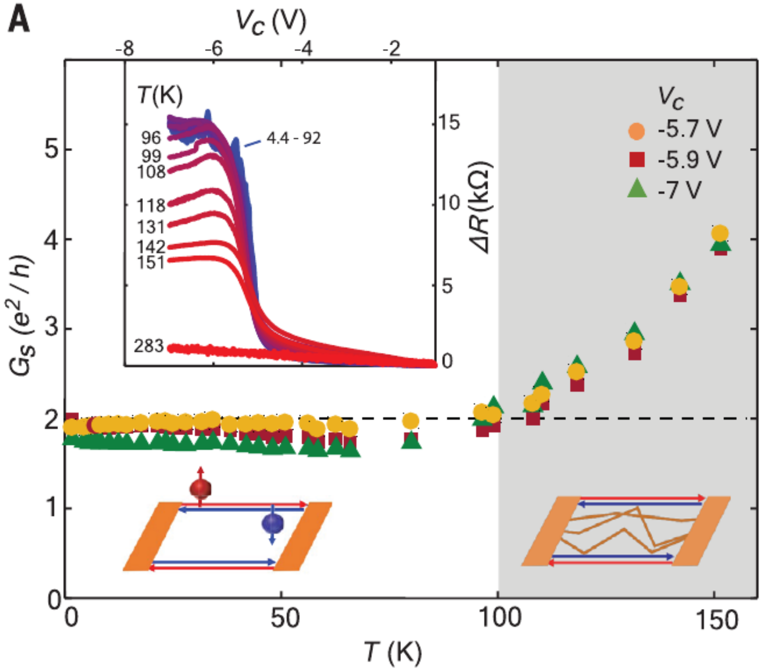


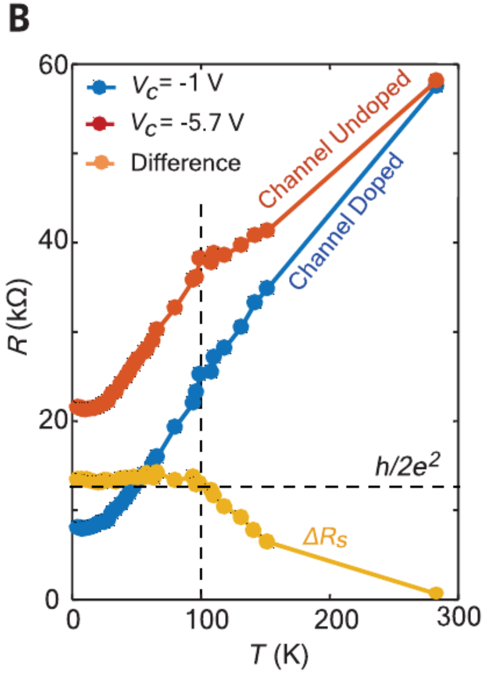
König *et al.* Science **318,** 766 (2007)

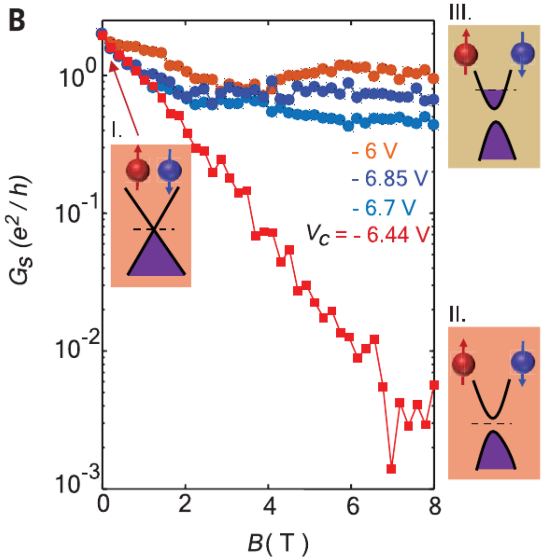
Quantum Spin Hall Effect in WTe2







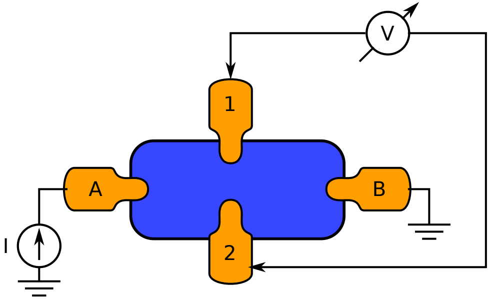




Wu et al., Science **359**, 76 (2018)

The blue sample is an insulator with Chern number Q.
We drive a current I from contact A to B.
The measured Hall voltage between contacts 1 and 2 is V.

What is the Hall conductance I/V ?



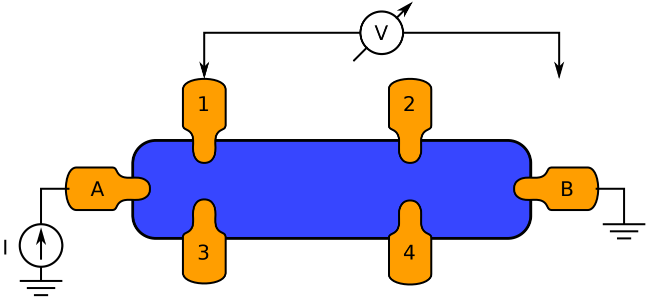
a) 0, since the setup is symmetric

b) e^2/h

c) Qe^2/h

d) none of the above

The blue sample is a Chern insulator.
Which contact is on the same potential as contact 1?



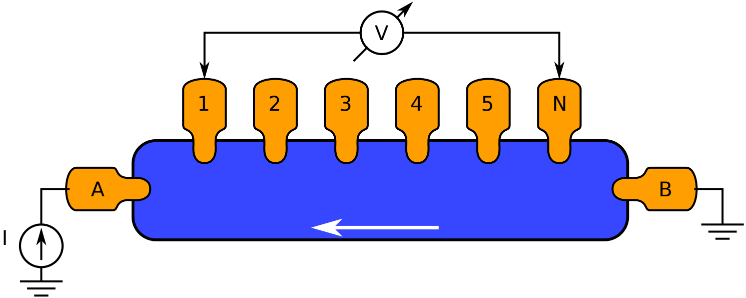
a) 2

b) 3

c) 4

d) none of the above

An edge state on the Chern insulator is marked by an arrow.
As the number N of floating contacts increases, the voltage V...



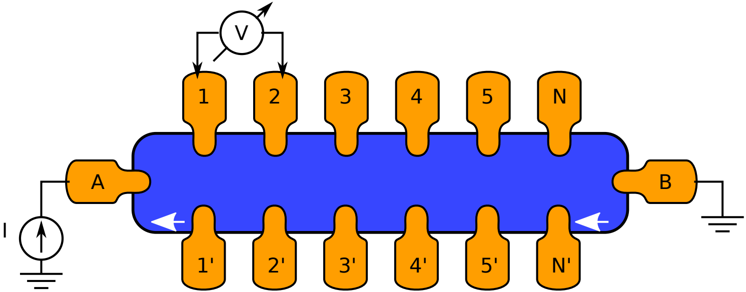
a) does not change

b) decreases

c) increases

d) The question is not well defined

An edge state on the Chern insulator is marked by an arrow.
As the number N of floating contacts increases, the voltage V...



a) does not change

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c) increases

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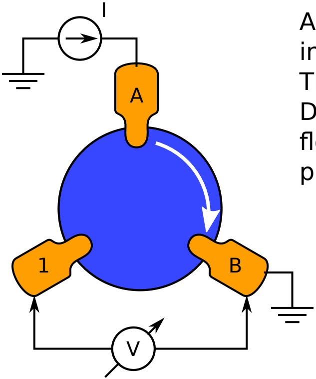


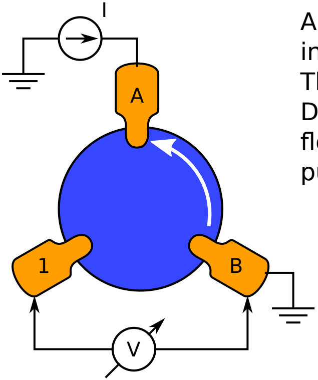
A protected edgestate on a Chern insulator is denoted by the white arrow. The function V(I), i.e., the DC voltage V measured on the floating contact as a function of the pumped DC current I, is ...

a) even

b) odd

c) neither

d) both



A protected edgestate on a Chern insulator is denoted by the white arrow. The function V(I), i.e., the DC voltage V measured on the floating contact as a function of the pumped DC current I, is ...

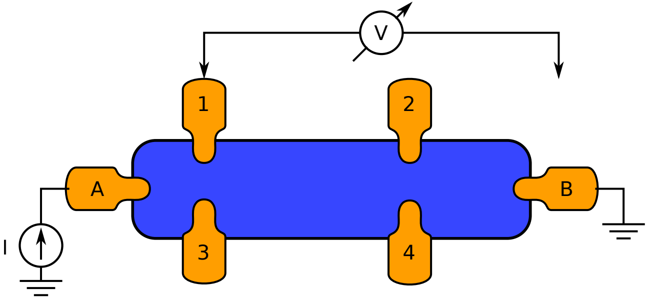
a) even

b) odd

c) neither

d) both

The blue sample is a nontrivial Z_2 insulator.
Which contact is on the same potential as contact 1?

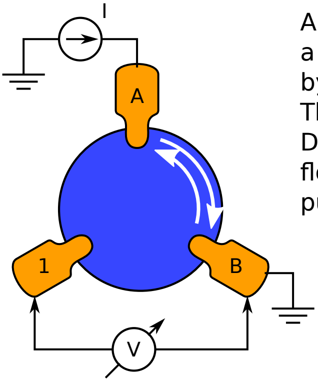


a) 2

b) 3

c) 4

d) none of the above



A pair of protected edge states on a Z2 topological insulator is denoted by the white arrows.
The function V(I), i.e., the DC voltage V measured on the floating contact as a function of the pumped DC current I, is ...

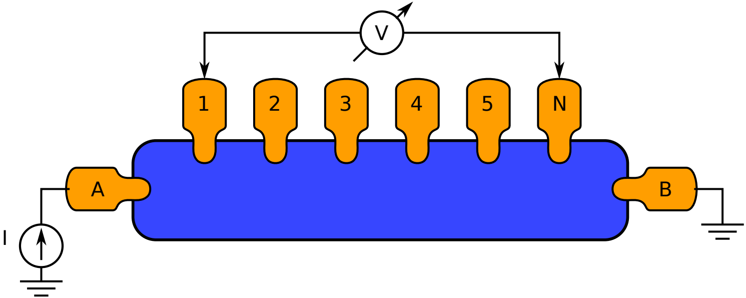
a) even

b) odd

c) neither

d) both

Take a nontrivial Z_2 topological insulator.
As the number N of floating contacts increases, the voltage V...



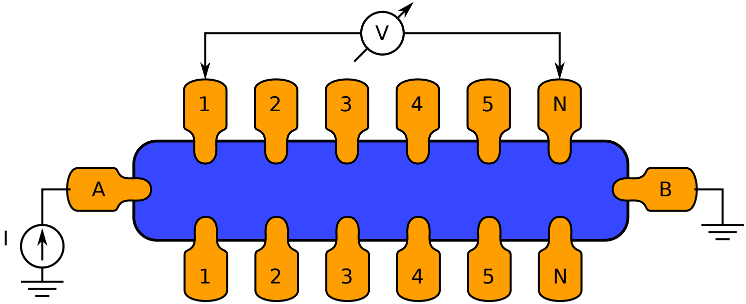
a) does not change

b) decreases

c) increases

d) The question is not well defined

Take a nontrivial Z_2 topological insulator.
As the number N of floating contacts increases, the voltage V...

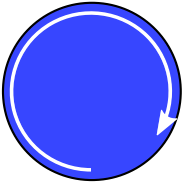


a) does not change

b) decreases

c) increases

d) The question is not well defined



The blue sample is a Chern insulator in thermal equilibrium.
Which statement is true ?

a) The electric current density is zero everywhere in the sample
since there cannot be any current in equilibrium.

b) A nontrivial Chern insulator can be in equilibrium
but then the current density is not zero.

c) Nontrivial Chern insulators cannot be in equilibrium since
edge state currents generate Joule heating.

d) None of the above statements is true.