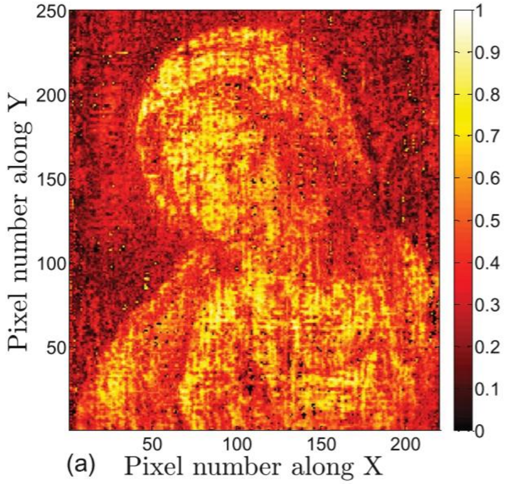
Introduction

Global mapping of stratigraphy of an old-master painting using sparsity-based terahertz refectometry

Dong, J., Locquet, A., Melis, M., & Citrin, D. S. (2017). Global mapping of stratigraphy of an old-master painting using sparsity-based terahertz reflectometry. Scientific reports, 7(1), 15098.

Madonna in Preghiera (17th cent., oil)

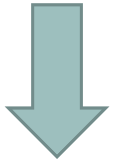


Outline:

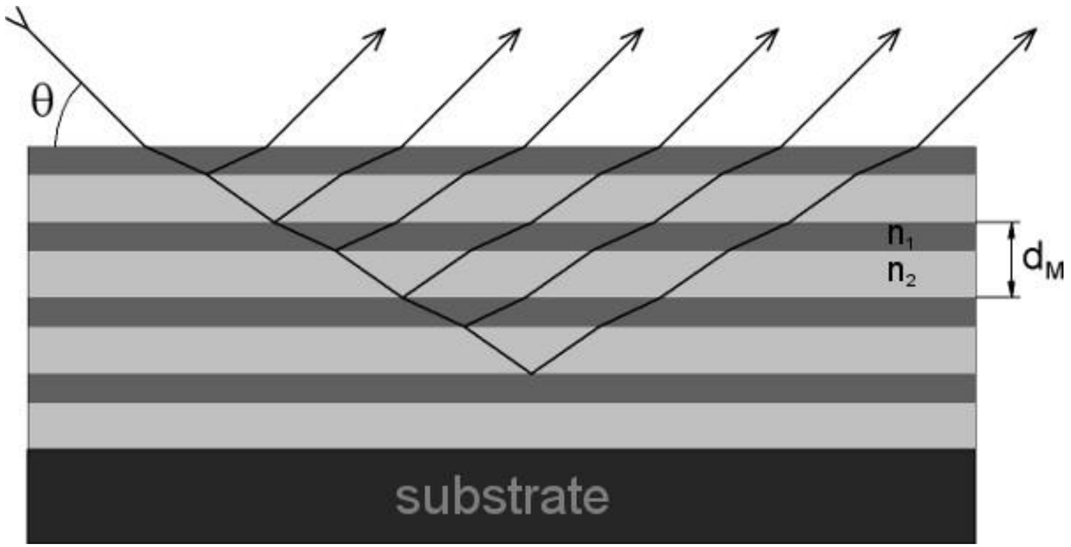
- THz reflectometry and the problem with paintings
- Mathematical solution
- Demonstration and results
- Conlcusions

THz reflectometry

- Useful for *stratigraphy:* maping the layered structure of the sample.
- We expose the sample to a THz pulse (100 GHz – 3 THz).
- The pulse is partially reflected from the interfaces between layers.
- We record the reflected signal, which now has multiple peaks, each with a time-delay and different amplitude.



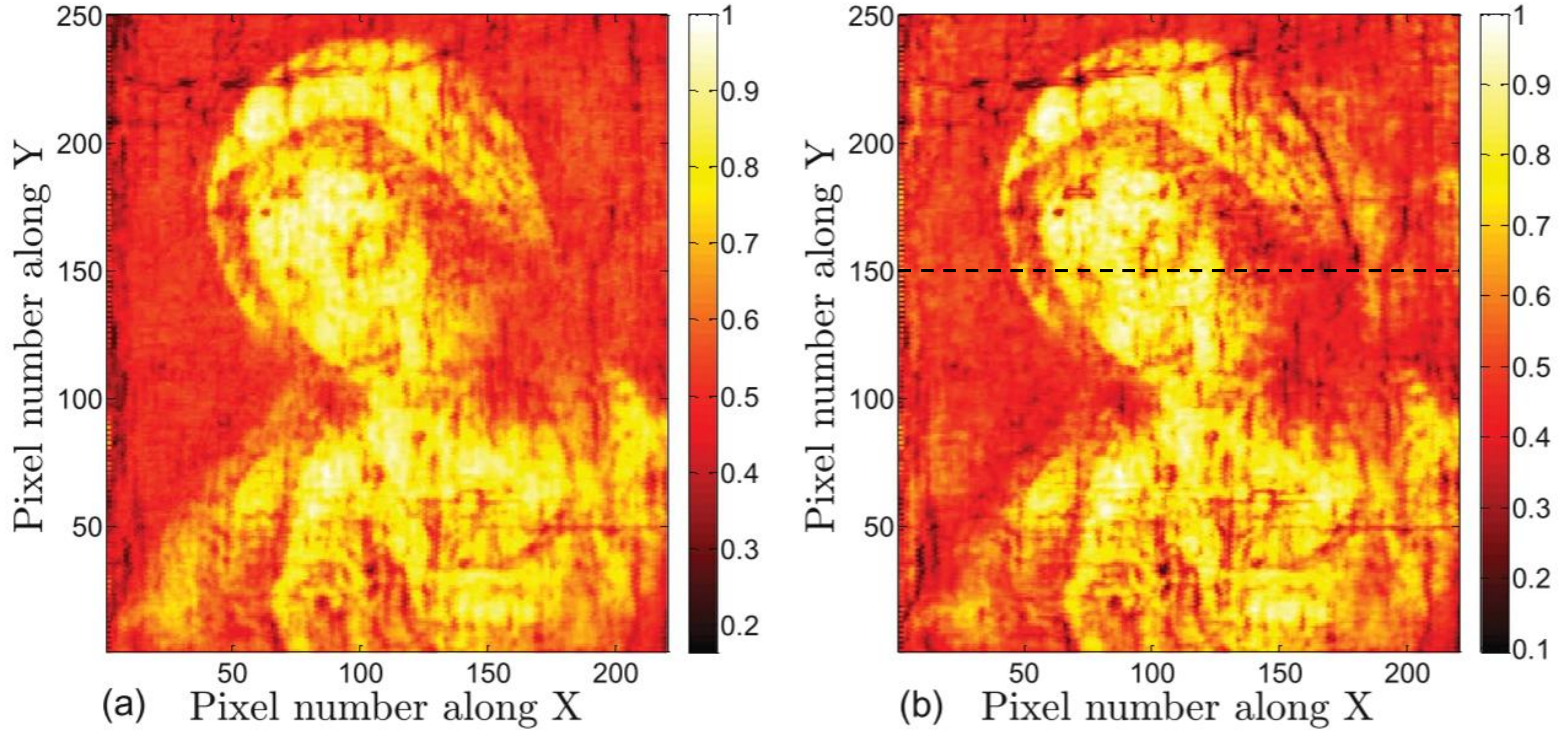
We get the thickness, and reflectiveness of the layers.



Other methods lack the penetration, or similarily the depth resolution.
(e.g. x-ray fuorescence, nuclear magnetic resonance, optical coherence tomography)

THz reflectometry

C-scan: 2D plot of the reflected signal contrasted by a particular measure.

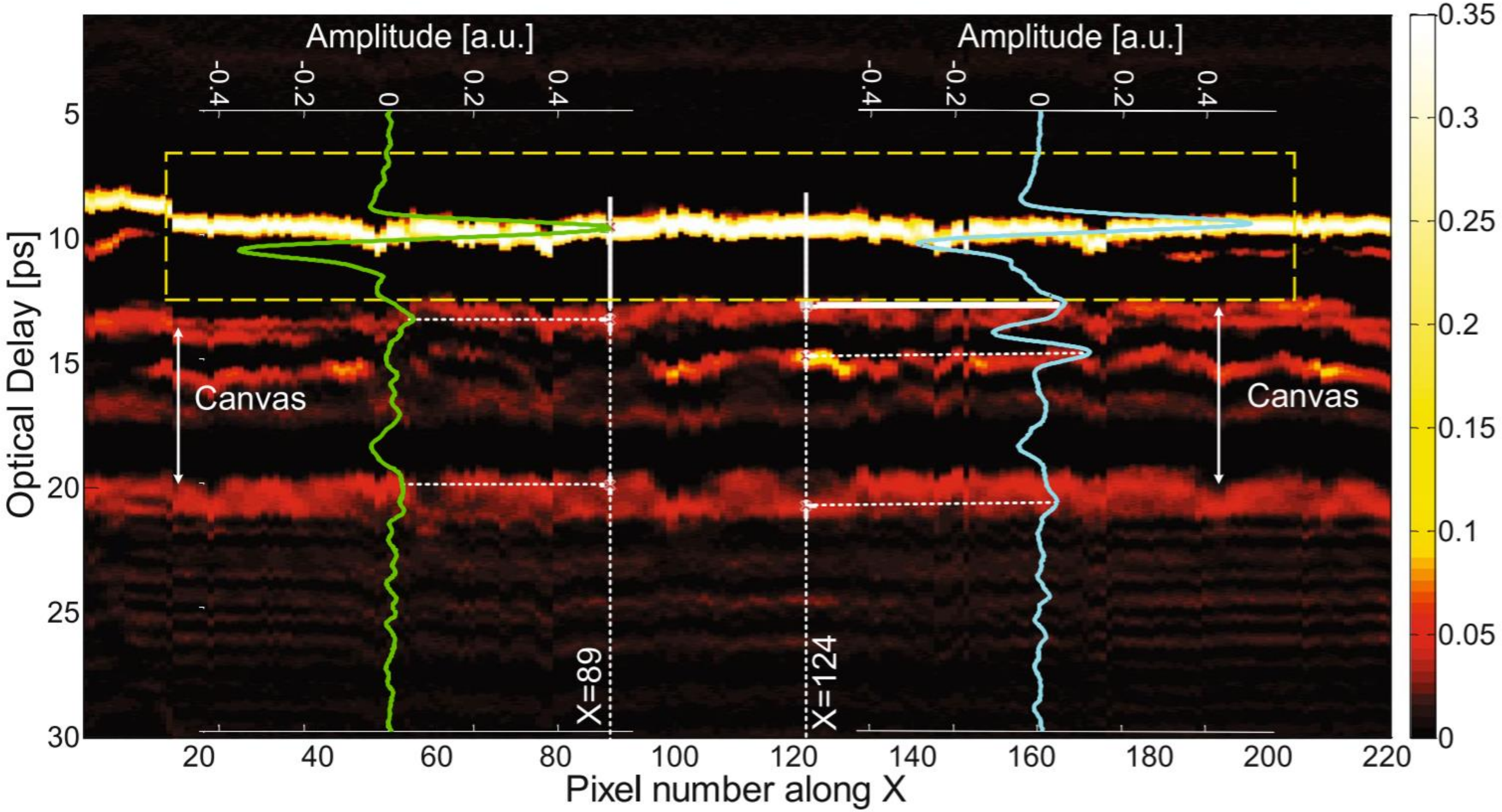


Peak-to-valley amplitude

Integrated spectrum between 0.5 and 1 THz

THz reflectometry

B-scan: reflected signal in time domain along one dimension (here along X, with Y=150).



THz reflectometry

Wavelength of 3 THz signal: $\sim 100\ \mu m$

Thickness of a layer of paint: $\sim 50\ \mu m$

The layers cannot be resolved.

Solution

The reflected signal is the result of a convolution:

reflected signal → $r(t)$; incident signal → $i(t)$; response function → $h(t)$

$$r(t) = i(t) \otimes h(t) = \int_{-\infty}^{+\infty} i(\tau)h(t-\tau)d\tau$$

In practice it is a summation:

$$r_n = \sum_{m=0}^{M-1} i_m h_{n-m} + e_n$$

e_n ← measurement error

Conventional deconvolution:

$$\mathrm{R}(\omega) = I(\omega)H(\omega) \Longrightarrow H(\omega) = \frac{R(\omega)}{I(\omega)}$$

This requires the filtering of high and low frequencies, in order to get a meaningful solution.

Solution

The convolution can be written in the form of the following matrix equation:

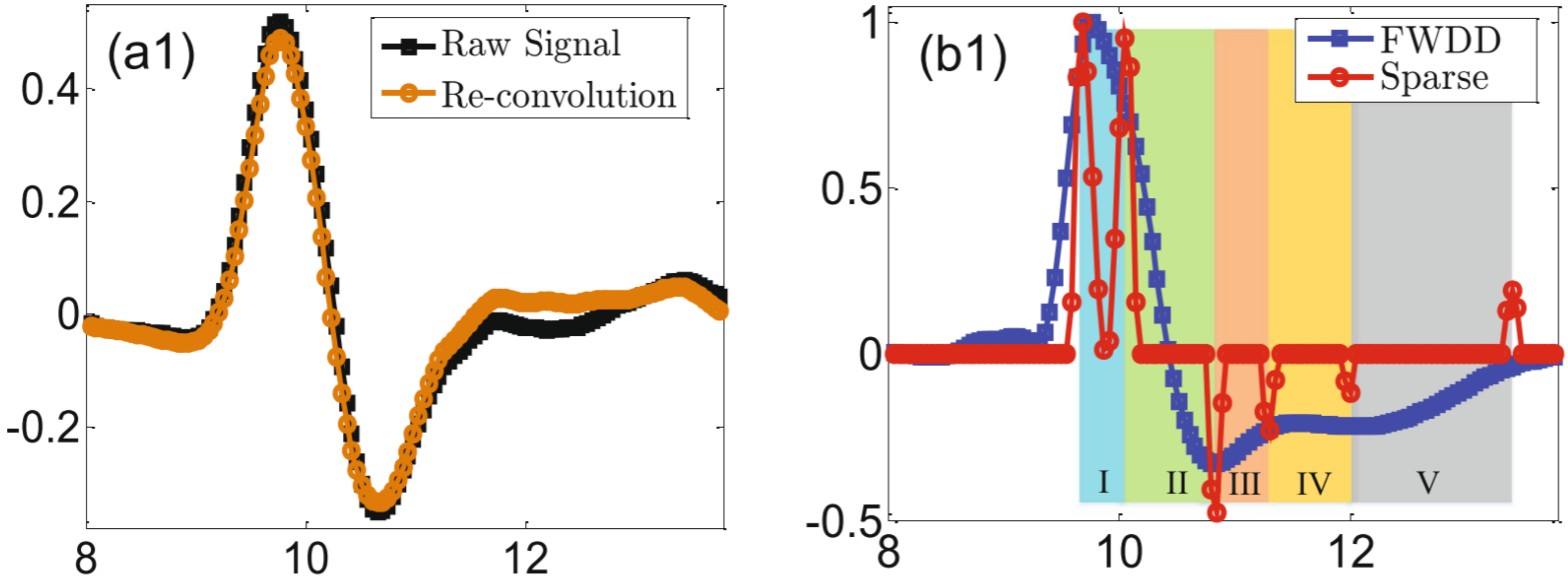
$$r_n = \sum_{m=0}^{M-1} i_m h_{n-m} + e_n \Longrightarrow \mathbf{r} = \mathbf{A}\mathbf{h} + \mathbf{e}$$

We can exploit the fact that $\boldsymbol{h}$ is sparse: Due to the sparsity, $\boldsymbol{h}$ can be found by $\boldsymbol{l_0}$ **regularized optimization**. The l_0 norm is the number of components in $\boldsymbol{h}$ that are not zero. The optimization is carried out by iteration.

$$\min_{\mathbf{h}} \frac{1}{2} \|\mathbf{A}\mathbf{h} - \mathbf{r}\|_2^2 + \lambda \|\mathbf{h}\|_0$$

Results

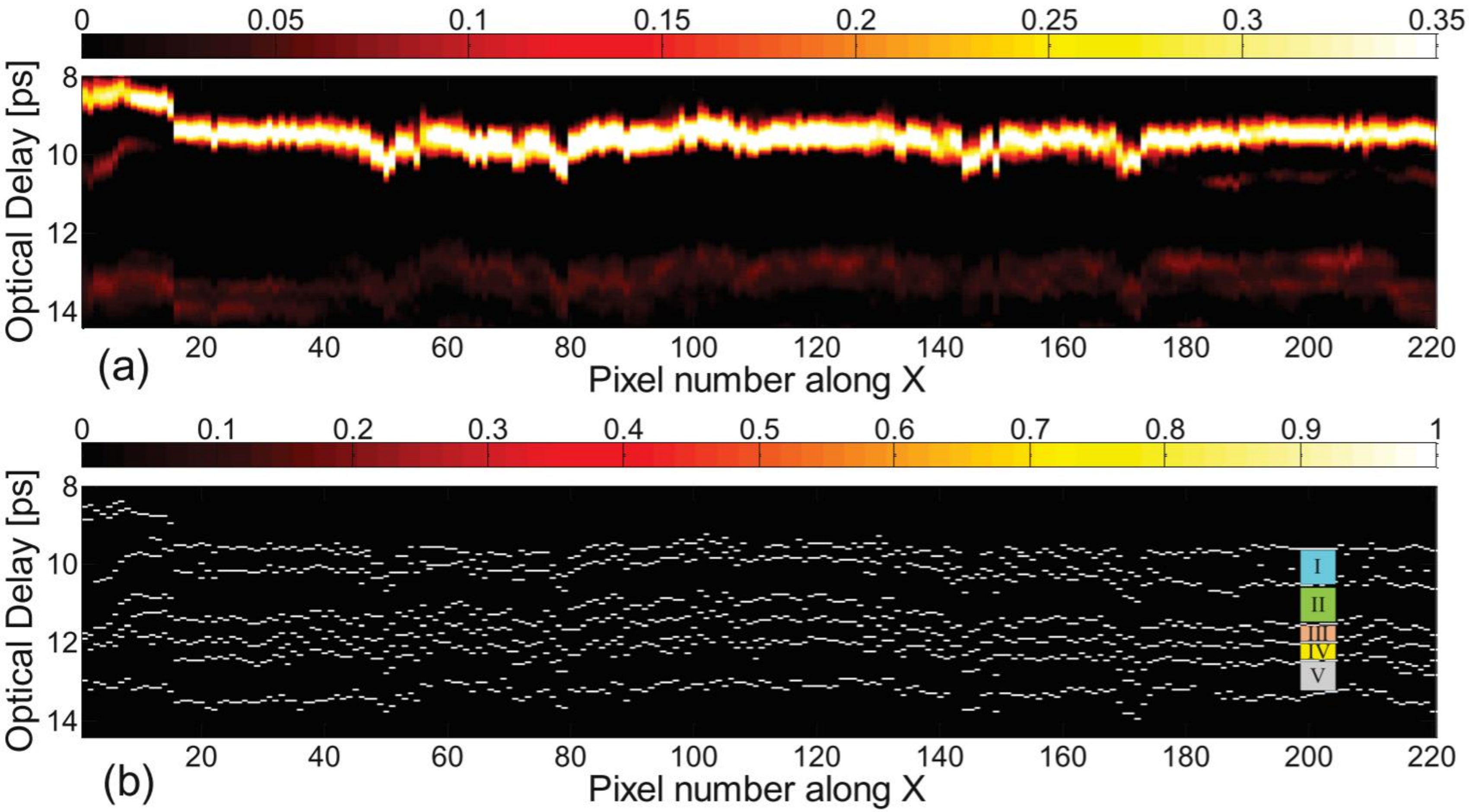
The nonzero elements of h indicate the individual reflections from the various interfaces.

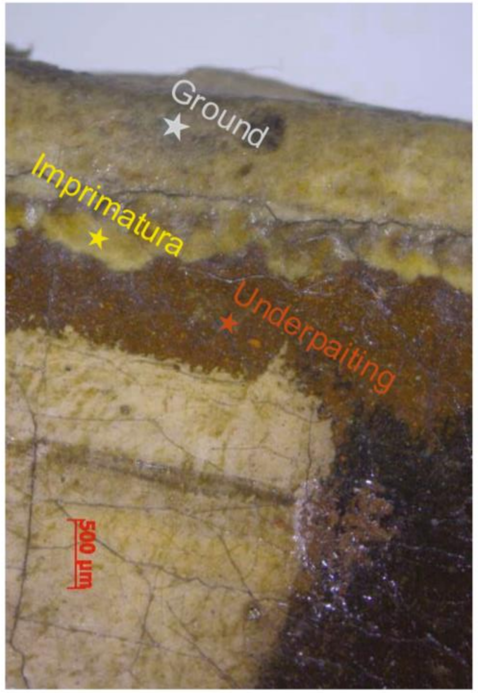


FWDD: Frequency-wavelet domain deconvolution, it filters higher frequencies, but those are what we need.

Results

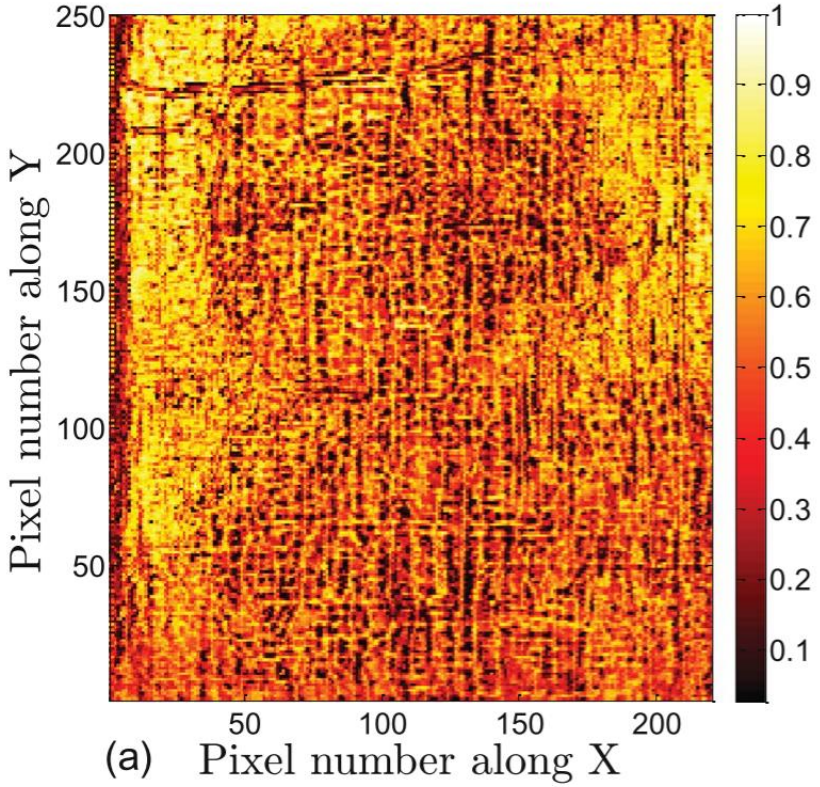
With this method the B-scans can show us the individual layers:



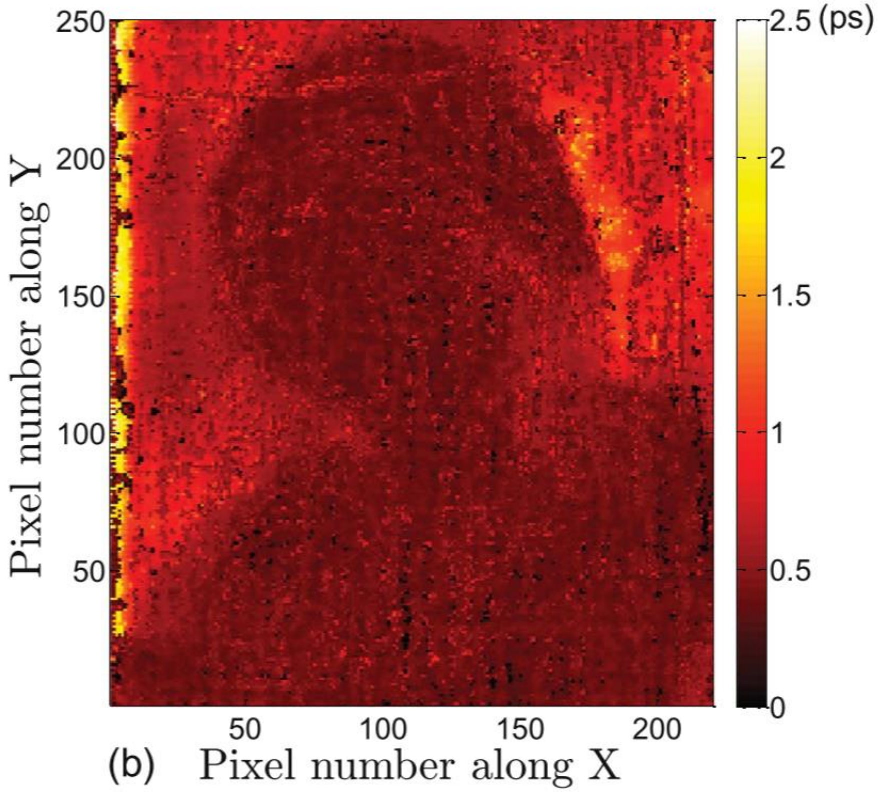


I. **Varnish**
II. **Pictorial**
III. **Underpainting**
IV. **Imprimatura**
V. **Ground**

Varnish

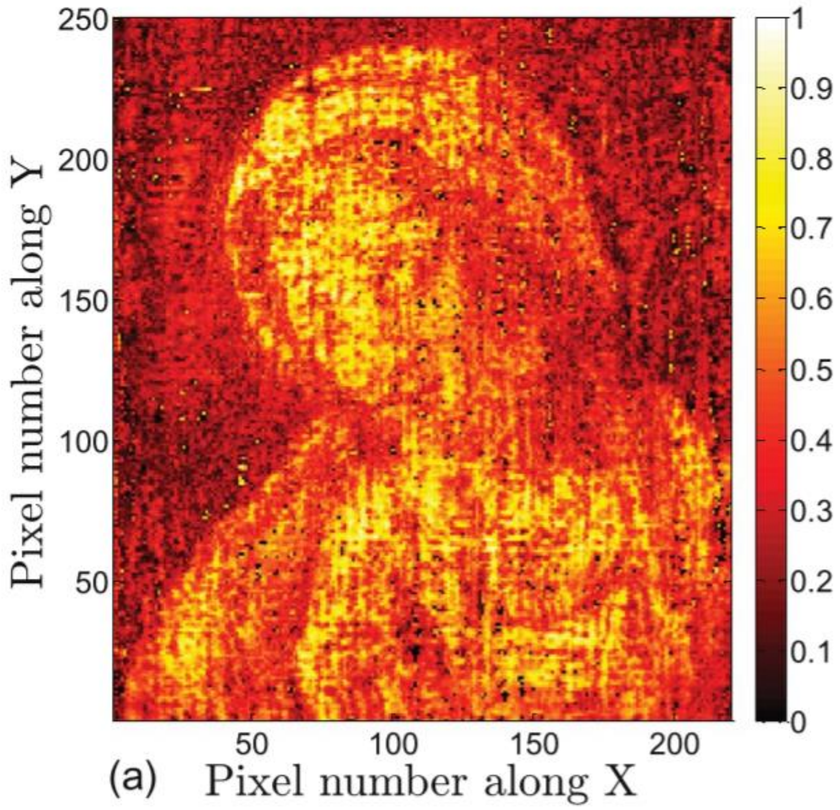


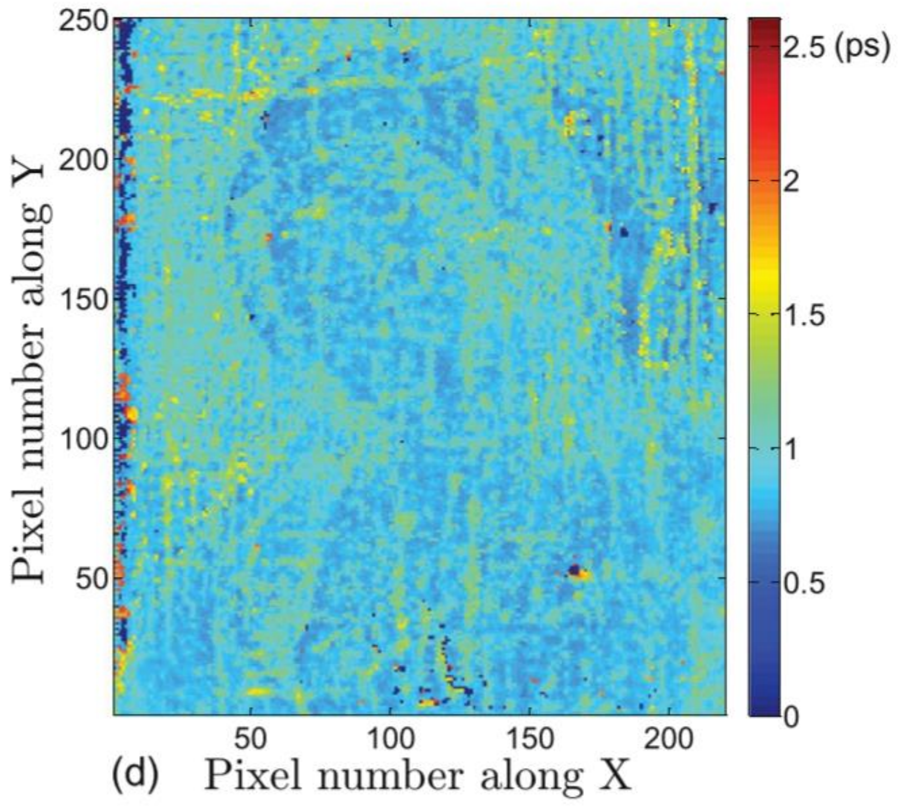
Based on the amplitude of the first peak, the varnish is smoother around the *Madonna's* head.



Thickness of varnish shows signs of retouching.

Pictorial

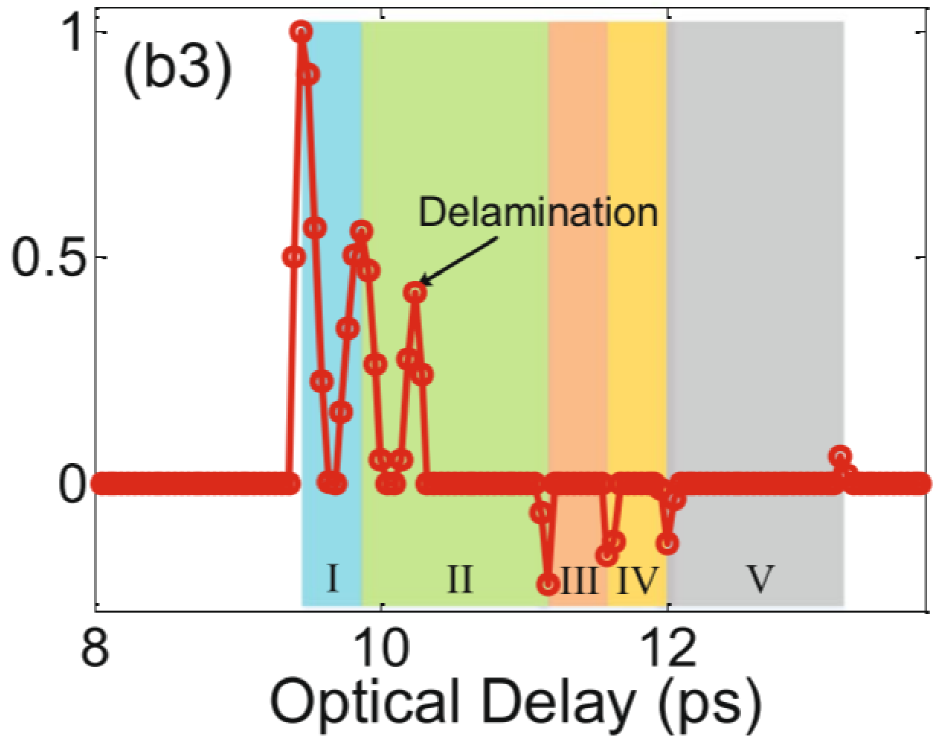




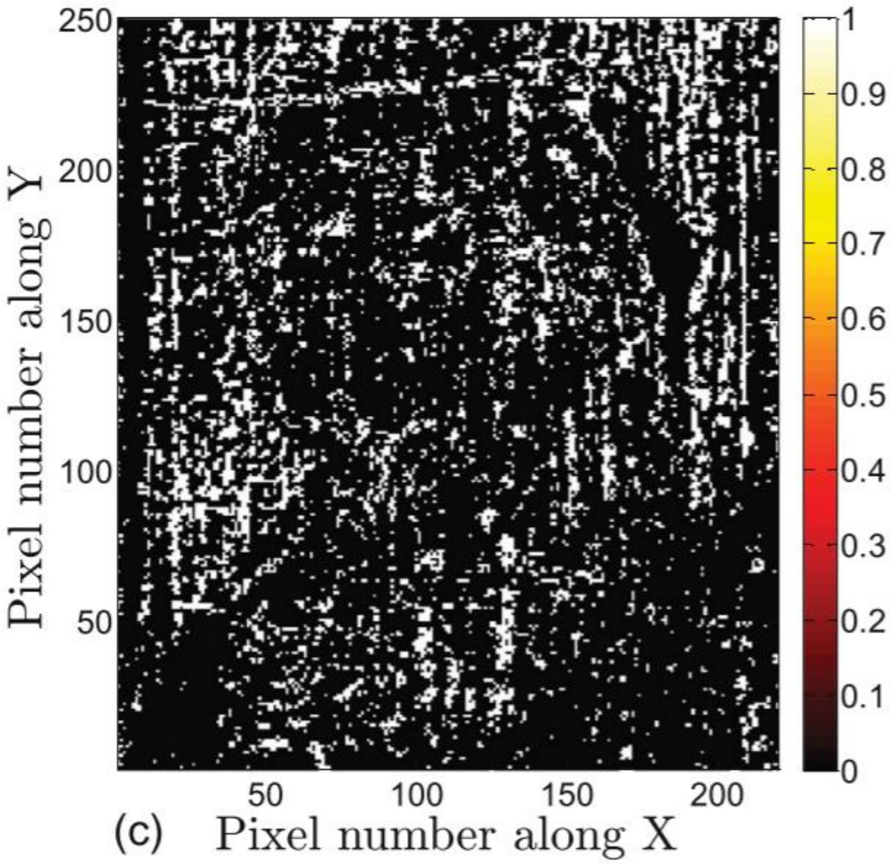
The amplitude of the pictorial layer's spike shows the difference in reflectiveness.

Seemingly, the thickness of the pictorial layer corresponds with craquelure.

Delamination



The reason is delamination, which occurs more often at cracks.



Binary C-scan showing the locations of delamination.

Conclusions

- **Successful reconstruction of stratigraphy with high depth resolution.**
- **Both quantitative (thickness) and qualitative (reflectancy) information extracted.**
- **Useful method for authentication, restoration and study of a broad range of cultural heritage objects.**